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# Composing Complete and Partial Knowledge

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**Abstract**

We study theory revision and composition in the context of OLP-FOL logic. In this logic, the representation of knowledge is split into two parts: writing definitions for completely known concepts (OLP part), and writing constraints and expressing partial knowledge on other concepts (FOL part).

In a previous work, the composition of two OLP-FOL theories with nonintersecting sets of defined predicate symbols was studied. It was argued that the composition is given by the set of common models.

Here, we consider the case of two OLP-FOL theories, defining the same predicate $p$ and introducing a form of theory revision that makes possible their composition in the sense previously proposed. We introduce two operators for theory revision: the $p$-opening operator and the conditional $p$-opening operator. After applying one of these operators to both theories, the knowledge they represent about $p$ is not complete anymore, and the theories can (under certain conditions) be composed.

## 1 Introduction

The *representation of knowledge* in the logic OLP-FOL is split in two parts: writing definitions for known concepts, and writing constraints and expressing partial knowledge on other concepts. To clarify this, consider a situation in which an expert wants to represent knowledge on a subdomain of the world in an OLP-FOL theory $\mathcal{T}$. Some concepts are completely known by the expert, whereas on other concepts, the expert has only partial knowledge or no knowledge at all. This is reflected in the theory $\mathcal{T}$, which consists of two parts: $\mathcal{T} = (\mathcal{T}_d, \mathcal{T}_c)$. The *definition part*, $\mathcal{T}_d$, is a normal open logic program (OLP), representing the definitions for the known predicates. The *FOL part*, $\mathcal{T}_c$, is a set of first-order logic (FOL) axioms, representing partial knowledge on the other predicates. So the theory $\mathcal{T}$ divides the set of predicates in two disjoint subsets: the *defined* predicates, which occur in the head of a clause of $\mathcal{T}_d$, and the *open* predicates, which occur at the most in the body of the clauses of $\mathcal{T}_d$.

The semantics of OLP-FOL, which is a generalisation of the well-founded semantics [VRS91], is a *possible state semantics*. This means that a model of a theory represents a state in which the problem domain might occur according to the (incomplete) expert knowledge. A theory with several models expresses the fact that the expert who wrote the theory has no complete knowledge about the problem area. A theory in which all the predicates are defined can at most have one model (namely, the well-founded model [VRS91]).

In a previous work ([VDDar], an extended version of [VDD97]), the *composition* of two OLP-FOL theories $\mathcal{T}_1$ and $\mathcal{T}_2$ was investigated. It was argued that the composition of two such theories should contain exactly the sum of the knowledge of the components. By continuing our intuitive interpretation, this means that we have two experts whose knowledge is reflected in the two theories, and we want to represent the situation in which these two experts put together their knowledge. Formally, this means that the set of models of the composition $\mathcal{T}$ of the theories $\mathcal{T}_1$ and $\mathcal{T}_2$ is exactly equal to the intersection of the sets of models of $\mathcal{T}_1$ and $\mathcal{T}_2$: $\mathcal{M}od(\mathcal{T}) = \mathcal{M}od(\mathcal{T}_1) \cap \mathcal{M}od(\mathcal{T}_2)$. In [VDDar], two theories with nonintersecting sets of defined predicate symbols are considered and several conditions on $\mathcal{T}_1$ and $\mathcal{T}_2$ are given, such that their composition is simply the union, that is, such that $\mathcal{M}od(\mathcal{T}_1) \cap \mathcal{M}od(\mathcal{T}_2) = \mathcal{M}od(\mathcal{T}_1 \cup \mathcal{T}_2)$. Note that in [VDDar], the composition of $\mathcal{T}_1$ and $\mathcal{T}_2$ is taken only if $\mathcal{T}_1$ and $\mathcal{T}_2$ define disjoint sets of predicate symbols. The reason for this is that a definition for a predicate expresses the fact that the expert has complete knowledge about this predicate. So the fact that both theories define the same predicate means that both experts are convinced to have complete knowledge on this predicate. They do not put their knowledge together unless they first revise their beliefs.

In this paper, we consider such a situation. More specifically, we consider the case of two experts who both believe to have complete knowledge about a predicate $p$ but, coming together, realise that their knowledge about $p$ is not complete and thus first revise their beliefs and then sum their expertise.

On theories, this means that we consider two theories, $\mathcal{T}_1$ and $\mathcal{T}_2$, which both give a definition for the same predicate symbol $p$, and we study how to revise and compose them so that the set of models of the composition is exactly equal to the intersection of the sets of models of the revised theories.

We introduce two operators for *theory revision*, and we show that, after one of these operators is applied to both theories, the theories represent the situation in which there is no complete knowledge about $p$ anymore, and under certain conditions, they can be composed. The first operator for theory revision we introduce is the *$p$-opening operator*: The $p$-opening operator $\Theta_p$, applied on a theory that has a definition for the predicate $p$, makes the definition of $p$ indirectly open, by letting $p$ depend on a new open predicate. We prove that applying the $p$-opening operator on a theory is the same as reading the definition of $p$ as a set of FOL axioms, that is, putting the definition of $p$ in the FOL part of the theory. The second operator for theory revision is the *conditional $p$-opening operator*: The conditional $p$-opening operator $\Theta_p^{cond,\tau}$, applied on a theory that has a definition for the predicate $p$, splits the definition of $p$ into two parts: one part remains closed and can be used only if a certain condition holds; the other part becomes open and can be used only if the negation of the given condition holds.

We show that we can *compose* two theories that both have an *open definition* for the same predicate $p$, or that both have a *conditional open definition* for $p$, if the conditions do not overlap.

In Section 2 we first discuss the logic OLP-FOL and knowledge representation in OLP-FOL (Subsection 2.1). In Subsection 2.2, we introduce some more notation and assumptions, and in Subsection 2.3, we recall the compositionality criterion of [VDDar]. The two operators for theory revision are introduced in Section 3. In Subsection 3.1, the opening operator for theory revision is introduced. We give some properties and, as a result, we show in Subsection 3.2 that, after applying the $p$-opening operator on two theories that both define the predicate $p$, their knowledge about $p$ is not complete anymore, and the theories can be composed. Next, in Subsection 3.3, we introduce the conditional opening operator, and, in Subsection 3.4, we give the conditions under which two theories can be composed after applying the conditional $p$-opening operator. In Section 4, a simple example illustrates the use of the opening and conditional opening operators. We end with a conclusion in Section 5, where we give some related works. The proofs of the propositions and theorems can be found in [VB98].

# 2 Preliminaries

## 2.1 The logic OLP-FOL

We assume familiarity with the basic concepts of logic programming [Llo87] and with well-founded semantics [VRS91].

The truth function of a Herbrand interpretation $I$ (i.e., the function that maps ground atoms to $\{\mathbf{f}, \mathbf{u}, \mathbf{t}\}$) is denoted by $\mathcal{H}_I$. The truth function $\mathcal{H}_I$ of a Herbrand interpretation $I$ is described as a set of tuples of ground atoms together with their truth value under $\mathcal{H}_I$ (e.g., $\{p^{\mathbf{f}}, q^{\mathbf{u}}, r^{\mathbf{t}}\}$, meaning that $\mathcal{H}_I(p) = \mathbf{f}, \mathcal{H}_I(q) = \mathbf{u}, \mathcal{H}_I(r) = \mathbf{t}$).

We work in the logic OLP-FOL [DD93, Den95], which is an expressive logic to represent uncertainty on definitions of concepts and on the problem domain. A *theory* $\mathcal{T}$ in this logic is a pair $(\mathcal{T}_d, \mathcal{T}_c)$ of a first-order logic (FOL) theory $\mathcal{T}_c$, the *FOL part* of $\mathcal{T}$, and a normal open logic program (OLP) $\mathcal{T}_d$, the *definition part* of the theory. A normal open logic program is a set of normal program clauses $p(t) \leftarrow q_1(t_1), \ldots, q_n(t_n), \neg r_1(s_1), \ldots, \neg r_m(s_m)$ ($t, t_i, s_j$ are tuples of terms). Atomic rules are denoted $p(t) \leftarrow$. Predicates occurring in the head of a clause of $\mathcal{T}_d$ are called *defined*. With $\mathcal{D}ef(\mathcal{T})$, we denote the set of defined predicate symbols of the theory $\mathcal{T}$. If $\mathcal{D}ef(\mathcal{T}) = \{q_1, \ldots, q_k\}$, then we also write the definition part of $\mathcal{T}$ as the following disjoint union: $\mathcal{T}_d = \mathcal{T}_d^{q_1} \cup \ldots \cup \mathcal{T}_d^{q_k}$, where $\mathcal{T}_d^{q_i}$ is the set of clauses of $\mathcal{T}_d$ defining $q_i$, that is, the *definition* of $q_i$. The predicates that are not defined, and hence occur at the most in the body of program clauses, are called *open*. Intuitively, they represent concepts for which no definitions are given. Partial knowledge about these predicates can be expressed in the set of FOL axioms $\mathcal{T}_c$. A definition for a predicate in a theory expresses the fact that this predicate is completely known by (the expert who wrote) the theory. But, it is of course possible that the definition of a predicate depends on one or more open predicates, and hence the expert has only complete knowledge about that predicate with respect to the interpretation of the open predicates. This becomes more clear below, where we give the definition of the model semantics of OLP-FOL. For simplicity, we restrict ourselves here to Herbrand interpretations only. In the general definition of the model semantics of OLP-FOL [DD93], also non-Herbrand interpretations are considered, hence taking into account uncertainty on the domain of discourse. We want to remark that everything in this paper carries over in a trivial way to the case of general interpretations.

**Definition 1** *A* model *of a normal logic program $\mathcal{T}_d$ is a Herbrand interpretation $M$, such that $M$ is the well-founded model [VRS91] of the grounding of $\mathcal{T}_d$ augmented with some ground unit clauses of open predicates representing the interpretation of the open predicates in $M$ (i.e., $M$ is the extended well-founded model of $\mathcal{T}_d$ as defined in [PAA91]).*

*A* model *of a theory $(\mathcal{T}_d, \mathcal{T}_c)$ is a Herbrand interpretation $M$, such that $M$ is a model of $\mathcal{T}_d$, as defined above, and $M$ satisfies the set of FOL axioms $\mathcal{T}_c$ (in the classical FOL sense). A formula is a* consequence *of a theory $\mathcal{T}$ if and only if it is true in every model of $\mathcal{T}$.*

Note that a model is necessarily 2-valued on the open predicates. With $\mathcal{M}od(\mathcal{T})$, we denote the set of models of a theory $\mathcal{T}$.

With a *partial* Herbrand interpretation, we mean a Herbrand interpretation $I$ of which the truth function is defined only on some part of the predicate symbols. We say that a Herbrand interpretation $M$ *extends* $I$ if $\mathcal{H}_M$ coincides with $\mathcal{H}_I$ on the predicates to which $I$ gives truth values. As a consequence of Definition 1, we have that, for each 2-valued partial Herbrand interpretation $I$ on the open predicates of a normal logic program $\mathcal{T}_d$, there exists a unique model $M$ of $\mathcal{T}_d$ extending $I$. Note that, if all predicate symbols of $\mathcal{T}$, except $=$, are defined, $M$ is the unique well-founded model of $\mathcal{T}_d$. Possibly $M$ is a model of the full theory $\mathcal{T} = (\mathcal{T}_d, \mathcal{T}_c)$, namely, if $M$ satisfies the FOL axioms $\mathcal{T}_c$.

OLP-FOL has a *possible state semantics*; that is, a model corresponds to a state in which the problem domain might occur according to the (incomplete) expert knowledge. At the level of the semantics, uncertainty on the definition of a concept is modeled by allowing models that give to the open predicates an arbitrary interpretation satisfying the set of FOL axioms $\mathcal{T}_c$ (and not, e.g., by having the third truth value **u** for these open predicates as in a belief set semantics). The occurrence of an undefined ground atom (i.e., an atom with truth value **u**) in a model of $\mathcal{T}_d$ reveals an ambiguity or a local inconsistency in the definition. This motivates the following definition.

**Definition 2** *An open normal logic program $\mathcal{T}_d$ is called a* correct definition *(or* correct*) if and only if each model of $\mathcal{T}_d$ is 2-valued. A theory $\mathcal{T} = (\mathcal{T}_d, \mathcal{T}_c)$ is* correct *if and only if $\mathcal{T}_d$ is a correct definition.*

In [DD98], the abductive proof procedure, called SLDNFA, for open normal programs is defined. This proof procedure is applied on an open program and initial query, and generates a set of open (abducible) atoms which imply the initial query. In the case when there is partial knowledge on open predicates, that is, when there is an FOL part, then for each FOL axiom $F$ in this FOL part, the implication $\mathit{false} \leftarrow \neg(F)$ is transformed into clausal form (*false* is a new predicate) and the initial query $\leftarrow Q$ is transformed into the clause $\mathit{false} \leftarrow \neg Q$. The SLDNFA proof procedure is then applied to the program extended with these new clauses and to the query $\leftarrow \neg \mathit{false}$. In this way, the generated set of open atoms satisfies the FOL part.

## 2.2 Some notation and assumptions

We use the notion of *dependency graph* of the predicates in a logic program. For the definition, we refer to [Llo87]. A predicate $q$ is said to *depend on* a predicate $p$ if and only if there is a path from $q$ to $p$ in the dependency graph. A predicate $q$ is said to depend *only oddly* (resp., *only evenly*) on a predicate $p$ if and only if in each path from $q$ to $p$ in the dependency graph, there is an odd (resp., even) number of negative edges.

With $\rho_{r \rightarrow s}$ we denote the *renaming operator*, which replaces each occurrence of the predicate symbol $r$ by the predicate symbol $s$. Renaming operators can be applied to (conjunctions of) literals, program clauses, or theories.

We consider a fixed alphabet $\mathcal{A}$ with a finite number of predicate symbols. For each predicate symbol $q/n$ of $\mathcal{A}$, we introduce a new and unique predicate symbol $q^*/n$. This results in an extension $\mathcal{A}^* = \mathcal{A} \cup \{q^*/n \mid q/n$ is a predicate symbol of $\mathcal{A}\}$ of the alphabet $\mathcal{A}$.

We consider theories whose definition parts are based on the alphabet $\mathcal{A}$. A predicate symbol $p^*$ of $\mathcal{A}^* \setminus \mathcal{A}$ can only occur in the definition part of a theory as a consequence of the application of the $p$-opening operator $\Theta_p$ or the conditional $p$-opening operator $\Theta_p^{cond,\tau}$, which we introduce in Subsections 3.1 and 3.3, respectively.

We assume that the FOL part of each theory contains the axioms $p(X) \Longleftrightarrow p^*(X)$ for each predicate symbol $p$ of $\mathcal{A}$ (and assuming that predicates of $\mathcal{A}^* \setminus \mathcal{A}$ do not appear in another axiom of $\mathcal{T}_c$). Because of this assumption, in each model of a theory, the truth values of the instantiations of $p$ and $p^*$ are the same. Hence, in all the following examples, when writing down a model of a theory, we do not explicitly write down the interpretation of the predicates $p^* \in \mathcal{A}^* \setminus \mathcal{A}$. Note also that adding these axioms imposes the restriction on the models of a theory to be 2-valued. Since we only consider correct theories, and since the definition parts of the theories, where we start from, are based on the alphabet $\mathcal{A}$, these axioms do not add any relevant information.

We want to remark that the axioms of the form $p(X) \Longleftrightarrow p^*(X)$ become relevant once we have applied the $p$-opening or conditional $p$-opening operator on a theory. More precisely, these axioms are added to deal in a clear way with theory composition (see Subsections 3.2 and 3.4). We suppose for the rest of this paper that the FOL parts of all the theories contain these axioms $p(X) \Longleftrightarrow p^*(X)$, without explicitly writing these axioms down.

## 2.3 The compositionality criterion

We recall the compositionality criterion of [VDDar]. In [VDDar], a situation is considered where two (or more) experts have more or less disjunct subdomains of expertise and represent their knowledge independently in two theories, $\mathcal{T}_1$ and $\mathcal{T}_2$, with nonintersecting sets of defined predicate symbols, that is, $\mathcal{D}ef(\mathcal{T}_1) \cap \mathcal{D}ef(\mathcal{T}_2) = \emptyset$. The problem considered in [VDD97] is how to combine these two theories in one united theory $\mathcal{T}$. They argue that the *composition* $\mathcal{T}$ of two theories $\mathcal{T}_1$ and $\mathcal{T}_2$ should contain exactly the sum of the knowledges of the separate theories. Because OLP-FOL has a *possible state semantics*, the compositionality criterion has the following natural formulation: The set of models of $\mathcal{T}$ is precisely the intersection of the sets of models of $\mathcal{T}_1$ and $\mathcal{T}_2$:

$$\mathcal{M}od(\mathcal{T}) = \mathcal{M}od(\mathcal{T}_1) \cap \mathcal{M}od(\mathcal{T}_2).$$

In [VDDar, VDD97], several conditions on $\mathcal{T}_1$ and $\mathcal{T}_2$ are given such that their composition can be obtained by simply taking the union, $\mathcal{T}_1 \cup \mathcal{T}_2 = (\mathcal{T}_{1d} \cup \mathcal{T}_{2d}, \mathcal{T}_{1c} \cup \mathcal{T}_{2c})$, that is, such that $\mathcal{M}od(\mathcal{T}_1 \cup \mathcal{T}_2) = \mathcal{M}od(\mathcal{T}_1) \cap \mathcal{M}od(\mathcal{T}_2)$. We give a slightly modified version of [VDD97, Theorem 5.2] (taking into account the assumptions we made in Subsection 2.2).

**Theorem 3** *We are given two theories $\mathcal{T}_1$ and $\mathcal{T}_2$, such that $\mathcal{D}ef(\mathcal{T}_1) \cap \mathcal{D}ef(\mathcal{T}_2) = \emptyset$. Let $D$ be the dependency graph of the defined predicates of $\mathcal{T}_1 \cup \mathcal{T}_2$. With $C$ we denote a cycle in $D$.*

*If $\forall C$, $\exists i \in \{1, 2\}$, such that all the predicates in $C$ are defined in $\mathcal{T}_i$, then*

$$\mathcal{M}od(\mathcal{T}_1) \cap \mathcal{M}od(\mathcal{T}_2) = \mathcal{M}od(\mathcal{T}_1 \cup \mathcal{T}_2).$$

Note that the restriction $\mathcal{D}ef(\mathcal{T}_1) \cap \mathcal{D}ef(\mathcal{T}_2) = \emptyset$ is imposed on the theories in order to compose them. This is because a definition for a predicate $p$ in a theory expresses the fact that the expert who wrote the theory has complete knowledge about $p$. If one were not sure about $p$, one would not have written a definition for it. Consider, for instance, the following two programs: $\mathcal{T}_1 = \{p(a) \leftarrow\}$ and $\mathcal{T}_2 = \{p(b) \leftarrow\}$. Then, $\mathcal{M}od(\mathcal{T}_1) = \{\{p(a)^{\mathbf{t}}, p(b)^{\mathbf{f}}\}\}$ and $\mathcal{M}od(\mathcal{T}_2) = \{\{p(a)^{\mathbf{f}}, p(b)^{\mathbf{t}}\}\}$. There is no natural way to compose these two definitions of $p$, since one expert says that only $p(a)$ is true and no more, and the other says that only $p(b)$ is true and no more. This is because the definitions determine completely the interpretation of $p$.

In this paper, we show how we can relax this requirement and compose theories that define the same predicate, provided they are first revised by means of the operators for theory revision introduced in the next section.

# 3 Open definitions

Consider the following scenario. There is an expert who initially is certain to have complete knowledge about a predicate $p$ and writes a theory in which $p$ is defined. Later, the expert realises that this knowledge about $p$ is not complete and that the theory needs to be revised. We offer two methods of performing such a revision. The first form of revision can be used when the expert wants to preserve his or her positive knowledge, while the second one allows the expert also to partially retract his or her previous beliefs. The following two simple examples illustrate the two situations.

**Example 4** *Let $\mathcal{T}$ be the following theory:*

$$\left\{ \begin{array}{l} bird(tweety) \leftarrow \\ fly(X) \leftarrow bird(X), \neg abn(X) \\ abn(X) \leftarrow \neg has\_wings(X) \end{array} \right.$$

*with*

$$\begin{array}{l} \mathcal{M}od(\mathcal{T}) = \\ \quad \{M_1 = \{has\_wings(tweety)^{\mathbf{f}}, abn(tweety)^{\mathbf{t}}, bird(tweety)^{\mathbf{t}}, fly(tweety)^{\mathbf{f}}\}, \\ \quad\ M_2 = \{has\_wings(tweety)^{\mathbf{t}}, abn(tweety)^{\mathbf{f}}, bird(tweety)^{\mathbf{t}}, fly(tweety)^{\mathbf{t}}\}\}. \end{array}$$

The expert who defined the theory of Example 4 was convinced to have complete knowledge on the predicate $abn$. As a consequence, in $\mathcal{T}$ there is no way to be abnormal except that of not having wings. This is reflected in the fact that, in the two possible models of $\mathcal{T}$, either *tweety* is not abnormal or it does not have wings. The expert, when realising that the knowledge on *abn* is incomplete, does not want to discharge the implication $abn(X) \leftarrow \neg has\_wings(X)$. The expert just needs to enlarge the set of possible models by opening the predicate *abn*.

To illustrate the other form of revision, we give a very simple example in genetics.

**Example 5** *There is a disease D, which is under research by a number of experts. One researcher studies male persons with disease D and discovers that a man has disease D if one of his parents has disease D. The researcher writes down this knowledge about disease D in the theory*

$$\mathcal{T}_1 = \{ disD(X) \leftarrow parent(Y, X), disD(Y) \} .$$

Later, the expert of Example 5 realises that in the female population disease D has a completely different behaviour: it is not inherited by women. The studies are still valid, but only in the context of the male population. The knowledge is not only incomplete, but also must be revised by inserting the beliefs into the right context.

## 3.1 The opening operator

Let us consider again Example 4. The expert, who realises the knowledge on abnormal birds is incomplete, has a simple way of revising theory $\mathcal{T}$. The expert can just transfer in the FOL part the definition of the predicate *abn*.

**Example 6** *Recall theory $\mathcal{T}$ of Example 4. Consider the theory obtained from $\mathcal{T}$ by putting the definition of abn/1 in the FOL part,* $\mathcal{T}_r = (\mathcal{T}_d \setminus \mathcal{T}_d^{abn}, \mathcal{T}_c \cup \mathcal{T}_d^{abn})$*:*

$$\begin{cases} bird(tweety) \leftarrow \\ fly(X) \leftarrow bird(X), \neg abn(X) \\ FOL: \\ abn(X) \leftarrow \neg has\_wings(X). \end{cases}$$

*It is easy to see that both models $M_1$ and $M_2$ of Example 4 are also models of $\mathcal{T}_r$, but there is one more model, $M_3$, in which tweety is abnormal although it has wings:* $M_3 = \{has\_wings(tweety)^{\mathbf{t}}, abn(tweety)^{\mathbf{t}}, bird(tweety)^{\mathbf{t}}, fly(tweety)^{\mathbf{f}}\}$.

In this paper we propose a semantically equivalent (see next Theorem 9), but syntactically different, operation for opening a predicate in a theory. The motivation underlying the next definition is made clear after Theorem 9.

The application of the $p$-opening operator $\Theta_p$ on a theory $\mathcal{T}$, which defines the predicate $p$, results in the theory $\Theta_p(\mathcal{T})$, which has an open definition for $p$.

**Definition 7** *We are given a theory $\mathcal{T} = (\mathcal{T}_d, \mathcal{T}_c)$ in which the predicate $p/n$ is defined. Applying the* $p$-opening operator $\Theta_p$ *to $\mathcal{T}$ results in the theory $\Theta_p(\mathcal{T})$:*

$$\mathcal{T} = \begin{cases} \mathcal{T}_d \setminus \mathcal{T}_d^p \\ p(t_1) \leftarrow D_1 \\ \dots \\ p(t_r) \leftarrow D_r \\ FOL: \\ \begin{cases} \mathcal{T}_c \\ p^*(X) \iff p(X) \end{cases} \end{cases} \quad \rightarrow \quad \Theta_p(\mathcal{T}) = \begin{cases} \rho_{p\to p^*}(\mathcal{T}_d \setminus \mathcal{T}_d^p) \\ p(t_1) \leftarrow \rho_{p\to p^*}(D_1) \\ \dots \\ p(t_r) \leftarrow \rho_{p\to p^*}(D_r) \\ p(X) \leftarrow p^*(X) \\ FOL: \\ \begin{cases} \mathcal{T}_c \\ p^*(X) \iff p(X) \end{cases} \end{cases}$$

*where $D_i, i = 1, \dots, r$, is a conjunction of literals, $t_i, i = 1, \dots, r$, is an $n$-tuple of terms, and $X = (X_1, \dots, X_n)$. We say that $\Theta_p$ makes the definition of $p$ in $\mathcal{T}$* open. *The definition of $p$ (or briefly $p$) in $\Theta_p(\mathcal{T})$ is called* open.

When we apply the $p$-opening operator on a theory $\mathcal{T}$, we apply the renaming operator $\rho_{p\to p^*}$ to the bodies of all the clauses of $\mathcal{T}_d$, and add the clause $p(X) \leftarrow p^*(X)$ to $\mathcal{T}_d$. So, $p$ is still defined in $\Theta_p(\mathcal{T})$, but its definition depends on the new, open predicate $p^*$.

**Example 8** *Let $\mathcal{T}$ be the theory of Example 4. When we apply $\Theta_{abn}$ to $\mathcal{T}$, we obtain the theory $\mathcal{T}^* = \Theta_{abn}(\mathcal{T})$:*

$$\begin{cases} bird(tweety) \leftarrow \\ fly(X) \leftarrow bird(X), \neg abn^*(X) \\ abn(X) \leftarrow \neg has\_wings(X) \\ abn(X) \leftarrow abn^*(X). \end{cases}$$

*We see that $\mathcal{T}^*$ has the same models as the theory $\mathcal{T}_r$ of Example 6: $\mathcal{M}od(\mathcal{T}^*)$ $= \{M_1, M_2, M_3\}$.*

The next theorem formalises the fact that opening the definition of a predicate $p$ in a theory $\mathcal{T}$ is equivalent to putting the definition of $p$, $\mathcal{T}_d^p$, in the FOL part of the theory and, hence, making the predicate $p$ open. This justifies the name of the operator $\Theta_p$: *p-opening* operator.

**Theorem 9** *Let $\mathcal{T}$ be a theory that defines the predicate $p$. Then*

$$\mathcal{M}od(\Theta_p(\mathcal{T})) = \mathcal{M}od((\mathcal{T}_d \setminus \mathcal{T}_d^p, \mathcal{T}_c \cup \mathcal{T}_d^p)).$$

Hence, there are two syntactically different ways to revise a theory by changing a predicate $p$ from defined to open. The two revisions differ in the presentation. The theory $\Theta_p(\mathcal{T})$ defines exactly the same predicate symbols as the theory $\mathcal{T}$, that is, $\mathcal{D}ef(\Theta_p(\mathcal{T})) = \mathcal{D}ef(\mathcal{T})$, and has the same FOL part. The main difference between the two approaches lies in the fact that the $p$-opening operator can be inverted. In fact, a *p-closing operator* $\mathcal{C}_p$ can be defined (see [Ver98]) that just removes the clause $p(X) \leftarrow p^*(X)$ and applies the renaming operator $\rho_{p^* \to p}$ to the bodies of all clauses of the definition part. It is obvious that the following equality holds: $\mathcal{M}od(\mathcal{T}) = \mathcal{M}od(\mathcal{C}_p(\Theta_p(\mathcal{T})))$. A similar result cannot be obtained if we open the definition of $p$ by just putting its defining clauses in the FOL part. That is because the FOL part could contain other axioms for $p$ that would be no more distinguishable.

The last two propositions of this subsection study the relation between the models of a theory and the models of the $p$-opening of that theory. We already noticed in Example 6 that the models of the theory $\mathcal{T}$ are also models of $\Theta_{abn}(\mathcal{T})$. This holds in general. Namely, if we open the definition of a predicate $p$ in a theory $\mathcal{T}$, the models of $\mathcal{T}$ remain models of $\Theta_p(\mathcal{T})$. We just enlarge the set of models.

**Proposition 10** *Let $\mathcal{T}$ be a theory that defines the predicate $p$. Then,*

$$\mathcal{M}od(\mathcal{T}) \subseteq \mathcal{M}od(\Theta_p(\mathcal{T})).$$

The next proposition gives us a more detailed relation between $\mathcal{M}od(\mathcal{T})$ and $\mathcal{M}od(\Theta_p(\mathcal{T}))$. It tells us something about the possible truth values of atoms in the models of $\Theta_p(\mathcal{T})$, in comparison with their truth values in the models of $\mathcal{T}$.

Before going into any details, let us first return to the theory $\mathcal{T}$ of Example 4. Recall that $\mathcal{M}od(\mathcal{T}) = \{M_1, M_2\}$ and $\mathcal{M}od(\Theta_{abn}(\mathcal{T})) = \{M_1, M_2, M_3\}$ (see Example 8). Consider $I = \{has\_wings(tweety)^{\mathbf{t}}\}$, a partial Herbrand interpretation of the open predicate *has_wings*/1. There is only one model of $\mathcal{T}$ extending this partial Herbrand interpretation $I$, namely, $M_2$, whereas $\Theta_{abn}(\mathcal{T})$ has two models extending $I$, $M_2$, and also $M_3$. If we look at the truth values of *abn*(*tweety*) and *fly*(*tweety*) in the model $M_3$ and compare them with their truth values in $M_2$, we see that the truth value of *abn*(*tweety*) in $M_3$ is greater than its truth value in $M_2$, whereas the truth value of *fly*(*tweety*) in $M_3$ is smaller than its truth value in $M_2$. Note that the predicate *fly*/1 depends negatively on *abn*/1.

**Proposition 11** *Let $\mathcal{T}$ be a theory defining the predicate $p$. Let $I$ be a partial 2-valued Herbrand interpretation of the open predicates of $\mathcal{T}$ that belong to $\mathcal{A}$. Suppose there exists a model $M$ of $\mathcal{T}$ extending $I$. Then $M$ is the unique model of $\mathcal{T}$ extending $I$. Denote the set of models of $\Theta_p(\mathcal{T})$ that extend $I$ with $\mathcal{M}$ (then $M \in \mathcal{M}$). Consider the dependency graph of $\mathcal{T}_d$. Suppose $p$ depends only evenly on itself. Let $t$ be an arbitrary element of the Herbrand universe. Then the following hold:*

- $\forall M' \in \mathcal{M} : \mathcal{H}_{M'}(p(t)) \geq \mathcal{H}_M(p(t))$,
- $\forall q$ *defined in $\mathcal{T}$, which depends only evenly on $p$,*
  $\forall M' \in \mathcal{M} : \mathcal{H}_{M'}(q(t)) \geq \mathcal{H}_M(q(t))$,
- $\forall r$ *defined in $\mathcal{T}$, which depends only oddly on $p$,*
  $\forall M' \in \mathcal{M} : \mathcal{H}_{M'}(r(t)) \leq \mathcal{H}_M(r(t))$,
- $\forall s$ *defined in $\mathcal{T}$, which does not depend on $p$,*
  $\forall M' \in \mathcal{M} : \mathcal{H}_{M'}(s(t)) = \mathcal{H}_M(s(t))$.

## 3.2 Composing open definitions

In this subsection, we prove that, as a consequence of Theorem 9, we can compose two theories that both have an open definition for $p$. We consider the following situation. There are two experts representing their knowledge independently in distinct theories, $\mathcal{T}_1$ and $\mathcal{T}_2$, and both theories define the predicate $p$. The experts, who initially thought they had complete knowledge about $p$, revise their theories, by making the definition of $p$ open; this results in the theories $\mathcal{T}_1^* = \Theta_p(\mathcal{T}_1)$ and $\mathcal{T}_2^* = \Theta_p(\mathcal{T}_2)$. Because of Theorem 9, the predicate $p$ can be considered as open in $\mathcal{T}_1^*$ and $\mathcal{T}_2^*$. Hence, the knowledge of these two theories can be composed. The *composition* of $\mathcal{T}_1^*$ and $\mathcal{T}_2^*$ is given by the intersection of their sets of models:

$$\mathcal{M}od(\mathcal{T}_1^*) \cap \mathcal{M}od(\mathcal{T}_2^*).$$

The next theorem is a direct consequence of Theorems 9 and 3. It gives a condition on $\mathcal{T}_1^*$ and $\mathcal{T}_2^*$, such that the composition is simply the union of both theories.

**Theorem 12** *We are given two theories* $\mathcal{T}_1^* = \Theta_p(\mathcal{T}_1)$ *and* $\mathcal{T}_2^* = \Theta_p(\mathcal{T}_2)$*, such that* $\mathcal{D}ef(\mathcal{T}_1^*) \cap \mathcal{D}ef(\mathcal{T}_2^*) = \{p\}$*. Let* $D$ *be the dependency graph of the defined predicates of* $(\mathcal{T}_{1d} \setminus \mathcal{T}_{1d}^p) \cup (\mathcal{T}_{2d} \setminus \mathcal{T}_{2d}^p)$*. With* $C$ *we denote a cycle in* $D$*. If* $\forall C$*,* $\exists i \in \{1,2\}$*, such that all the predicates of* $C$ *are defined in* $\mathcal{T}_{id} \setminus \mathcal{T}_{id}^p$*, then*

$$\mathcal{M}od(\mathcal{T}_1^*) \cap \mathcal{M}od(\mathcal{T}_2^*) = \mathcal{M}od(\mathcal{T}_1^* \cup \mathcal{T}_2^*).$$

**Example 13** *Let* $\mathcal{T}_1^* = \Theta_{abn}(\mathcal{T}_1)$ *and* $\mathcal{T}_2^* = \Theta_{abn}(\mathcal{T}_2)$ *be the following revised theories:*

$$\mathcal{T}_1^* = \left\{ \begin{array}{l} bird(tweety) \leftarrow \\ bird(fleety) \leftarrow \\ fly(X) \leftarrow bird(X), \neg abn^*(X) \\ abn(X) \leftarrow \neg has\_wings(X) \\ abn(X) \leftarrow abn^*(X) \end{array} \right. \qquad \mathcal{T}_2^* = \left\{ \begin{array}{l} penguin(tweety) \leftarrow \\ abn(X) \leftarrow penguin(X) \\ abn(X) \leftarrow abn^*(X) \end{array} \right.$$

*Theories* $\mathcal{T}_1^*$ *and* $\mathcal{T}_2^*$ *both have an open definition for* $abn/1$*, which means that both theories do not have complete knowledge about abnormal birds with respect to flying, and the theories are left "open" for adding knowledge about the* $abn/1$ *predicate. To obtain the sum of the knowledges of these two theories, we take the intersection of their sets of models:* $\mathcal{M}od(\mathcal{T}_1^*) \cap \mathcal{M}od(\mathcal{T}_2^*)$*. This intersection contains six models. In all the models tweety does not fly, because it is a penguin. In four of the six models, fleety does not fly. In two of them this is due to its lack of wings, but in the other two fleety does not fly because it is abnormal, although it has wings.* $\mathcal{T}_1^*$ *and* $\mathcal{T}_2^*$ *satisfy the conditions of Theorem 12; hence their composition is given by their union,* $\mathcal{M}od(\mathcal{T}_1^*) \cap \mathcal{M}od(\mathcal{T}_2^*) = \mathcal{M}od(\mathcal{T}_1^* \cup \mathcal{T}_2^*)$*.*

Note that when the experts of Example 13 think that there is no more knowledge about $abn/1$ to add, they can close by means of the closing operator, the definition of $abn/1$ in the composition of their theories.

In a natural way, we can extend the results of Subsection 3.1 and this subsection, by considering the case in which we open the definition of more than one predicate symbol in a theory and compose theories that have several open definitions. Due to the lack of space, we omit these results and refer to [VB98] instead.

## 3.3 The conditional opening operator

We introduce a second operator for theory revision, the conditional opening operator, to deal with those situations in which the definition of a predicate has to be opened only partially. Recall theory $\mathcal{T}_1$ of Example 5. The expert who wrote theory $\mathcal{T}_1$ had a complete (and correct) knowledge of disease D only for the male population. Suppose $male/1$ is the predicate that identifies male persons. We show that the application of the conditional opening operator yields the following theory $\mathcal{T}_{1r}$:

$$\mathcal{T}_{1r} = \begin{cases} disD_h(X) \leftarrow parent(Y, X), disD(Y) \\ disD(X) \leftarrow disD_h(X), male(X) \\ disD(X) \leftarrow disD^*(X), \neg male(X) \end{cases}$$

The clause $disD(X) \leftarrow disD^*(X), \neg male(X)$ allows us to open the definition of $disD$ in the cases not studied by the expert. An auxiliary predicate $disD_h$ is used to make a copy of the previous (wrong) knowledge on disease D, and the clause $disD(X) \leftarrow disD_h(X), male(X)$ introduces the correct partial knowledge on disease D in the context of the male population. Note that we could easily eliminate the auxiliary predicate $disD_h$ by unfolding it in the body of the second clause. We introduce it for the sake of clearness.

Note also that, in the previous example, the constraining predicate $male$ is applied exactly to the arguments of the head. This might not always be the case; we could, for instance, consider a predicate $p/n$ and want to constrain it with a predicate $cond/m$, with $n \neq m$. We may also want to apply $cond/m$ to an $m$-tuple of terms instead of variables. For this reason we use, in Definition 14, a mapping $\tau$ from $n$-tuples of variables to $m$-tuples of terms.

The application of the conditional $p$-opening operator $\Theta_p^{cond,\tau}$ on a theory $\mathcal{T}$, which defines the predicate $p$, results in the theory $\Theta_p^{cond,\tau}(\mathcal{T})$, which has a conditional open definition for $p$. We denote with $Term$ the set of all terms, and for a term $s$, we denote with $Var(s)$ the set of variables occurring in $s$.

**Definition 14** *Let $\mathcal{T} = (\mathcal{T}_d, \mathcal{T}_c)$ be a theory in which the predicate $p/n$ is defined. Let $cond/m$ be a predicate that is either open or defined in $\mathcal{T}$ such that, in the dependency graph of $\mathcal{T}_d$, $cond/m$ does not depend on $p/n$. Let $X = (X_1, \ldots, X_n)$ be an $n$-tuple of distinct variables and $\tau$ be a mapping from $X$ to $Term^m$, $\tau(X) = (s_1, \ldots, s_m)$, such that for each $i \in \{1, \ldots, m\}$, $Var(s_i) \subseteq \{X_1, \ldots, X_n\}$.*

*The application of the* conditional $p$-opening operator $\Theta_p^{cond,\tau}$ *on the theory $\mathcal{T}$ yields the theory $\Theta_p^{cond,\tau}(\mathcal{T})$:*

$$\mathcal{T} = \left\{ \begin{array}{l} \mathcal{T}_d \setminus \mathcal{T}_d^p \\ p(t_1) \leftarrow D_1 \\ \ldots \\ p(t_r) \leftarrow D_r \\ FOL: \left\{ \begin{array}{l} \mathcal{T}_c \\ p^*(X) \iff p(X) \end{array} \right. \end{array} \right. \quad \rightarrow \quad \Theta_p^{cond,\tau}(\mathcal{T}) = \left\{ \begin{array}{l} \mathcal{T}_d \setminus \mathcal{T}_d^p \\ p_h(t_1) \leftarrow D_1 \\ \ldots \\ p_h(t_r) \leftarrow D_r \\ p(X) \leftarrow p_h(X), cond(\tau(X)) \\ p(X) \leftarrow p^*(X), \neg cond(\tau(X)) \\ FOL: \left\{ \begin{array}{l} \mathcal{T}_c \\ p^*(X) \iff p(X) \end{array} \right. \end{array} \right.$$

*where $p_h/n$ is a new, auxiliary predicate symbol, $D_i$, $i = 1, \ldots, r$, is a conjunction of literals, and $t_i$, $i = 1, \ldots, r$, is a $n$-tuple of terms.*

*We say that the predicate $p$ has a* conditional open definition *in the theory $\Theta_p^{cond,\tau}(\mathcal{T})$. The definition of $p$ (or briefly $p$) in $\Theta_p^{cond,\tau}(\mathcal{T})$ is called* conditional open.

We already noted that the auxiliary predicate $p_h$ in Definition 14, which depends on the theory $\mathcal{T}$, could easily be eliminated in $\Theta_p^{cond,\tau}(\mathcal{T})$ by unfolding it in the body of the clause $p(X) \leftarrow p_h(X), cond(\tau(X))$. We introduce it for the sake of clarity.

**Example 15** *Consider the theory $\mathcal{T}_1$ of Example 5. In this case, the constraining predicate cond is $male/1$. Since just one variable, $X$, occurs in the head of the clause defining disD in $\mathcal{T}_1$, the function $\tau$ is the identity function $I$. The theory $\Theta_{disD}^{male,I}(\mathcal{T}_1)$ is then exactly the theory $\mathcal{T}_{1r}$ illustrated before Definition 14.*

**Remark 16** *Note that we impose the condition that, in the dependency graph of $\mathcal{T}_d$, $c/m$ does not depend on $p/n$. As a consequence, we can split $\mathcal{T}_d$ into the two parts $(\mathcal{T}_{cond})_d$ and $\mathcal{T}'_d$, defining disjoint sets of predicates. Let*

*$(\mathcal{T}_{cond})_d$ be the set of definitions of the predicate cond/m and of the predicates on which cond/m depends (if cond/m is an open predicate in $\mathcal{T}$, then $(\mathcal{T}_{cond})_d = \emptyset$), and let $\mathcal{T}'_d$ contain all the other definitions of $\mathcal{T}_d$ (in particular, the definition of p/n). Put $(\mathcal{T}_{cond})_c = \mathcal{T}'_c = \mathcal{T}_c$. Then, $\mathcal{T} = \mathcal{T}_{cond} \cup \mathcal{T}'$ and $\Theta_p^{cond,\tau}(\mathcal{T}) = \Theta_p^{cond,\tau}(\mathcal{T}') \cup \mathcal{T}_{cond}$. Moreover, by Theorem 3, also the equality $\mathcal{M}od(\Theta_p^{cond,\tau}(\mathcal{T})) = \mathcal{M}od(\Theta_p^{cond,\tau}(\mathcal{T}')) \cap \mathcal{M}od(\mathcal{T}_{cond})$ holds.*

*We return to this remark in Subsection 3.4, where we compose two theories which both have a conditional open definition for p.*

When we apply the conditional $p$-opening operator $\Theta_p^{cond,\tau}$ on a theory $\mathcal{T}$, we restrict the definition of $p$ in $\mathcal{T}$ to the case that $cond(\tau(X))$ holds, and in the other case the predicate $p$ is completely open (and via $p^*$, more true $p$-atoms can be added). Actually, we retract some cases for $p$, in which we do not know the predicate $p$ and make $p$ open in these cases. Note that, in the special case that the condition $cond = true$, the theory $\Theta_p^{true}(\mathcal{T})$ is semantically equivalent with the original theory $\mathcal{T} : \mathcal{M}od(\Theta_p^{true}(\mathcal{T})) = \mathcal{M}od(\mathcal{T})$. Note also that if the condition $cond = false$, the definition of $p$ is constrained by *false*, just as if it had been deleted.

The next proposition gives a relation between the sets $\mathcal{M}od(\mathcal{T})$ and $\mathcal{M}od(\Theta_p^{cond,\tau}(\mathcal{T}))$. Note that, as opposed to Proposition 10, it is not the case that the models of $\mathcal{T}$ are also models of $\Theta_p^{cond,\tau}(\mathcal{T})$. But, informally, if $M$ is a model of $\mathcal{T}$ such that, when a $p$-atom $p(t)$ is true then $cond(\tau(t))$ is true, then $M$ is also a model of $\Theta_p^{cond,\tau}(\mathcal{T})$.

**Proposition 17** *If $M \in \mathcal{M}od(\mathcal{T})$, $M \models \forall X(p(X) \rightarrow cond(\tau(X)))$, and $M \models \forall X(p(X) \Longleftrightarrow p_h(X))$, then $M \in \mathcal{M}od(\Theta_p^{cond,\tau}(\mathcal{T}))$.*

## 3.4 Composing conditional open definitions

In the previous subsection, we introduced the conditional $p$-opening operator $\Theta_p^{cond,\tau}$. Different as in the case of the $p$-opening operator $\Theta_p$, the predicate $p$ cannot be seen as completely open in a theory with a conditional open definition for $p$. Hence, in general, we cannot compose two theories $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, because they both contain a definition for $p$ (by means of the predicate $p_{h_1}$, resp., $p_{h_2}$), which is applied if the condition $c_1(\tau_1(X))$ (resp., $c_2(\tau_2(X))$), holds. But, if the conditions $c_1(\tau_1(X))$ and $c_2(\tau_2(X))$ do not overlap, that is, the definitions for $p$ in the two theories describe a different case, we can compose $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$. We next show how this can be done.

First, we make more formal what is meant by *the two conditions do not overlap*. We already noticed, in Remark 16, that we can write a theory $\Theta_p^{cond,\tau}(\mathcal{T})$ as a union $\Theta_p^{cond,\tau}(\mathcal{T}') \cup \mathcal{T}_{cond}$, such that $p$ is defined in $\Theta_p^{cond,\tau}(\mathcal{T}')$, *cond* is defined in $\mathcal{T}_{cond}$ (or $\mathcal{T}_{cond} = (\emptyset, \mathcal{T}_c)$ if *cond* is open), and the defined predicates of $\mathcal{T}_{cond}$ do not depend on the defined predicates of $\Theta_p^{cond,\tau}(\mathcal{T}')$. In this section, where we deal with two theories $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, we suppose that $c_1$ and $c_2$ are open in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and in $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, and that we have a third theory $\mathcal{T}_{cond}$ in which both predicates $c_1$ and $c_2$ are (possibly) defined. More formally, we make the following assumptions:

**A1:** $c_1$ and $c_2$ are open in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and in $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$.

**A2:** $\mathcal{D}ef(\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)) \cap \mathcal{D}ef(\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)) = \{p\}$, $\mathcal{D}ef(\Theta_p^{c_i,\tau_i}(\mathcal{T}_i)) \cap \mathcal{D}ef(\mathcal{T}_{cond}) = \emptyset$, for $i = 1, 2$.

**A3:** The defined predicates of $\mathcal{T}_{cond}$ do not depend on the predicates that are defined in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ or in $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$.

The fact that the definitions for $p$ in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$ describe a different case—that is, $c_1$ and $c_2$ do not overlap—can now be expressed as follows:

**A4:** $\mathcal{T}_{cond} \models \forall X \neg (c_1(\tau_1(X)) \wedge c_2(\tau_2(X)))$.

If the four assumptions A1–A4 hold, we can *compose* $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ *and* $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$ *with respect to* $\mathcal{T}_{cond}$. This is because, although $p$ is defined in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, the definitions of $p$ describe a different case—namely, $c_1(\tau_1(X))$, $c_2(\tau_2(X))$, respectively. And $p$ is open in $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$, $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, respectively, in the case when $c_2(\tau_2(X))$, $c_1(\tau_1(X))$, respectively, holds. The meaning of the composition is given by the intersection of the models of $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$, $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, and $\mathcal{T}_{cond}$:

$$\mathcal{M}od(\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)) \cap \mathcal{M}od(\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)) \cap \mathcal{M}od(\mathcal{T}_{cond}).$$

In Subsection 3.2, we asked the question, When is the composition of $\Theta_p(\mathcal{T}_1)$ and $\Theta_p(\mathcal{T}_2)$ given by the union of these two theories? Here, we ask the question, When is the composition of $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$ with respect to $\mathcal{T}_{cond}$ given by the theory $\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1,\mathcal{T}_2)$ defined as follows:

$$\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1,\mathcal{T}_2) = \left\{ \begin{array}{l} (\Theta_p^{c_1,\tau_1}(\mathcal{T}_1))_d \setminus \{p(X) \leftarrow p^*(X), \neg c_1(\tau_1(X))\} \\ (\Theta_p^{c_2,\tau_2}(\mathcal{T}_2))_d \setminus \{p(X) \leftarrow p^*(X), \neg c_2(\tau_2(X))\} \\ (\mathcal{T}_{cond})_d \\ \{p(X) \leftarrow p^*(X), \neg c_1(\tau_1(X)), \neg c_2(\tau_2(X))\} \\ \text{FOL} : \left\{ \begin{array}{l} \mathcal{T}_{1c} \cup \mathcal{T}_{2c} \cup (\mathcal{T}_{cond})_c \\ p(X) \iff p^*(X)\,? \end{array} \right. \end{array} \right.$$

This theory is obtained by taking the union of the theories $\mathcal{T}_{cond}$, $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$, and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, but in which we replace the two clauses $p(X) \leftarrow p^*(X)$, $\neg c_1(\tau_1(X))$ and $p(X) \leftarrow p^*(X), \neg c_2(\tau_2(X))$ by one clause,

$$p(X) \leftarrow p^*(X), \neg c_1(\tau_1(X)), \neg c_2(\tau_2(X))$$

($p$ is only open in the case when $\neg c_1(\tau_1(X))$ *and* $\neg c_2(\tau_2(X))$ hold).

The next theorem gives conditions on $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$, $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, and $\mathcal{T}_{cond}$, such that the composition of $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$ and $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$ with respect to $\mathcal{T}_{cond}$ is given by $\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1,\mathcal{T}_2)$.

**Theorem 18** *Given are the theories $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)$, $\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$, and $\mathcal{T}_{cond}$, satisfying A1–A4. Let $D$ be the dependency graph of the defined predicates of $\Theta_p^{c_1,\tau_1}(\mathcal{T}_1) \cup \Theta_p^{c_2,\tau_2}(\mathcal{T}_2)$. With $C_{\overline{p}}$, we denote a cycle in $D$ not containing the predicate $p$. With $C_p$, we denote a cycle in $D$ containing the predicate $p$. If*

1. *$\forall C_{\overline{p}}$, $\exists i \in \{1,2\}$, such that all the predicates in $C_{\overline{p}}$ are defined in $\Theta_p^{c_i,\tau_i}(\mathcal{T}_i)$,*
2. *$\exists i \in \{1,2\}$, $\forall C_p$, such that all the predicates in $C_p$ are defined in $\Theta_p^{c_i,\tau_i}(\mathcal{T}_i)$,*

*then*

$$\begin{array}{l} \mathcal{M}od(\Theta_p^{c_1,\tau_1}(\mathcal{T}_1)) \cap \mathcal{M}od(\Theta_p^{c_2,\tau_2}(\mathcal{T}_2)) \cap \mathcal{M}od(\mathcal{T}_{cond}) \\ \quad = \ \mathcal{M}od(\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1,\mathcal{T}_2)) \cap \mathcal{M}od(\mathcal{T}_{cond}) \\ \quad = \ \mathcal{M}od(\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1,\mathcal{T}_2) \cup \mathcal{T}_{cond}). \end{array}$$

The second equality is a direct consequence of Theorem 3 and assumption A3.

**Example 19** *Let $\mathcal{T}_1$ be the theory defined in Example 5. Assume that another researcher studies women with disease D. He finds out that for women, disease D is a consequence of disease E. His results are expressed in the following theory:*

$$\Theta_{disD}^{female,I}(\mathcal{T}_2) = \begin{cases} disD_{h_2}(X) \leftarrow disE(X) \\ disD(X) \leftarrow disD_{h_2}(X), female(X) \\ disD(X) \leftarrow disD^*(X), \neg female(X). \end{cases}$$

*Suppose that there is a theory $\mathcal{T}_{cond}$ such that $\mathcal{T}_{cond} \models \forall X \neg(male(X) \wedge female(X))$. Then, the composition of $\Theta_{disD}^{male,I}(\mathcal{T}_1)$ and $\Theta_{disD}^{female,I}(\mathcal{T}_2)$ with respect to $\mathcal{T}_{cond}$ is given by*

$$\mathcal{M}od(\Theta_{disD}^{male,I}(\mathcal{T}_1)) \cap \mathcal{M}od(\Theta_{disD}^{female,I}(\mathcal{T}_2)) \cap \mathcal{M}od(\mathcal{T}_{cond}).$$

*In this composition, it is, for instance, the case that if a woman has disease E (and hence also disease D), and she is the mother of a boy, then this boy has disease D. Since the conditions of Theorem 18 are satisfied, the composition with respect to $\mathcal{T}_{cond}$ is given by the theory $\Theta_{disD}^{(male,I),(female,I)}(\mathcal{T}_1, \mathcal{T}_2)$ :*

$$\begin{cases} disD_{h_1}(X) \leftarrow parent(Y, X), disD(Y) \\ disD_{h_2}(X) \leftarrow disE(X) \\ disD(X) \leftarrow disD_{h_1}(X), male(X) \\ disD(X) \leftarrow disD_{h_2}(X), female(X) \\ disD(X) \leftarrow disD^*(X), \neg female(X), \neg male(X). \end{cases}$$

A special case to consider is when $\mathcal{T}_{cond} \models \forall X(c_1(\tau_1(X)) \Longleftrightarrow \neg c_2(\tau_2(X)))$. Both cases do not overlap, and together they cover the whole space. In the composition, the predicate $p^*$ becomes useless, since the case $\neg c_1(\tau_1(X)) \wedge \neg c_2(\tau_2(X))$ never occurs. In the case when the composition is given by the theory $\Theta_p^{(c_1,\tau_1),(c_2,\tau_2)}(\mathcal{T}_1, \mathcal{T}_2)$, then we can just as well remove the clause $p(X) \leftarrow p^*(X), \neg c_1(\tau_1(X)), \neg c_2(\tau_2(X))$. For instance, this can be done in Example 19 if also $\mathcal{T}_{cond} \models \forall X\ (male(X) \vee female(X))$.

We mention that all this can be extended to the case where we want to compose $n$ theories, all with a conditional open definition for the predicate $p$, and each describing a different case. Due the lack of space, we do not go into detail.

# 4 A simple example

By means of a simple example, we show the use of the opening and conditional opening operators in the context of theory revision and theory composition.

**Example 20** *Suppose there is an Italian person who expresses knowledge about eating customs in the following theory:*

$$\mathcal{T}_{1a}: \ \ eats\_a\_lot(X, Y) \leftarrow pasta(Y).$$

*Now let us suppose another Italian person wants to add some knowledge about the eats_a_lot/2-predicate. Namely, the person wants to add*

$$\mathcal{T}_{1b}: \ \ eats\_a\_lot(X, Y) \leftarrow pizza(Y).$$

*Therefore, in both theories* $\mathcal{T}_{1a}$ *and* $\mathcal{T}_{1b}$ *the definition of eats_a_lot/2 must be opened. The resulting theories are* $\Theta_{eats\_a\_lot}(\mathcal{T}_{1a})$ *and* $\Theta_{eats\_a\_lot}(\mathcal{T}_{1b})$*. The composition of these theories is given by the union* $\Theta_{eats\_a\_lot}(\mathcal{T}_{1a}) \cup \Theta_{eats\_a\_lot}(\mathcal{T}_{1b})$ *(Theorem 12):*

$$\Theta_{eats\_a\_lot}(\mathcal{T}_{1a}) \cup \Theta_{eats\_a\_lot}(\mathcal{T}_{1b}) : \left\{ \begin{array}{l} eats\_a\_lot(X, Y) \leftarrow pasta(Y) \\ eats\_a\_lot(X, Y) \leftarrow pizza(Y) \\ eats\_a\_lot(X, Y) \leftarrow eats\_a\_lot^*(X, Y). \end{array} \right.$$

*Denote the theory* $\mathcal{C}_{eats\_a\_lot}(\Theta_{eats\_a\_lot}(\mathcal{T}_{1a}) \cup \Theta_{eats\_a\_lot}(\mathcal{T}_{1b}))$ *(the definition of eats_a_lot/2 is closed) by* $\mathcal{T}_1$*:*

$$\mathcal{T}_1 : \left\{ \begin{array}{l} eats\_a\_lot(X, Y) \leftarrow pasta(Y) \\ eats\_a\_lot(X, Y) \leftarrow pizza(Y). \end{array} \right.$$

*Now, note that this theory* $\mathcal{T}_1$ *may be applicable to Italian persons, but for instance not to Belgian persons. There is another theory* $\mathcal{T}_2$*, written by a Belgian person:*

$$\mathcal{T}_2: \ \ eats\_a\_lot(X, Y) \leftarrow chips(Y).$$

*So the definition of eats_a_lot/2 in* $\mathcal{T}_1$ *needs to be restricted to Italian people and opened in the other cases. Likewise, the definition of eats_a_lot/2 in* $\mathcal{T}_2$ *needs to be restricted to Belgian people and opened otherwise. Let* $\pi_1^2$ *be the function that projects the first of two arguments:* $\pi_1^2(X, Y) = X$*. We apply* *the conditional opening operator* $\Theta^{italian,\pi_1^2}_{eats\_a\_lot}$ *(resp.,* $\Theta^{belgian,\pi_1^2}_{eats\_a\_lot}$*) to the theory* $\mathcal{T}_1$ *(resp.,* $\mathcal{T}_2$*). The resulting theories are*

$$\Theta^{italian,\pi_1^2}_{eats\_a\_lot}(\mathcal{T}_1) : \begin{cases} eats\_a\_lot_{h_1}(X,Y) \leftarrow pasta(Y) \\ eats\_a\_lot_{h_1}(X,Y) \leftarrow pizza(Y) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot_{h_1}(X,Y), italian(X) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot^*(X,Y), \neg italian(X) \end{cases}$$

$$\Theta^{belgian,\pi_1^2}_{eats\_a\_lot}(\mathcal{T}_2) : \begin{cases} eats\_a\_lot_{h_2}(X,Y) \leftarrow chips(Y) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot_{h_2}(X,Y), belgian(X) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot^*(X,Y), \neg belgian(X). \end{cases}$$

*Now, let* $\mathcal{T}_{cond}$ *be a theory such that* $\mathcal{T}_{cond} \models \forall X \neg(italian(X) \wedge belgian(X))$*; that is,* $\mathcal{T}_{cond}$ *implies that one cannot be an Italian and Belgian person simultaneously. Then, we can compose the theories*

$$\Theta^{italian,\pi_1^2}_{eats\_a\_lot}(\mathcal{T}_1) \quad and \quad \Theta^{belgian,\pi_1^2}_{eats\_a\_lot}(\mathcal{T}_2)$$

*with respect to* $\mathcal{T}_{cond}$*. By Theorem 18, the composition is given by*

$$\Theta^{(italian,\pi_1^2),(belgian,\pi_1^2)}_{eats\_a\_lot}(\mathcal{T}_1,\mathcal{T}_2) :$$

$$\begin{cases} eats\_a\_lot_{h_1}(X,Y) \leftarrow pasta(Y) \\ eats\_a\_lot_{h_1}(X,Y) \leftarrow pizza(Y) \\ eats\_a\_lot_{h_2}(X,Y) \leftarrow chips(Y) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot_{h_1}(X,Y), italian(X) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot_{h_2}(X,Y), belgian(X) \\ eats\_a\_lot(X,Y) \leftarrow eats\_a\_lot^*(X,Y), \neg italian(X), \neg belgian(X). \end{cases}$$

# 5 Conclusions

We studied theory revision and composition in the logic OLP-FOL [DD93, Den95].

Concerning *theory revision*, we introduced two operators: the $p$-opening operator and the conditional $p$-opening operator. Let us summarise the main ideas of these operators.

- *The opening operator:* The expert realises his or her knowledge on a given predicate $p$ is incomplete but does not retract it entirely. As Theorem 9 shows, previous knowledge on $p$ is transferred as if it were a constraint in the FOL part.

- *The conditional opening operator:* The expert not only realises his or her knowledge on $p$ is incomplete but also retracts it whenever the given condition *cond* is false. The two extreme cases of conditional opening operator are (1) $cond = true$: The expert retracts nothing and no revision is actually done. The revised theory has the same models as the original one. (2) $cond = false$: The expert retracts all knowledge on the considered predicate, which becomes completely open. Only the constraints on $p$ expressed in the FOL part of the original theory are preserved in the revised one.

Let us show the difference on a small example. Let the theory $\mathcal{T}$ be the program of Example 4 together with the unit clause $has\_wings(tweety) \leftarrow$. In the unique model of $\mathcal{T}$, we have $abn(tweety)^{\mathbf{f}}$. With the opening operator, the revised theory $\Theta_{abn}(\mathcal{T})$ also has a model with $abn(tweety)^{\mathbf{t}}$. With the conditional opening operator and $cond = true$, we obtain a semantically equivalent theory to $\mathcal{T}$. With the conditional opening operator and $cond = false$, we obtain a theory with a model with $abn(tweety)^{\mathbf{f}}$ and a model with $abn(tweety)^{\mathbf{t}}$.

A dual approach to the $p$-opening operator is to consider the $p$-closing operator (mentioned in Subsection 3.1), which closes the open definition of the predicate $p$. The $p$-closing operator puts the set of clauses with the predicate $p$ in the head, from the FOL part in the definition part of a theory (Theorem 9). When we close the definition of a predicate in a theory, we reduce the set of models of the theory (Proposition 10); that is, we consider only a subset of the set of models of the theory. Actually, in general, a semantics for logic programs can be seen as a closing operator. In [BLMM92], these closing operators are studied. There, each normal logic program is denoted by the set of Herbrand models of its positive version. Mappings (closing operators) are given from this set to some existing semantics of normal logic programs. Note that here we consider the case in which we close (or, dually, open), the definition of only one predicate (and possibly more).

Concerning *theory composition*, we argued that two theories, which both have an open definition for the predicate $p$, can be composed in the sense of [VDD97]. Also in [BLMM92], and further in [BLM92], the compositionality of normal logic programs is studied. In [BLM92], a compositional model-theoretic semantics for positive logic programs is presented, where the composition of programs is modeled by the composition of the admissible Herbrand models of the programs. These results, together with the results of [BLMM92] (which we mentioned above), provide normal logic programs with a compositional semantics. We also studied the composition of two theories which both have a conditional open definition for the same predicate, in case the conditions do not overlap. The compositionality issue is also studied in [BGLM92, GF95, Eta98, Bry96, LT94]. For more references and a detailed discussion on theory composition, we refer to [VDD97]. A survey of the compositionality issue in logic programming is given in [BLM94].


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