The International Journal of Analytical and Experimental Modal Analysis v 7 n 3 p 169-178 Jul 1992

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ABSTRACT

In conventional modal testing, accelerometers are used to sense structural response data which is processed to obtain the natural frequencies, damping and mode shapes of the structure under test. In the case of light-weight structures like composites where mass loading and other local effects of these transducers are not negligible, optical instruments like the laser doppler vibrometer (LDV) are used. The availability of real-time scanning LDV's has introduced many interesting measurement possibilities. By applying a time-domain sorting algorithm, we have recently demonstrated the use of such a scanning LDV to simulate multiple discrete sensors distributed over the test structure. In the method developed in this paper, we process the scanning LDV velocity output signal in the frequency domain to obtain directly the deflection shape of the vibrating structure in a functional (series) form. The technique is illustrated by measuring the second mode shape of a light-weight cantilever beam. A discussion of the limitations of the method and comparisons with theoretical predictions are also included.

List of Symbols

A_j	Chebyshev coefficients of $\phi(x)$
B_j	Chebyshev coefficients of $\psi(x)$
T_k	k^{th} Chebyshev polynomial
t	time coordinate
$V(t)$	scanning LDV velocity output
$v(x,t)$	space-time distribution of velocity
$w(x)$	weighting function in error integral
x	spatial coordinate along beam

Greek Symbols

α	relative phase between velocity samples
$\phi(x)$	spatial velocity distribution function (sine component)
$\psi(x)$	spatial velocity distribution function (cosine component)
ω_b	structure (beam) vibration frequency
ω_m	LDV scanning frequency

1. Introduction

The design criterion for many structural elements is based on performance under dynamic conditions and thus, accurate dynamic characterization of such structures is of paramount importance. The standard approach is to predict the response based on a mathematical model, typically a finite element description.

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An earlier version of this paper was to have been presented at the 9th International Modal analysis Conference, Florence, Italy, April 1991.

Final manuscript received: March 19, 1992

Frequently, the model is required to be validated and this function is accomplished through an iterative interaction with experiments. The use of such an approach allows the designer to observe the effects of potentially destructive loads without destroying the structure itself. One of the building blocks in this technique is the ability to measure the dynamic characteristics of structures quickly and accurately.

In conventional vibration testing, accelerometers are commonly used to sense the acceleration time-histories at selected locations. The acceleration (response) and the load (excitation) signals are processed using various analysis techniques[1], usually involving digital signal processing, to estimate the system dynamic characteristics. The dynamic characteristics of the system are usually described by the eigenvalues (natural frequencies and damping ratios) and corresponding eigenvectors (mode shapes) obtained through such analyses. The natural frequencies and damping ratios are obtained through some type of curve-fitting procedure. The mode shape information is obtained by using multiple exciters and/or sensors or by successively moving the exciters and/or the sensors to different points in the structure. In the case where there is little interaction between the modes, the structure can simply be excited at a selected resonance frequency and the resulting response vector used as the approximate mode shape. A similar technique can be used even in the presence of significant modal interaction by using multiple exciters and special control logic. There are many sources for error, and in testing light-weight structures, the mass loading of the response sensors can often distort the measurement so that one has to seek alternate non-contact sensors like the laser Doppler vibrometer (LDV).

The LDV is based on the measurement of the Doppler shift of the frequency of laser light scattered by a moving object. The magnitude of the Doppler shift is related to the optical geometry and the velocity of the scattering object. Application of the LDV to the field of vibration measurements in solids was initially in the context of rotating systems[2]. Subsequently, the technique has attained popularity in special situations requiring the use of a non-contacting optical sensor such as in biomedical vibration analysis[3,4]. The instrument is perceived to be useful enough that now there are several commercial LDV systems available for vibration measurements in solids, including a portable model based on a novel frequency shifting scheme[5].

The measurement of vibration mode shapes requires the sensing of structural response at a series of locations on the structure. When using an LDV system, the straightforward solution is to translate the test object or the complete LDV system so that various points of interest can be probed. While this is simple for small test objects and small LDV systems, it is not always convenient. Various modifications have been introduced into LDV optical systems to make multi-point measurements easier, including the use of a fiber optic link which allows most of the LDV components and the test object to remain fixed while only a few components have to be moved. Such a system has been demonstrated for the measurements of vibration frequency and velocity amplitude[6]. An alternate approach introduced by Bendick[7] involved the translation of a single mirror to move the probe area along the optical axis of the LDV. There are also commercial LDV systems specifically oriented towards vibration measurements wherein the traverse is automated[†].

There are two main types of LDV, namely, the reference or single-beam and differential or dual-beam arrangements[8]. A single-beam LDV measures velocities along the line of sight and introduction of scanning causes a response due to the varying distance between the surface and the sensor. Most commercial LDV systems intended for structural applications are of this type, apparently, since scanning during measurements is not anticipated. On the other hand, a two beam arrangement is sensitive only to transverse velocities in the plane of the beams and is, therefore, free of this range sensitivity effect. The following discussion assumes the use of a dual-beam LDV. For measurements on opaque solid surfaces, the LDV beams have to impinge on the surface and this requires the optical axis of the system to be at an angle to the surface. Then, axial motion of the probe volume is of limited use since the probe volume will

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[†]*The Ometron Vibration Pattern Imager VPI 9000, Ometron Inc., Herndon VA 22070 and Polytec OFV-050, Polytec Optronics Inc., Costa Mesa CA 92626.*

have to be accommodated either inside the solid (which is impossible since the object is opaque) or away from the surface (in which case the velocity measured is not the solid surface velocity). The surface can be scanned only by moving the LDV probe transverse to the optical axis or in a combination of axial and transverse motion. A transverse scanning technique has been devised by Durst, et al [9] with scanning motion derived from an oscillating mirror in the laser beam path. The scanning rates were limited in this system by inertial effects in the mirror to about 15 Hz (lines/sec) for scan angles of about 10°. Chehroudi and Simpson[10] have improved upon the concept by incorporating a commercial scanning device and strip mirrors to obtain scan rates up to 150 Hz over linear scan lengths of about 400 mm.

A scanning LDV can cause errors in fluid flow measurements where the scattered light signal arrives in intermittent bursts corresponding to particles crossing the LDV probe volume[9]. In the case of solid surfaces, light is scattered continuously as long as the probe volume intersects the scattering surface, and thus the Doppler signal is continuous, providing for good spatial resolution. Then, a scanning system can be used effectively on vibrating surfaces to map the spatial velocity distribution (mode shapes) accurately. The use of digital signal processing techniques means that the velocity at a given point is required only during the sampling intervals. Between samples, the sensor system is idle. A scanning LDV can make use of this idle time to measure the response at other spatial locations. In previous work[11], we addressed this possibility and constructed a scanning LDV system that accomplishes this task of measuring the response at multiple locations in an interlaced fashion within the sampling interval of the signal processing system. One of the problems encountered therein was the inability to use anti-aliasing filters due to interaction with the spatial scanning. In this paper we develop an alternate approach that harnesses this interaction to infer the spatial velocity distribution. The theory of Chebyshev demodulation, as this approach is called, is outlined and test data from a light-weight cantilever beam excited at its second bending mode is presented for verification.

2. Experimental Setup and Procedure

The experimental setup will be described only briefly here since a detailed description has been presented previously[11]. A TSI 9100-7 high-power two velocity component LDV operated in a single velocity component measurement mode was used. The single component mode was enabled by operating the laser in a single-mode configuration radiating at a 514.5 nm wavelength (green). The actual laser power output that was used during the tests was in the range of 10-20 mW, which was sufficient to produce good signals from the vibrating beam. Scanning capability was added to the LDV by directing the emerging laser beams onto a scanning mechanism, as illustrated in Fig. 1. In the figure, the beam is assumed to vibrate in and out of the plane of the paper, matching the sensitivity direction of the LDV. The scanning mechanism consisted of a pivoted front-surface mirror driven through a lever arm by an electrodynamic shaker. The lever arm arrangement converted the rectilinear motion of the shaker head into angular motion of the mirror. A mirror position sensing transducer was also built into the mechanism though it was not used for the tests reported here.

Using the above described configuration, the mirror scanner can be placed on a mounting independent of the rest of the system. Thus, vibrations due to the oscillating mirror can be isolated from the rest of the optics as well as the test structure. The working distance from the scanning mirror to the test object was 1.0 m. The optics limited the scan to a ±7.5° range due to defocussing effects. The maximum scan rate was restricted by inertial effects in the mirror to about 150 Hz for the full range scan, with higher rates attainable for shorter scans. These range and rate limitations can be readily overcome using correcting lenses and/or commercial scanning devices.

The test structure was an acrylic plastic rectangular bar of cross section 26 mm x 2.8 mm. One end of the bar was clamped between two aluminum blocks to produce a cantilever beam of length 130 mm. The

density of the beam material was so low (86.0 g/m linear mass density) that even a sub-miniature accelerometer (1 g typical mass) caused significant mass loading. The beam was driven at its tip by a B&K 4810 exciter and the force input was measured with a PCB 208A02 load cell. The excitation force, scanning mirror position, and velocity output signal from the scanning LDV were fed into a Preston analog-to-digital converter system controlled by an HP 1000 minicomputer. The overall instrument response time was measured to be in the range 200 - 250 μsec. The velocity measurement resolution and range were limited by the LDV electronics to 0.0004 m/sec and ±0.5 m/sec respectively. In order to provide a stringent test of the measurement system and to illustrate the effect of errors, the excitation level was set to provide only about 0.01 m/sec peak response or 2% of the full-scale reading. In terms of the more common acceleration units, the measurement resolution and range were 0.05 g and ±80 g, respectively, while the peak response signal was about 1.6 g, all assuming a vibration frequency of 255 Hz.

Initially, the tip response was sensed with the LDV operating in a non-scanning mode and the load and response signals were routed to a GENRAD 2515 modal analysis system. The first two natural (bending) modes were estimated to be at 23.6 Hz and 255 Hz, with damping ratios of about 0.08. These frequencies may appear to be excessively separated considering the fact that for a cantilever beam the ratio of the first two natural frequencies should be about 6.3. However, the test beam had an accelerometer (PCB type 303A) mounted near the tip. It can be shown that for a cantilever beam with such a tip mass, the spacing between the first two natural frequencies increases significantly[12], a trend consistent with the measured data. The well separated natural frequencies with low damping suggested that the modes could be considered to be non-interacting. Thus, the deflection shape of the beam at resonance could be used to approximate the corresponding mode shape. In order to sense this approximate mode shape, the beam was excited using a 255 Hz sine wave while the LDV was configured to scan the entire length of the beam. The scan rate was set in accordance with the data analysis requirements as described subsequently. If the LDV were operated in a non-scanning mode, the velocity signal would also be a 255 Hz sine wave with an amplitude corresponding to the deflection magnitude at the measurement location. However, with the scanning action, the measurement location is not fixed and, thus, the velocity signal is multiplexed or modulated by the spatial deflection distribution. In the following section, we describe a method of demodulating this multiplexed signal to obtain the spatial velocity distribution.

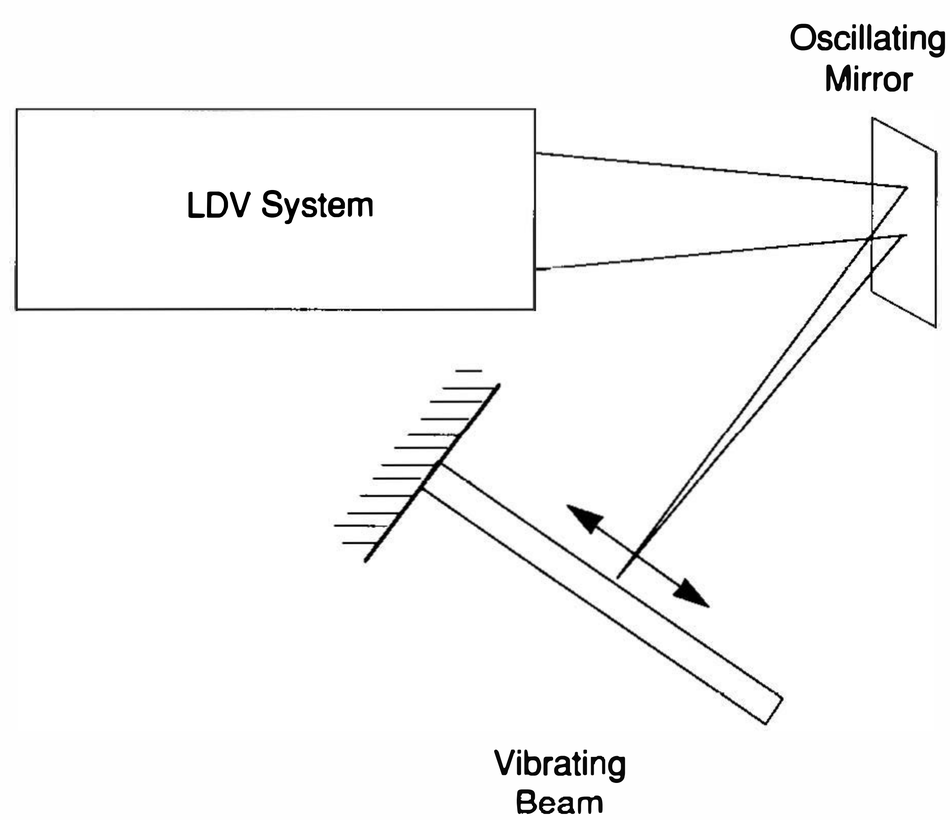

Fig. 1 Scanning LDV schematic

3. Theory of [illegible]

It is assumed for convenience that a beam-like vibrating structure is situated in the region $-1 \leq x \leq 1$. Even in the case of complex three-dimensional structures, the method applies in the domain represented by the scan region. The structure is assumed to be excited at one of its resonance frequencies, ω_b. The velocity distribution within the domain of interested is written as

$$v(x,t) = \phi(x)\sin\omega_b t + \psi(x)\cos\omega_b t \tag{1}$$

where $\phi(x)$ and $\psi(x)$ are mutually orthogonal components of the vibratory response velocity distribution, allowing for the general case of complex mode shapes. If the mode shape under examination was real, these two functions would differ only by a scale factor and the equation can be written using a single harmonic term. The case where there is a non-oscillatory velocity field (for example, as in a rotating system) can be dealt with in a similar fashion and this has been reported on elsewhere [13]. The first measurement is arbitrarily assigned as the reference, implying a zero phase. Subsequent samples of the velocity distribution have to include an unknown phase difference in the argument. This problem will be dealt with subsequently. The problem posed now is to be able to detect $\phi(x)$ and $\psi(x)$. A non-scanning LDV senses velocity at a given point (x = constant) where the LDV beams are incident on the structure. These beams are now assumed to be scanned in a controlled manner by a normalized scan function of the form

$$x = \cos\omega_m t \tag{2}$$

where ω_m is assumed as the scan rate. At any instant of time t, the scanning LDV output corresponds to the velocity at the position x given by Eq. 2. This output, denoted $V(t)$, is

$$V(t) = \phi(\cos\omega_m t)\sin\omega_b t + \psi(\cos\omega_m t)\cos\omega_b t \tag{3}$$

The functionals ϕ and ψ in this equation are even and periodic by virtue of the $\cos\omega_m t$ argument. Hence, these can be expanded in their Fourier cosine series as

$$\phi(\cos\omega_m t) = A_0 + \sum_{k=1}^{\infty} A_k \cos k\omega_m t \tag{4}$$

$$\psi(\cos\omega_m t) = B_0 + \sum_{k=1}^{\infty} B_k \cos k\omega_m t \tag{5}$$

Then, $$V(t) = \left(A_0 + \sum_{k=1}^{\infty} A_k \cos k\omega_m t\right)\sin\omega_b t + \left(B_0 + \sum_{k=1}^{\infty} B_k \cos k\omega_m t\right)\cos\omega_b t \tag{6}$$

By writing the trigonometric products in terms of sums, we obtain

$$\begin{aligned} \mathrm{V(t)} = {} & A_0 \sin\omega_b t + \sum_{k=1}^{\infty}\frac{A_k}{2}\sin(\omega_b + k\omega_m)t + \sum_{k=1}^{\infty}\frac{A_k}{2}\sin(\omega_b - k\omega_m)t \\ & + B_0\cos\omega_b t + \sum_{k=1}^{\infty}\frac{B_k}{2}\cos(\omega_b + k\omega_m)t + \sum_{k=1}^{\infty}\frac{B_k}{2}\cos(\omega_b - k\omega_m)t \end{aligned} \tag{7}$$

It is to be noted that Eqs. 4 and 5 can be written as

$$\phi(x) = A_0 + \sum_{k=1}^{\infty} A_k \cos\left(k \cos^{-1} x\right) \tag{8}$$

$$\psi(x) = B_0 + \sum_{k=1}^{\infty} B_k \cos\left(k \cos^{-1} x\right) \tag{9}$$

Recalling the definition of the Chebyshev polynomials T_k

$$T_k(x) = \cos\left(k \cos^{-1} x\right) \tag{10}$$

it can be recognized that Eqs. 8 and 9 represent the Chebyshev series expansions for $\phi(x)$ and $\psi(x)$. The quantities A_k and B_k are the Chebyshev series coefficients. If a finite number of these coefficients are available, a Chebyshev approximation to the velocity distribution is obtained. A Fourier transform of $V(t)$ will exhibit peaks at the frequencies ($\omega_b \pm k\omega_m$). Measurement of the complex amplitudes at these frequencies will, therefore, yield the A_k and B_k coefficients which determine the spatial distributions $\phi(x)$ and $\psi(x)$, and hence $v(x,t)$. The location of these peaks in the frequency domain is illustrated in Fig. 2.

If discrete Fourier analysis is used, it is essential to perform the Fourier transform such that the spectral lines of interest occur on the analysis spectral lines so that the required amplitudes are estimated accurately. This can be readily accomplished in practice through a frequency matching scheme, namely, by setting the scan rate ω_m to a 1/(n+.5) fraction of the excitation frequency ω_b where (n+1) terms in the approximation are sought and having an integral number of analysis spectral lines in a bandwidth of ω_m. An additional advantage of this choice is that apparent velocity signals due to vibration of the scanning stage (which occur at the harmonics of ω_m) can be readily isolated from the true beam vibration signals. The need to isolate and suppress such apparent velocity signals severely restricts usage of windowing techniques. The frequency matching scheme will be numerically illustrated while discussing the test results.

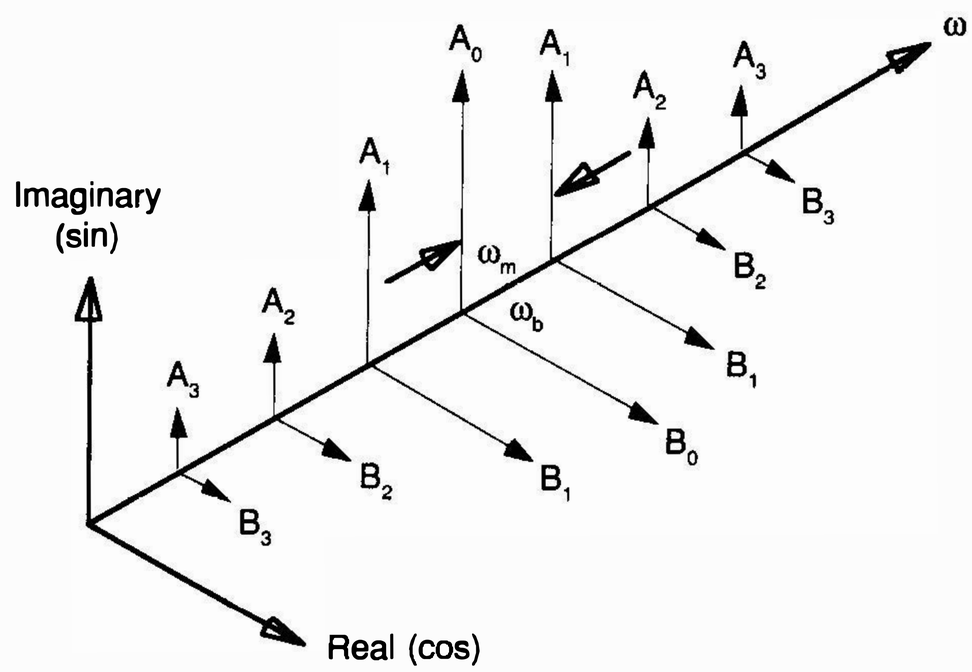

Fig. 2 Frequency components of scanning LDV signal

The foregoing procedure can be used to obtain a single estimate of $\phi(x)$ and $\psi(x)$. In order to obtain statistically significant results, many such estimates have to be averaged. This is not straightforward due to the unknown phase difference between samples. The following error integral minimization procedure was used to account for this. Consider two samples separated by an unknown phase difference α. Using subscripts to denote successive samples and dropping out the functional arguments,we have

$$v_1 = \phi_1 \sin \omega_b t + \psi_2 \cos \omega_b t \tag{11}$$

$$v_2 = \phi_2 \sin(\omega_b t + \alpha) + \psi_2 \cos(\omega_b t + \alpha) \tag{12}$$

Now, v_2 can be rotated to the same phase as v_1 by using

$$v_2 = \phi_2 * \sin \omega_b t + \psi_2 * \cos \omega_b t \tag{13}$$

where

$$\phi_2 * = \phi_2 \cos \alpha - \psi_2 \sin \alpha \tag{14}$$

and

$$\psi_2 * = \psi_2 \cos \alpha + \phi_2 \sin \alpha \tag{15}$$

The rotated v_2 can be directly averaged with v_1 and a similar process used for each sample. The task is to find a suitable α which is accomplished as follows. We choose α such that the integrated error between v_1 and v_2 is a minimum. The integrated error is expressed as

$$\varepsilon_{12} = \int_{-1}^{1} (\phi_1 - \phi_2 \cos \alpha + \psi_2 \sin \alpha)^2 dx + \int_{-1}^{1} (\psi_1 - \psi_2 \cos \alpha - \phi_2 \sin \alpha)^2 dx \tag{16}$$

By setting the first derivative of this integral with respect to α to be zero, we obtain

$$\tan \alpha = \frac{\int (\phi_2 \psi_1 - \phi_1 \psi_2) dx}{\int (\phi_1 \phi_2 + \psi_1 \psi_2) dx} \tag{17}$$

where the limits of integration are (-1,1). This leads to two principal values for α which differ by 180°. From the nature of the problem, one of these corresponds to the minimum error and the other to maximum error. The value of α corresponding to the minimum is easily determined by evaluating the second derivative of the error integral. The integrals can be simplified if a weighting function $w(x)$ is used while evaluating the error integrals. For example, if we set

$$w(x) = \frac{1}{\sqrt{1 - x^2}} \tag{18}$$

the integrals in Eq. 17 degenerate into summations of products of Chebyshev coefficients of ϕ and ψ (due to the orthogonality property of Chebyshev polynomials).

4. Test Results and Discussion

The second mode of the test cantilever beam was excited using a 255Hz sine wave. A simple analysis showed that a five-term Chebyshev approximation to the second mode shape would have less than 1% error and this was set as the target. Thus, the scan frequency ω_m was set to 255/4.5 = 56.67 Hz. Choosing to have six lines in the ω_m bandwidth leads to a required spectral resolution Δf of 9.444 Hz. For a block length of 256 samples, then, the required sampling rate is half the number of samples times Δf or 1208.9 Hz. The sampling rate was set accordingly. Under these conditions, the first Chebyshev terms were at ω_b or the 27th spectral line with the successive terms at ($\omega_b \pm k\omega_m$) falling on the 21st and 33rd lines (for k=1), 15th and 39th lines (k=2), 9th and 45th lines (k=3) and 3rd and 51st lines (k=4). The scan harmonics were at the multiples of ω_m, i.e., the 6th, 12th, 18th... lines, clearly isolated from the required Chebyshev terms.

Initial tests showed that the fidelity of the test data depended to a large extent on the accuracy and stability of the frequency ratios. Ideally, one would solve the problem by deriving all signals from a common clock source. However, it was found that two signal generators could be tuned by hand with sufficient accuracy while observing one waveform on an oscilloscope triggered by the other. This was the method used.

In order to demonstrate the overall validity of the analysis technique, the auto-spectrum of the scanning LDV output was estimated using ten samples. This is presented in Fig. 3. A wideband spectral noise floor of about 0.05 mm/sec is clearly evident in the figure along with several other spectral peaks. The highest peak can be easily located at 255 Hz, corresponding to the first terms in the Chebyshev series. Further terms in the series are also visible in the figure, along with several scan harmonics (especially the first one at 56.67 Hz), though it is somewhat challenging to identify the peaks without the aid of a clear frequency scale. In order to easily compare the test results with theoretical predictions, the mode shape was assumed to be real and thus corresponding ϕ and ψ terms were combined to obtain the experimental mode shape. Both theoretical and experimentally obtained modes shapes were normalized with respect to the corresponding tip deflections.

A close inspection of the data in Fig. 3 reveals that the fourth term is barely above the noise floor while the fifth term is virtually indistinguishable. Thus, one can attach a fair degree of confidence to a four-term approximation whereas a five-term approximation is bound to be contaminated. In Fig. 4, the experimentally obtained three-term approximation is presented along with the theoretical second mode shape of the beam.

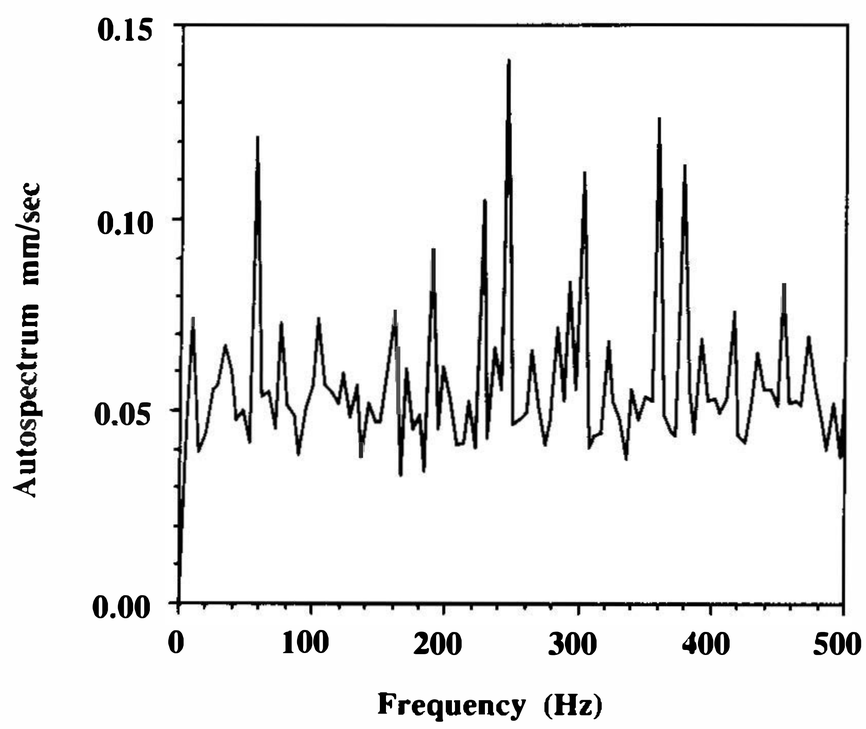

Fig. 3 Auto-spectrum of scanning LDV output signal

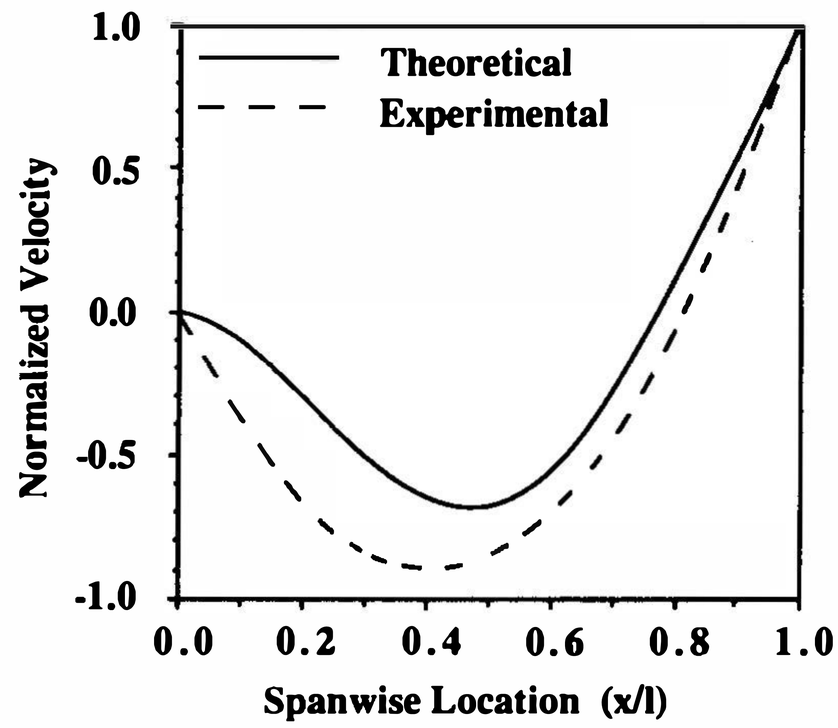

Fig. 4 Three-term approximation of beam mode shape

Clearly, the experimental data is inadequate. However, it is interesting to note that a three-term Chebyshev approximation of the theoretical mode shape has an 8% rms error while the corresponding experimental value is 24%. This is close enough to be encouraging, considering the fact that the signal levels were only around two percent of the full-scale values.

The four-term experimental approximation is compared with the theoretical mode shape in Fig. 5. Good overall agreement is observed, validating the measurement technique. The effect of using noise contaminated terms is illustrated in Fig. 6, where the experimental five-term value is compared with the theoretical mode shape. The agreement is significantly poorer than in the case of the four-term approximation, testifying to the danger of using contaminated data. Mutual comparisons between the three-, four- and five-term data also suggest a simple test to determine when contamination has occurred. While the four-term approximation is relatively smooth (similar to the three-term value), the addition of the fifth term does not follow the trend, indicating possible errors.

5. Concluding Remarks

In this paper, we have shown the feasibility of using a scanning Laser Doppler Velocimeter to measure the vibrating mode shape of a structural dynamic system. The technique of simultaneously gathering vibration velocity information from multiple locations along the scan has been demonstrated. This makes it possible to obtain the velocity from multiple points while using only a single sensor system. In particular, the new data processing technique allows measurement of a Chebyshev approximation to the velocities of a structural dynamic system modeled as a distributed parameter system. While not the subject of the present paper, the basic scanning technique can also be used to simulate a finite number of discrete transducers[11]. The scanning LDV provides a measuring tool that is non-contacting and adds no additional mass to a light-weight structure. The modal data extracted from measurements using the scanning LDV exhibit good agreement with theoretical predictions. Additional work is necessary to exploit the benefits of this development and to apply it to other important practical problems like separation of interacting modes and non-harmonic or random vibrations.

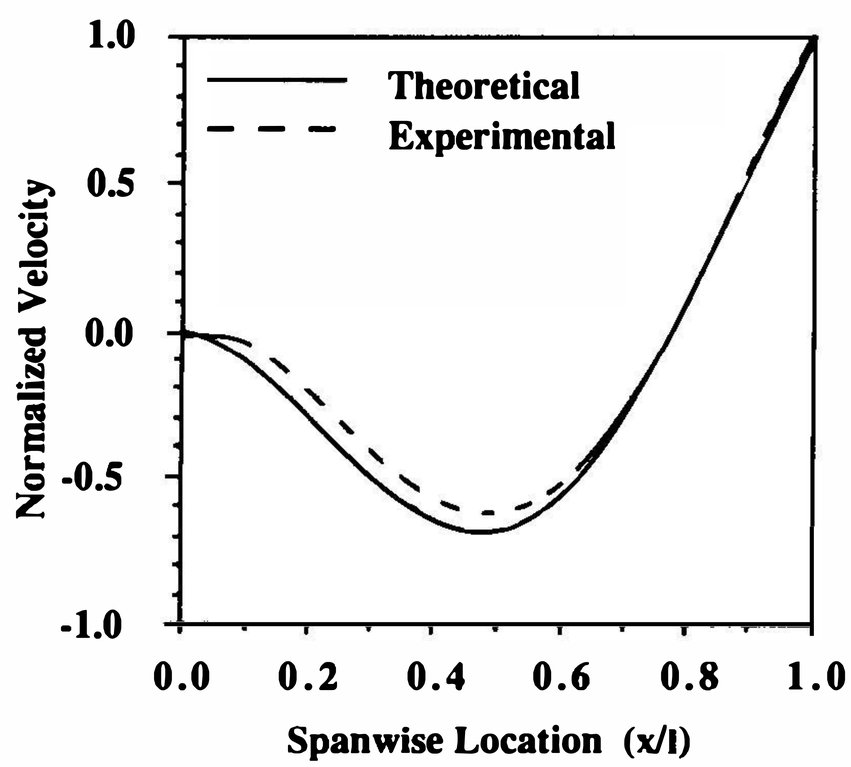

Fig. 5 Four-term approximation of beam mode shape

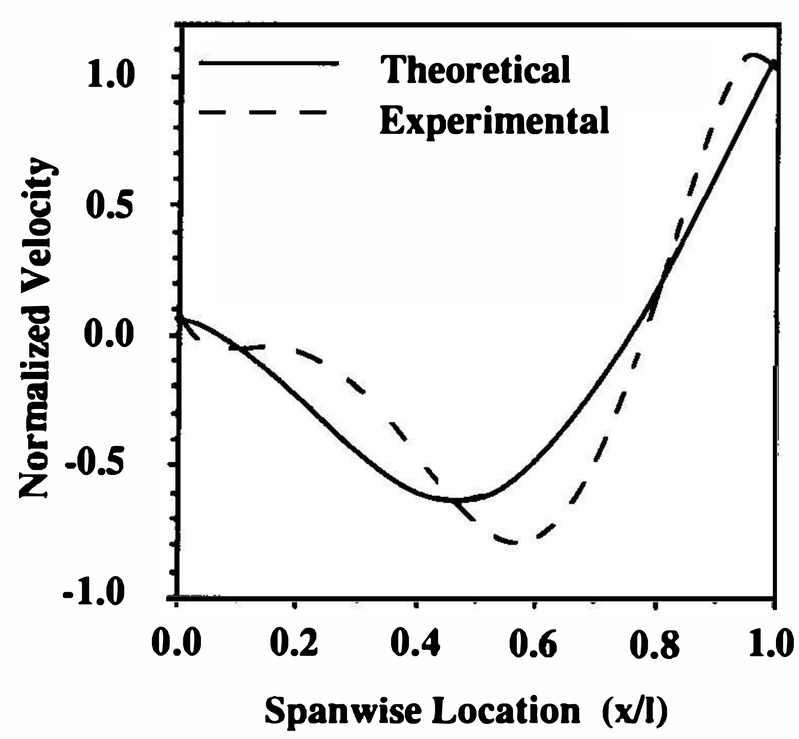

Fig. 6 Five-term approximation of beam mode shape

Acknowledgments

The authors gratefully acknowledge the support received for this research from the US Army Research Office as part of the Georgia Tech Center of Excellence for Rotary Wing Aircraft Technology (GT-CERWAT). The authors would also like to thank Dr. N. M. Komerath and Dr. S. G. Liou for their suggestions and assistance in using the LDV system.

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