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ABSTRACT

The purpose of this paper is to introduce a model refinement method realized by the updating FORTRAN program MORES (Modal Residuals) for time-invariant non-gyroscopic and viscously damped elastomechanical systems, which identifies adjustment parameters of submatrices related to the uncertain substructures of the associated discrete model. The parameters are estimated by minimizing a cost function of modal residuals using a weighted least-squares method to solve an ill-conditioned and overdetermined system of linear equations. The program has been applied to a simulated non-proportionally damped vibrator chain with increasing model order. Even for model paramter errors of 50% the parameter estimate errors are less than 0.1% unless additive measurement noise has to be taken into account. The errors of the parameter estimates caused by erroneous measurements have been investigated statistically for elastomechanical models of 8 and 20 degrees-of-freedom by running 50 simulated measurement-data samples with uniformly distributed additive random errors of zero mean. It is shown that the biased parameter estimates caused by disturbed data still yield a satisfactory update of the model. The program execution time and storage request with regard to the increasing size of the problem are discussed briefly.

List of Symbols

$[A_0]$ matrix of stiffness; symmetric positive definite of order n

$[A_1]$ matrix of damping; symmetric positive semidefinite of order n

$[A_2]$ matrix of inertia; symmetric positive definite of order n

$\left[A_{0k_0}\right]$ n_0 uncertain submatrices of stiffness; symmetric positive semidefinite of order n

$\left[A_{1k_1}\right]$ n_1 uncertain submatrices of damping; symmetric positive semidefinite of order n

$\left[A_{2k_2}\right]$ n_2 uncertain submatrices of inertia; symmetric positive semidefinite of order n

$[\acute{A}_0]$ constant (exact) submatrix of stiffness; symmetric positive semidefinite of order n

$[\acute{A}_1]$ constant (exact) submatrix of damping; symmetric positive semidefinite of order n

$[\acute{A}_2]$ constant (exact) submatrix of inertia; symmetric positive semidefinite of order n

$\{a\}$ $(n_2+n_1+n_0=:n_p)$-dimensional vector of parameters

a_{0k_0} $(k_0 = 1,..,n_0)$ dimensionless, positive parameters of stiffness to be multiplied by the respective submatrices A_{0k_0}

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a_{1k_1} ($k_1 = 1,..,n_1$) dimensionless, positive parameters of damping to be multi-plied by the respective submatrices A_{1k_1}

a_{2k_2} ($k_2 = 1,..,n_2$) dimensionless, positive parameters of inertia to be multiplied by the respective submatrices A_{2k_2}

c_r normalizing factor of the rth eigenvector

cor(..) correlation coefficients of (..)

cov(..) covariance of (..)

$D(..)$ partial differentiation of (..) with respect to the parameters

$[D^2J]$ Hessian matrix; real-valued, symmetric positive definite of order n_p

d_r damping ratio associated with the rth eigenmode

$E\{..\}$ expectation value of $\{..\}$

$\{e\}$ n_p-dimensional vector containing n_p ones

$[G]$ positive definite weighting matrix of order $n_f(n+1)$

$[G_r]$ positive definite weighting matrix of order $(n + 1)$ for the rth eigenmode

g_r generalized inertia associated with the rth eigenmode

$\Im(..)$ imaginary part of (..)

J cost function to be minimized

$\mathrm{j} := \sqrt{-1}$ (imaginary unit)

$\{K\}$ $n_p(n + 1)$-dimensional complex vector denoting the negative of that part of the vector of residual $\{v\}$ which is independent of the adjustment parameters

$M_{ik}\left(\mathring{X}, \hat{X}\right)$ MAC (Modal Assurance Criterion) factors of the 'true' eigenvectors and the eigenvector estimates

n order of the discrete model

n_f number of eigenmodes used for updating

n_p total number of adjustment parameters to be estimated

n_0 number of parameters of stiffness

n_1 number of parameters of damping

n_2 number of parameters of inertia

p_r first component (normalizing part) of the rth residual vector $\{v_r\} =: \binom{p_r}{q_r}$

$\{q_r\}$ n-dimensional part of the rth residual vector $\{v_r\}$

$\Re(..)$ real part of (..)

$\{v\}$ n_f (n+1)-dimensional residual vector containing n_f vectors $\{v_r\}$ (r=1,..,$n_f \leq n$)

$\{v_r\}$ (n+1)-dimensional residual vector of the rth eigenmode

var(..) variance of (..)

[X] complex matrix of eigenvectors of order n

$\{x_{or}\}$ rth real-valued eigenvector of the undamped model

x_{oir} ith component of the rth real-valued n-dimensional eigenvector of the undamped model

$\{x_r\}$ rth complex eigenvector of the damped model

z random number uniformly distributed in the interval $\left[-\sqrt{3}, \sqrt{3}\right]$ and with unit standard deviation

Greek Symbols

$\Delta(..)$ additive error of (..)

λ_r rth complex eigenvalue

σ_H standard deviation of the parameter estimates obtained from the inverse Hessian matrix

σ_M estimates for the standard deviation of the parameter estimates

σ_Q estimates for the square roots of the quadratic deviation of the parameter estimates from the 'true' parameter values

ω_{or} rth eigenfrequency of the undamped model

ω_r rth eigenfrequency of the damped model

Superscripts

- $(..)^R$ Round (100·(..)) rounded values of (..) in percentage (%)
- $(..)^T$ the transpose of (..)
- (..) true' value of (..)
- $(\hat{..})$ estimates of (..)
- $(..)^*$ the conjugate of the compex quantity (..)
- $(..)^\dagger$ the conjugate transpose of (..)
- := defined as

1. Introduction

The estimation technique used in the progam MORES is based mainly on the works of Natke, Cottin, et. al. [1,2,3] and is an improvement of a method realized by the program MEARES in 1983/84 at the Curt-Risch-Institute, University of Hannover (Cottin, Zacharias : CRI-F-3/1984). The successful application of the experimental version and the increasing interest in identification procedures required a more powerful version, i.e., for large models with acceptable execution time, storage request and with a minimum loss of numerical precision. The improved version of the tested program is a first issue for external use and it has to be fitted by the user to the available hardware. The module-like structure of the program permits individual program optimization and modification.

The method presented here updates a discrete model of order n of a linear elastomechanical system by means of estimated modal quantities such as $n_f \leq n$ complex eigenvalues, λ_r, and eigenvectors, $\{x_r\}$, of the damped system or by means of estimated eigenfrequencies, ω_{or}, real-valued eigenvectors, $\{x_{or}\}$, and generalized inertias, g_r, of the associated undamped system and modal damping ratios, d_r, obtained by experimental modal analysis [1] or phase resonance test respectively. The standard deviations of these modal estimates also have to be estimated.

The discrete model can be the mathematical model of an elastomechanical multi-body system or a FEM (finite-element-model), in general with $n \gg n_f$, evaluated by programs like SAP or SOLVIA. In the latter case the number of measured degrees-of-freedom (DOFs) is often $m \ll n$, that is, $n_f \leq m \ll n$. Thus, the order of the model has to have been reduced from n to m. The reduction of the model order can be done by an incomplete modal transformation due to the band-limited excitation of the system that is described by the model. The reduced model, restricted to the pre-given frequency band, has eigenfrequencies and associated eigenvector components identical with the original model [4,5,6]. Since the adjustment parameters are attached to pre-selected submatrices as indicated in Eq. (3) the model reduction procedure mentioned above reduces the order of all submatrices but does not change the attachment of the adjustment parameters [4]. An essential advantage of this model reduction is that computer storage and execution time are saved. Since model reduction is not the object of the investigation presented here we assume that the reduction is already done, that is, we can set $n \equiv m$.

2. Theoretical Basis

Given the linear elastomechanical discrete n-DOF-model with viscous damping, from the equations of the associated eigenvalue problem [1,7] we define

$$\{q_r\} := \lambda_r^2[A_2]\{x_r\} + \lambda_r[A_1]\{x_r\} + [A_0]\{x_r\} \tag{1}$$

and from the normalizing equation for the rth eigenvector

$$p_r := 2\lambda_r \{x_r\}^T [A_2]\{x_r\} + \{x_r\}^T [A_1]\{x_r\} - c_r \tag{2}$$

with $[A_2]$, $[A_0]$ as symmetric positive definite real-valued matrices of inertia and stiffness respectively, and the symmetric positive semidefinite real-valued matrix of damping $[A_1]$. c_r is the normalizing constant, which in the case of real-valued eigenvectors (phase resonance test) is identical to the generalized inertias. Since the model matrices $[A_l]$ (l=0,1,2) are uncertain and erroneous, and since the estimated modal quantities are uncertain and erroneous due to measurement, the right-hand sides of Eq. (1) and Eq. (2) will not vanish for estimated eigenvalues and eigenvectors. We always assume the model matrices to be linear combinations of the uncertain submatrices $\left[A_{lk_l}\right]$ multiplied by n_p real-valued dimensionless adjustment parameters a_{lk_l} (k_l=1,..,n_l; $n_0+n_1+n_2 =: n_p$) to be estimated, plus one known submatrix $[\acute{A}_l]$ for each $l \in \{0,1,2\}$, i.e.,

$$[A_l] := [A_l'] + \sum_{k_l=1}^{n_l} a_{lk_l}\left[A_{lk_l}\right] \tag{3}$$

Each submatrix is related to a substructure of the model (e.g., a group of finite elements or lumped masses at the nodes of the elements). Those which are assumed to be known exactly, a priori, are represented by the submatrices $[\acute{A}_l]$.

Recalling the definition of the scalar p_r and the vector $\{q_r\}$ from Eq. (1) and Eq. (2) we can define nf $(n+1)$-dimensional complex residual vectors as linear functions of the parameters

$$\{v_r(a)\} := \begin{bmatrix} p_r(a) \\ q_r(a) \end{bmatrix} \qquad r = 1,\dots,n_f \tag{4}$$

where we have $\{a\} := \left[a_{21},..,a_{2n_2}, a_{11},..,a_{1n_1}, a_{01},..,a_{0n_0}\right]^T$. Assembling all the n_f residual vectors in one n_f(n+1)-dimensional complex residual vector $\{v(a)\}:=[v_1(a)^T,..,v_{n_f}(a)^T]^T$ and defining its partial derivatives with respect to the parameters by the complex matrix $[Dv]$ with n_f(n+1) rows and n_p columns, we can rewrite Eqs. (1) and (2) in a more condensed form

$$\{v(a)\} = [Dv]\{a\} - \{K\} \tag{5}$$

where the n_f(n+1)-dimensional complex vector (-$\{K\}$) denotes that part of the residual vector $\{v\}$, which is independent of the adjustment parameters.

We may now ask for a parameter vector $\{\hat{a}\}$, which satisfies

$$\{v(\hat{a})\} = [Dv]\{\hat{a}\} - \{K\} = 0 \tag{6}$$

A necessary condition for the existence of a real-valued solution $\{\hat{a}\}$ of Eq. (6) is

$$\operatorname{rank}\left(\begin{bmatrix} \Re(Dv) \\ \Im(Dv) \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} \Re(Dv) & \Re(K) \\ \Im(Dv) & \Im(K) \end{bmatrix}\right) \tag{7}$$

Since the residual vector consists of modal estimates, Eq. (7) generally will not be satisfied. To overcome the inconsistency of Eq. (6) we solve the problem by applying a weighted least square method (WLS) minimizing the cost function

$$J(a) := \frac{1}{2}\{v(a)\}^{\dagger}[G]v\{(a)\} \tag{8}$$

with the positive definite weighting matrix

$$[G] := \begin{bmatrix} [G_1] & & 0 \\ & \ddots & \\ 0 & & [G_{nf}] \end{bmatrix} \tag{9}$$

For statistical reasons [2] the rth Hermitian weighting matrix is defined by

$$[G_r]^{-1} := \operatorname{cov}(v_r(e)) \approx E\left\{v_r\left(\mathring{a}\right)v_r\left(\mathring{a}\right)^{\dagger}\right\} \tag{10}$$

where the n_p-dimensional vector $\{e\}=(1,..,1)^T$ denotes the vector of parameters apriori given and $\{\mathring{a}\}$ denotes the 'true' parameter vector. Assuming that the random errors of the modal estimates have zero mean values (i.e., they are without systematic errors or bias) and substituting $\lambda_r = \mathring{\lambda}_r + \Delta\lambda_r$ as well as $\{x_r\}=\{\mathring{x}_r\}+\Delta x_r$ and $c_r = \mathring{c}_r + \Delta c_r$ in Eq. (4), we obtain a linearized expression for the residual vector $\{v_r(\mathring{a})\}=\{v_r(\mathring{a},c_r,\lambda_r,x_r)\}$ by calculating its total differential with respect to the assumed deviations $\Delta\lambda_r$, Δx_r, Δc_r of the modal estimates from their 'true' values denoted by $(\mathring{.})$

$$\begin{aligned}
\left\{v_r\left(\mathring{a}, c_r, \lambda_r, x_r\right)\right\} &\approx \left[\frac{\partial v_r}{\partial c_r}, \frac{\partial v_r}{\partial \lambda_r}, \frac{\partial v_r}{\partial x_r^T}\right]\Bigg|_{\left(\mathring{a},\mathring{c},\mathring{\lambda}_r,\mathring{x}_r\right)} \cdot \begin{bmatrix} \Delta c_r \\ \Delta\lambda_r \\ \Delta x_r \end{bmatrix} \\
&= \begin{bmatrix} -1 & 2\left\{\mathring{x}_r\right\}^T [A_2]\left\{\mathring{x}_r\right\} & 2\left\{\mathring{x}_r\right\}^T\left[2\mathring{\lambda}_r A_2 + A_1\right] \\ 0 & \left[2\mathring{\lambda}_r A_2 + A_1\right]\left\{\mathring{x}_r\right\} & \left[\mathring{\lambda}_r^2 A_2 + \mathring{\lambda}_r A_1 + A_0\right] \end{bmatrix} \cdot \begin{bmatrix} \Delta c_r \\ \Delta\lambda_r \\ \Delta x_r \end{bmatrix} \\
&= \left[U_r\left(\mathring{a}\right)\right] \cdot \left\{\bar{\Delta}_r\right\}
\end{aligned} \tag{11}$$

The modal estimates obtained from measurements are gained by using modal analysis, i.e. by curve fitting procedures which are in general based on least squares estimation without calculating any cross-correlation between the resulting modal estimates.

Therefore neglecting the, in general, unknown cross-correlations between the errors in the rth eigenvalue and its associated eigenvector and generalized inertia from Eq. (11) we may write

$$E\left\{v_r\left(\mathring{a}\right)v_r^{\dagger}\left(\mathring{a}\right)\right\} \approx \left[U_r\left(\mathring{a}\right)\right] \begin{pmatrix} \sigma_{c_r}^2 & 0 & 0^T \\ 0 & \sigma_{\lambda r}^2 & 0^T \\ 0 & 0 & \operatorname{Diag}\operatorname{var}(x_r) \end{pmatrix} \left[U_r^{\dagger}\left(\mathring{a}\right)\right] \tag{12}$$

Since neither the 'true' parameters nor the 'true' modal quantities are known, the a priori values are used in Eq. (10). This is possible for sufficiently small changes $\{\Delta a\} := \{\mathring{a}\} - \{e\}$, because of the linearity of $[U_r(\mathring{a})] = [U_r(e)] + [U_r(\Delta a)]$. The inverse of this matrix, Eq. (12), is then used for a rough but sufficient approximation of the rth Hessian weighting matrix, Eq. (10), that has the effect of a whitening filter [8].

In general [9] the sufficient and necessary conditions for a strong local minimum of the quadratic cost function of Eq. (8) at the point $\{a\} = \{\hat{a}\}$ are

$$\begin{aligned} \{DJ(\hat{a})\} &= \Re\{Dv^\dagger G v(\hat{a})\} \\ &= \Re\{Dv^\dagger G Dv\}\{\hat{a}\} - \Re\{Dv^\dagger GK\} = 0 \\ &\Leftrightarrow \\ \Re\{Dv^\dagger G Dv\}\{\hat{a}\} &= \Re\{Dv^\dagger GK\} && (13) \\ [D^2 J] &:= \Re\{Dv^\dagger G Dv\} \text{ Hessian matrix positive definite} && (14) \end{aligned}$$

The first relation, Eq. (13), estimates the vector of adjustment parameters by

$$\{\hat{a}\} = \left[\Re\{Dv^\dagger G Dv\}\right]^{-1} \Re\{Dv^\dagger GK\} = \left[D^2 J\right]^{-1} \Re\{Dv^\dagger GK\}$$

Since the weighting matrix is positive definite the second condition, Eq. (14), is equivalent with

$$\operatorname{rank}\left(\begin{bmatrix} \Re(Dv) \\ \Im(Dv) \end{bmatrix}\right) = n_p \tag{15}$$

which includes the necessary condition for the total number of adjustment parameters $n_p \le 2n_f(n+1)$, since $[Dv]$ is a rectangular matrix.

For unbiased estimators the inverse Hessian matrix yields lower bounds (with reference to the Rao-Cramér theorem) for the variances and covariances of the parameter estimates [2,3]. If in the inverse Hessian matrix the element placed in row α and column β ($\alpha, \beta = 1,..,n_p$) is denoted by $h_{\alpha\beta}$ an estimate for the correlation coefficient of the parameter estimates $\hat{a}_\alpha$ and $\hat{a}_\beta$ is given by

$$\operatorname{cor}\left[\hat{a}_\alpha \hat{a}_\beta\right] := \frac{h_{\alpha\beta}}{\sqrt{h_{\alpha\alpha} h_{\beta\beta}}}$$ where $\sigma_\alpha := \sqrt{h_{\alpha\alpha}}$ is an estimate (lower bound) of the standard deviation of $\hat{a}_\alpha$.

3. Data Normalization and Parameter Estimation

In the general case, the estimation procedure realized by the program MORES requests complex eigenvalues and eigenvectors as well as their standard deviations $\sigma_{\lambda_r}, \sigma_{x_{ir}}$ presuming the normalization constants $\mathring{c}_r = 1$. To compute the weighting matrices, Eq. (10), the standard deviations σ_{c_r} are calculated from the first component of Eq. (11)

$$\Delta c_r = 2\left\{\mathring{x}_r\right\}^T [A_2]\left\{\mathring{x}_r\right\}\Delta\lambda_r + 2\left\{\mathring{x}_r\right\}^T\left[2\,\mathring{\lambda}_r A_2 + A_1\right]\{\Delta x_r\}$$

which yields approximately

$$\sigma_{c_r}^2 \approx 4\left[\left|\left\{\overset{\circ}{x}_r\right\}^T [A_2]\left\{\overset{\circ}{x}_r\right\}\right|^2 \sigma_{\lambda_r}^2 + \left\{\overset{\circ}{x}_r\right\}^T \left(2\overset{\circ}{\lambda}_r[A_2]+[A_1]\right) \text{ Diag var}(x_r)\left(2\overset{\circ}{\lambda}{}_r^*[A_2]+[A_1]\right)\left\{\overset{\circ}{x}_r\right\}^*\right]$$

If real-valued eigenvectors $\{x_{or}\}$ are available (e.g. by phase resonance test), MORES also needs the corresponding eigenfrequencies, ω_{or}, generalized inertias, g_r, and damping ratios, d_r, and the standard deviation for each of them. The real-valued eigenvectors are supposed to be valid for the undamped model, since in case of proportional (modal) damping a real-valued normalization is possible [1,7]. The condition

$$c_r = \{x_r\}^T (2\lambda_r[A_2]+[A_1])\{x_r\} = \{x_{or}\}^T [A_2]\{x_{or}\} = g_r$$

leads to the relations [1]

$$\{x_r\} := \frac{1}{2}\{x_{or}\}\omega_r^{-\frac{1}{2}}(1-j) \tag{16}$$

$$\lambda_r := \omega_r(j-d_r) \tag{17}$$

with $\omega_r := \omega_{or}\sqrt{1-d_r^2}$ $\quad (j^2 := -1)$ which are used for the calculation of complex modal quantities. Consequently, by a pre-process the program calculates approximately the standard deviations of the complex values by linearization of Eqs. (16) and (17) from the estimated real values

$$\sigma_{\lambda_r}^2 \approx \left(1-d_r^4\right)\sigma_{\omega_r}^2 + \left(\omega_r\sigma_{d_r}\right)^2\left[\frac{1+d_r^2}{1-d_r^2} - 4d_r^2\right], \text{ and}$$

$$\sigma_{xir}^2 \approx \omega_r^{-1}\left[\sigma_{xoir}^2 + \frac{1}{4}\left|x_{oir}\right|^2\left(\omega_{or}^{-2}\sigma_{\omega or}^2 + \frac{d_r^2}{\left(1-d_r^2\right)^2}\sigma_{d_r}^2\right)\right]$$

If only inertias and stiffnesses have to be identified, MORES works within the undamped model. In this case the residual vector and the weighting matrix are real-valued. With the truncated vector of parameters $\{a\} := \left(a_{21}, .., a_{2n_2}, a_{01}, .., a_{0n_0}\right)^T$ the relations between the modal quantities and the adjustment parameters indicated by Eqs. (1) and (2) are modified to

$$\{q_r\} := \left[-(\omega_{or})^2 A_2 + A_0\right]\{x_{or}\}$$
$$p_r := \{x_{or}\}^T [A_2]\{x_{or}\} - g_r$$

Using the same parameter estimation technique, all the relations are formally equal to the complex case.

4. Program Arrangements

The usual method of handling matrices of increasing order is a transformation to a sparse form with minimal bandwidth (skyline). This cannot be recommended in this case, since we want to preserve the relations between the parameters and the uncertain physical system quantities (stiffnesses, damping values and inertias). Thus, to turn the theoretical aspect into practical utility we have to balance three major subjects: storage request, execution time and numerical precision.

A measure for the problem size is given by the number of reals to establish $[G_r]$ and $[Dv_r]$ from Eq. (5) and Eq. (10), i.e. by doubling the order of the matrices the problem size we define as

$$p_s := 4(n+1)^2 + 2n_p(n+1) \tag{18}$$

which is a rough approach for the program storage request (mtot). As internal pre-processing, MORES calculates the actual precise value of "mtot" and the effective lengths of the necessary fields and records (scratch files).

The execution time is dominated by the n_f inversions indicated by Eq. (10) to calculate the weighting matrices, which are (like the Hessian) in general ill-conditioned. For numerical reasons each matrix is Lambda-shifted [10] before the fast Eskalator-iteration [11] is applied to the associated real-valued symmetric matrix of double order. While the inversion is carried out in single precision format, the result is post-iterated with double precision by Schulz's method [10,11]. Thus, depending on the working (double) precision (eps) of the available hardware, for ill-conditioned matrices the post-iteration will take more time to minimize the equation error. In spite of these techniques the inversion may still fail when the condition number of the matrix increases.

The condition number of a real-valued symmetric positive definite matrix $[A]$ of order n with positive eigenvalues $\lambda(A)$ is usually defined [12] by

$$\text{cond}\,(A) := \frac{\lambda(A)\max}{\lambda(A)\min} \geq 1$$

Its decadic logarithm provides a lower bound for the loss of significant digits due to rounding or truncating errors. Since it would take too much time to evaluate this condition number (i.e., solving an eigenvalue problem), for each weighting matrix and for the Hessian (to verifiy Eq. (14)) the Hadamard condition number [10] is calculated, which is invariant to the scaling of matrices and is maximum for orthogonal (unitary) matrices. Denoting $\{A_i\}$ the ith column vector of $[A]$ the Hadamard condition number is defined by

$$\text{cond}_H(A) := \frac{|det(A)|}{vol(A)} \leq 1 \text{ and } vol(A) := \prod_{i=1}^{n} |A_i|$$

5. Defining The Program Test

For demonstration purposes, MORES was crucially tested by a simulated theoretical model under the following conditions

1. ill-conditioned model matrices,
2. ill-conditioned eigenvalue problem (clustering of eigenvalues),

3. less sensitive parameters to identify,
4. non-proportional (non-modal) damping, i.e., the general complex case,
5. use of all the eigenmodes, i.e. $nf=n$ (maximum problem size),
6. erroneous a priori parameters with deviations up to 50%,
7. erroneous measurement data with various levels of additive noise.

To realize this with a moderate effort, we introduce a vibrator chain, clamped by a spring and a damper element, with n masses, n stiffness elements (springs) and k dampers in the left part (see Fig.1) . For identification we select the first k masses and k stiffness elements and the first damper at the boundary for a total of $2k + 1 = n_p$ adjustment parameters to be estimated. For better comparison the 'true' vector of adjustment parameters we set to $\{\mathring{a}\} = \{e\} = [1,...,1]^T$. The rest of the model is assumed to be exact, a priori, and is therefore represented by the three submatrices $[\acute{A}_l]$ of Eq. (3) and constitutes a part of the parameter independent term $\{K\}$ of the residual vector $\{v\}$ of Eq. (5). To assure increasing clustering of eigenvalues with increasing model order, we define for i=1,..,k,..,n and t=1,..,k (see Fig.1) $m_i := \frac{1}{i}$, $s_i := \frac{1}{i}$, $b_t := \frac{1}{t^2}$.

To evaluate the modal data, the eigenvalue problem formulated in the state space was solved with the software MATLAB on an IBM-PS/2, which provides a double precision of 14 digits (eps $\approx 10^{-14}$). For identification we used a CDC 990-E of the RRZN (Regionales Rechenzentrum Niedersachsen) with double precision eps $\approx 10^{-27}$.

For $(n,k) = (8,3)$ and $(n,k) = (20,7)$ the eigenvalues of the initial model are listed in Table 1. For $(n,k) = (50,17)$, the weighting matrices, Eq. (10), were almost singular ($\mathrm{cond}_H(G_r) \approx 10^{-95}$) and for $(n,k) = (100,36)$ the eigenvalue problem itself was beyond the working precision of the 386 unit. To simulate erroneous data, the eigenvector components and the eigenvalues were disturbed by uniformly distributed additive random errors

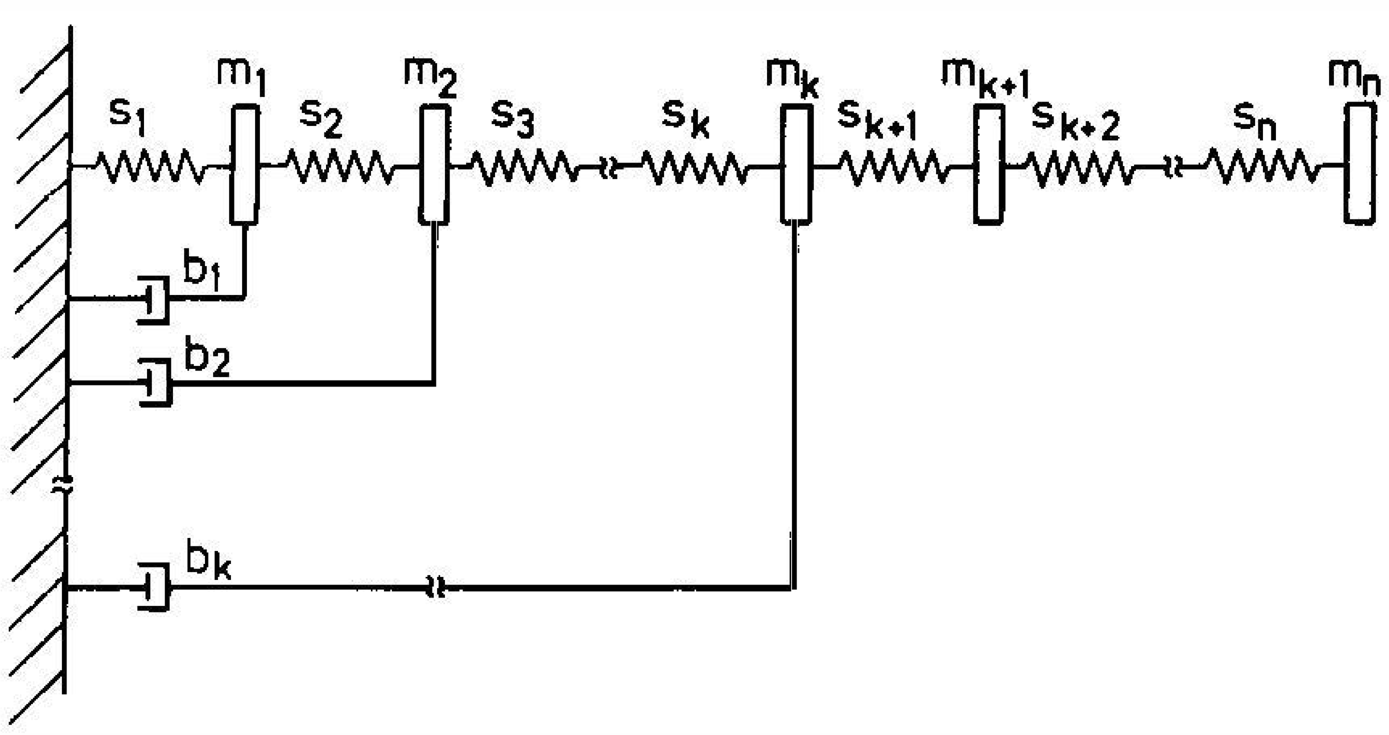

Fig. 1 The first k masses and k spring stiffnesses and the first damper at the boundary of the test model are chosen for a total of $n_p = 2k + 1$ adjustment parameters to be estimated

TABLE 1 EIGENVALUES OF THE TEST MODEL FOR MODEL ORDER $n = 8$ AND $n = 20$ WITH THE 'TRUE' PARAMETER VALUES $\mathring{a} = e$

mode	$n = 8$		$n = 20$	
r	$\Re(\lambda_r)$	$\Im(\lambda_r)$	$\Re(\lambda_r)$	$\Im(\lambda_r)$
1	-0.0185	0.2613	-0.0047	0.1135
2	-0.1390	0.6008	-0.0364	0.2654
3	-0.1674	0.8333	-0.0650	0.3940
4	-0.2323	1.1251	-0.0594	0.5387
5	-0.1326	1.2906	-0.0892	0.6892
6	-0.1460	1.5647	-0.0929	0.7965
7	-0.0714	1.6763	-0.0932	0.9577
8	-0.0095	1.8531	-0.1514	1.0188
9			-0.0946	1.1547
10			-0.1196	1.2346
11			-0.0706	1.3505
12			-0.0940	1.4664
13			-0.0664	1.5280
14			-0.0541	1.6445
15			-0.0791	1.6912
16			-0.0331	1.7681
17			-0.0681	1.8379
18			-0.0188	1.8579
19			-0.0051	1.9154
20			-0.0005	1.9523

$$\Delta x_{ir} = z \cdot c_x \max_{i,r} | \overset{\circ}{x}_{ir} |, \qquad \Delta \lambda_r = z \cdot c_\lambda \left| \Im\left(\overset{\circ}{\lambda}_r \right) \right| \tag{19}$$

where z is a uniformly distributed random number with $E\{z\} = 0, E\{z^2\} = 1$ for various factors $c_x, c_\lambda \in [0,1]$. Thus for the standard deviations of measurement errors we have

$\sigma_{x_{ir}} = c_x \sqrt{2} \max_{i,r} \left| \overset{\circ}{x}_{ir} \right|$, and $\sigma_{\lambda_r} = c_\lambda \sqrt{2} \left| \Im\left(\overset{\circ}{\lambda}_r \right) \right|$, or equivalently as signal-to-noise ratio (s/n in decibels)

$$\mathrm{x}\,\Delta\text{eval} := -20 \log\left(c_\lambda \sqrt{2}\right) \text{ and } \Delta\text{evec} := -20 \log\left(c_x \sqrt{2}\right)$$

For the two cases $(n,k) = (8,3)$ and $(n,k) = (20,7)$, the error behavior of the estimated parameters is investigated statistically by calculating 50 sets of random samples (runs of MORES) for each of two levels of biased model data. After the statistical runs the estimates of the standard deviations σ_M and the estimates of the square roots σ_Q of the quadratic deviations from the 'true' parameters are calculated in addition to the estimates of the mean standard deviations $\sigma_\alpha =: \sigma_H$ from the Hessian Eq. (14).

Since neither errors of the mathematical model structure nor discretization errors are the object of our investigation, the model errors are simulated only by deviations $\{\Delta a\}$ in the 'true' parameters, i.e., $\{a\} = \{e\} + \{\Delta a\}$. For numerical comparison the eigenvalues and eigenvectors of the erroneous models and the updated models (taking the mean estimates) have been calculated, and from this the relative errors

$$\Delta\Re(\lambda_r) := \frac{\Re\left(\lambda_r - \hat{\lambda}_r\right)}{\Re(\lambda_r)} \qquad \Delta\Im(\lambda_r) := \frac{\Im\left(\lambda_r - \hat{\lambda}_r\right)}{\Im(\lambda_r)}$$

and the MAC factors of the eigenvectors [13]

$$M_{ik}\left(\overset{\circ}{X}, \hat{X}\right) := \left(\frac{\left| \overset{\circ}{x}_i^{\dagger} \hat{x}_k \right|}{\left| \overset{\circ}{x}_i \right| \left| \hat{x}_k \right|} \right)^2 \qquad i,k \in \{1,\dots,n\}$$

6. Test Results

For erroneous adjustment parameters $\{a\} = \{a\} + \{\Delta a\}$ the mean values of the parameter estimates are listed in Tables 2a and 3a for $n = 8$ and $n = 20$ respectively. In spite of the particularly large errors (simulated by Eq. (19)) of modal data the parameter estimates update the model.

To verify this, the eigenvalue problem was solved for the erroneous, the estimated and the 'true' adjustment parameters. In Tables 2b and 3b the relative errors of the eigenvalues, and in Tables 2c and 3c the MAC-factors of the eigenvectors, are displayed in rounded percentage. Obviously, the frequencies are fitted better than the real parts of the eigenvalues. The remarkably large errors of the real parts in the upper frequency domain are due to interchanges of eigenvectors. Comparing the MAC-factors of Table 3c, it can be seen that even for biased estimates from highly noised data the interchange of eigenvectors is readjusted. This is possible with the estimation technique presented here, since by the definition of the residual vector there is no direct connection between the estimated and calculated modes. In the case of low noised data the relative errors of eigenvalues for $n = 8$ were below 0.5%, also for the MAC-factors for both model sizes (not listed).

TABLE 2A MEAN VALUES OF THE PARAMETER ESTIMATES AND THEIR STANDARD DEVIATIONS σ_M, THE ESTIMATES σ_Q FOR THE SQUARE ROOTS OF THE QUADRATIC DEVIATIONS OF THE PARAMETER ESTIMATES FROM THE 'TRUE' PARAMETER VALUES AND THE STANDARD DEVIATION OF THE PARAMETER ESTIMATES OBTAINED FROM THE INVERSE HESSIAN MATRIX σ_H

order of model n=8					
parameter	a priori	estimates	standard deviations		
$\{a\}$	$\{e+\Delta a\}$	$\{\hat{a}\}$	σ_M	σ_H	σ_Q
Δ eval ≈ 37 db (c_λ=0.01)	Δ evec ≈ 17 db (c_x=0.1)				
a_{21}	1.5	1.069	0.077	0.041	0.135
a_{22}	0.6	0.877	0.144	0.059	0.151
a_{23}	1.3	0.942	0.049	0.019	0.086
a_{11}	0.5	0.911	0.173	0.077	0.125
a_{01}	1.5	1.095	0.070	0.045	0.142
a_{02}	0.6	0.835	0.187	0.083	0.201
a_{03}	1.3	0.962	0.056	0.021	0.082
Δ eval ≈ 57 db (c_λ=0.001)	Δ evec ≈ 31 db (c_x=0.02)				
a_{21}	1.5	0.999	0.019	0.005	0.029
a_{22}	0.6	0.994	0.030	0.007	0.019
a_{23}	1.3	0.998	0.011	0.002	0.013
a_{11}	0.5	0.997	0.039	0.009	0.022
a_{01}	1.5	1.001	0.017	0.005	0.026
a_{02}	0.6	0.993	0.038	0.010	0.024
a_{03}	1.3	0.999	0.013	0.003	0.017

TABLE 2B THE RELATIVE ERRORS (IN ROUNDED PERCENTAGES) OF THE EIGENVALUES CALCULATED FOR THE TEST MODEL OF ORDER n=8 FOR THE ERRONEOUS PARAMETERS AND THE PARAMETER ESTIMATES.

order of model n = 8				
mode	$\{a\} = \{e + \Delta a\}$		$\{a\} = \{\hat{a}\}$: Δ eval≈37 db Δ evec≈17 db	
r	$\Delta\Re e(\lambda_r)^R$	$\Delta\Im(\lambda_r)^R$	$\Delta\Re(\lambda_r)^R$	$\Delta\Im(\lambda_r)^R$
1	-18	2	-5	1
2	20	7	6	1
3	52	-6	5	-1
4	42	4	5	
5	65	-1	3	
6	81	-4	13	-1
7	-294	-6	-32	
8	32		-24	

The execution time (Table 4) of MORES is mainly caused by the n_f inversions of the weighting matrices which are necessary, since without appropriate, statistically based weighting [2] the obtained parameter estimates are not acceptable. The program version was tested in a non-optimized compiling and execution mode, and because of the rough conditions of the test better results can be expected in normal practice.

7. Conclusions

A model refinement method is presented, which identifies adjustment parameters associated with erroneous substructures by minimizing a cost function of modal residuals. The disadvantage of biased parameters estimates caused by biased modal data is the price one has to pay for the linear and non-iterative technique, since the estimation is done by merely solving an ill-conditioned system of inhomogeneous

TABLE 2C THE MATRICES OF MAC (MODAL ASSURANCE CRITERION)-FACTORS $M_{i,k}$ (IN ROUNDED PERCENTAGES) CONCERNING THE EIGENVECTORS CALCULATED FOR THE TEST MODEL OF ORDER n=8 FOR THE ERRONEOUS PARAMETERS, THE PARAMETER ESTIMATES AND THE 'TRUE' PARAMETER VALUES ($\mathring{a}$=e).

order of model n = 8								
i/k	1	2	3	4	5	6	7	8
$\{a\} = \{e + \Delta a\}$								
1	100	16						
2	9	97	36	5			1	
3		43	89	36	4		4	
4		7	39	88	38	2	12	
5		1	10	49	86	6	14	
6			1	5	24	60	56	1
7				1	4	85	40	8
8						2	5	99
$\{a\} = \{\hat{a}\}$:	Δ eval ≈ 37 db, Δ evec ≈ 17 db							
1	100	10						
2	9	100	53	10	2	1		
3		54	100	44	11	2	1	
4		10	46	100	68	12	5	
5		2	12	68	100	27	8	
6		1	2	13	30	99	71	2
7			1	3	5	73	98	8
8						1	7	100
$\{a\} = \{e\}$								
1	100	9						
2	9	100	56	10	2	1		
3		56	100	45	11	2	1	
4		10	45	100	67	14	3	
5		2	11	67	100	32	6	
6		1	2	14	32	100	64	1
7			1	3	6	64	100	7
8						1	7	100

equations. One advantage of using an equation error as the residual vector is that there is no need for a direct one-to-one correspondence between estimated and calulated modal quantities. Another advantage of this estimator results in the case of phase resonance test where real-valued eigenvectors are estimated: no modeling of the damping matrix is necessary. As an improvement of the method described in [2] the estimated generalized inertias are taken into account additionally. For the realization of this estimation technique by the program MORES it is taken into account that the inverse problem of identification is ill-posed. This is done by special program arrangements, which provides a minimum loss of numerical precision. By statistical simulations it turned out, that even for highly noised data and a pre-given erroneous model the estimator leads to satisfactory results.

TABLE 3A MEAN VALUES OF THE PARAMETER ESTIMATES AND THEIR STANDARD DEVIATIONS (SEE ALSO TABLE 2A)

order of model $n = 20$					
parameter $\{a\}$	a priori $\{e + \Delta a\}$	estimates $\{\hat{a}\}$	standard deviations σ_M	σ_H	σ_R
Δ eval ≈ 37 db Δ evec ≈ 17 db	$(c_\lambda = 0.01)$ $(c_x = 0.1)$				
a_{21}	1.5	1.259	0.122	0.021	0.320
a_{22}	0.6	0.673	0.179	0.029	0.347
a_{23}	1.4	0.830	0.063	0.011	0.194
a_{24}	0.7	0.838	0.085	0.018	0.174
a_{25}	1.3	0.926	0.053	0.012	0.102
a_{26}	0.8	0.936	0.051	0.012	0.076
a_{27}	1.2	1.001	0.035	0.006	0.042
a_{11}	0.5	0.728	0.227	0.038	0.297
a_{01}	1.5	1.344	0.148	0.025	0.413
a_{02}	0.6	0.649	0.192	0.036	0.373
a_{03}	1.4	0.817	0.085	0.015	0.220
a_{04}	0.7	0.792	0.098	0.020	0.221
a_{05}	1.3	0.935	0.056	0.011	0.098
a_{06}	0.8	0.918	0.062	0.013	0.097
a_{07}	1.2	1.009	0.046	0.008	0.055
Δ eval ≈ 57 db Δ evec ≈ 31 db	$(c_\lambda = 0.001)$ $(c_x = 0.02)$				
a_{21}	1.5	1.046	0.049	0.003	0.087
a_{22}	0.6	0.954	0.083	0.005	0.068
a_{23}	1.4	0.969	0.028	0.002	0.050
a_{24}	0.7	0.989	0.041	0.003	0.031
a_{25}	1.3	0.992	0.021	0.002	0.029
a_{26}	0.8	0.999	0.017	0.002	0.014
a_{27}	1.2	0.999	0.009	0.001	0.011
a_{11}	0.5	0.971	0.068	0.006	0.045
a_{01}	1.5	1.062	0.062	0.004	0.113
a_{02}	0.6	0.948	0.094	0.006	0.077
a_{03}	1.4	0.976	0.033	0.002	0.052
a_{04}	0.7	0.980	0.052	0.003	0.041
a_{05}	1.3	0.995	0.024	0.002	0.031
a_{06}	0.8	0.996	0.022	0.002	0.018
a_{07}	1.2	1.003	0.012	0.001	0.015

TABLE 3B THE RELATIVE ERRORS (IN ROUNDED PERCENTAGES) OF THE EIGENVALUES CALCULATED FOR THE TEST MODEL OF ORDER $n = 20$ USING MODAL DATA WITH TWO DIFFERENT SIGNAL-TO-NOISE RATIOS.

order of model $n = 20$						
	$\{a\} = \{e + \Delta a\}$		$\{a\} = \{\hat{a}\}$: Δ eval $\approx$ 37 db Δ evec $\approx$ 17 db		$\{a\} = \{\hat{a}\}$: Δ eval $\approx$ 57 db Δ evec $\approx$ 31 db	
mode	$\Delta\,\Re(\lambda_r)^R$	$\Delta\,\Im(\lambda_r)^R$	$\Delta\,\Re(\lambda_r)^R$	$\Delta\,\Im(\lambda_r)^R$	$\Delta\,\Re(\lambda_r)^R$	$\Delta\,\Im(\lambda_r)^R$
1	-16	1	-27	2	-2	
2	-12	3	-14	2	-1	
3	5	3	-4	2	1	
4	-5	3	-7		2	
5	25	6	1	1	2	
6	36	1	9		2	
7	44	5	7	2		
8	81	-3	40	-3	9	-1
9	-58	5	-49	-1	2	-1
10	87	2	43	2	-5	
11	84	-1	36	-1	6	
12	89	-3	51	-2	10	
13	80	-7	-166	-3	-8	
14	65	-6	-14	1	1	
15	-276	-5	-5	1	-8	
16	-200	-2	2		-2	
17	79	-1	-3	-1		
18	-474	-2	-10		-2	
19	7		-13		-1	
20			-14		-2	

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TABLE 3C THE MATRICES OF MAC-FACTORS $M_{i,k}$ (IN ROUNDED PERCENTAGES) CONCERNING THE EIGENVECTORS CALCULATED FOR THE TEST MODEL OF ORDER $n = 20$ FOR THE ERRONEOUS PARAMETERS, THE PARAMETER ESTIMATES AND THE 'TRUE' PARAMETER VALUES

				order of model $n = 20$					
i/k	10	11	12	13	14	15	16	17	18
$\{a\} = \{e + \Delta a\}$									
10	46	14	2	1		5	2		
11	17	67	7	1		4	2		
12	3	23	42	7	2	15	10		2
13	1	6	63	19	3	12	11		2
14		1	7	74	17	13	29	2	7
15		1	3	35	41	21	61	5	19
16			1	3	84	5	55	15	20
17				2	17	5	55	47	77
18				1	3	1	20	93	28
$\{a\} = \{\hat{a}\}$:	Δ eval ≈ 37 db		Δ evec ≈ 17 db						
10	87	35	10	6	2	1			
11	32	95	18	21	4	3			
12	8	40	59	80	22	18	2	1	
13	3	12	59	87	42	29	3	2	
14		2	40	13	98	88	16	7	2
15		1	40	8	74	96	44	20	5
16			6	1	12	28	99	36	14
17			3	1	6	14	36	99	59
18			1		1	3	9	56	99
$\{a\} = \{e\}$									
10	100	51	18	6	1	1			
11	51	100	48	16	3	2			
12	18	48	100	76	15	12	2	1	
13	6	16	76	100	32	20	2	1	
14	1	3	15	32	100	78	14	7	2
15	1	2	12	20	78	100	40	20	4
16			2	2	14	40	100	41	11
17			1	1	7	20	41	100	54
18					2	4	11	54	100

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TABLE 4 STORAGE IN WORDS (8 BYTES) AND EXECUTION TIME MORES PROGRAM REQUESTS FOR THE TEST MODEL DEPENDING ON INCREASING PROBLEM SIZE $p_s = 4(n+1)^2+2n_p(n+1)$.

order of model n	8	20	50
number of modes n_f	8	20	50
number of parameters n_p	7	15	35
problem size p_s	450	2394	13974
mean Hadamard condition of $[G_n]$	10^{-11}	10^{-35}	10^{-95}
mean Hadamard condition of $[D^2J]$	10^{-4}	10^{-12}	10^{-38}
total field length mtot	5473	29593	174033
maximum record length	64	400	2500
maximum scratch volume	448	600	87500
cpu seconds	0.7	21.3	1190.8