Studies in Nonlinear Dynamics and Econometrics

Quarterly Journal
October 1996, Volume 1, Number 3
The MIT Press

Studies in Nonlinear Dynamics and Econometrics (ISSN 1081-1826) is a quarterly journal published electronically on the Internet by The MIT Press, Cambridge, Massachusetts, 02142. Subscriptions and address changes should be addressed to MIT Press Journals, 55 Hayward Street, Cambridge, MA 02142; (617)253-2889; e-mail: journals-orders@mit.edu. Subscription rates are: Individuals $40.00, Institutions $130.00. Canadians add additional 7% GST. Prices subject to change without notice.

Detecting Asymmetries in Observed Linear Time Series and Unobserved Disturbances

Jeong-Ryeol Kim
Stefan Mittnik
Institute of Statistics and Econometrics
Christian Albrechts University at Kiel
Olshausenstr. 40, D-24098 Kiel, Germany
kim@stat-econ.uni-kiel.de
mittnik@stat-econ.uni-kiel.de

Svetlozar T. Rachev
Department of Statistics and Applied Probability
University of California
Santa Barbara, CA 93106-3110, USA
rachev@pstat.ucsb.edu

Abstract. *This paper investigates the problem of testing for the symmetry of linear time series driven by asymmetric innovations. In particular, we examine the performance of alternative symmetry tests when innovations are fat tailed. Among the tests considered, only the test based on the tail estimator of the spectral measure yields satisfactory results in the presence of fat-tailed innovations.*

Keywords. Symmetry, Multivariate symmetry, Symmetry test, Characteristic symmetry function, Spectral measure

Acknowledgments. We thank an anonymous referee for comments that helped to improve the paper. The research of the first two authors was supported by the Deutsche Forschungsgemeinschaft. It was initiated while the third author visited the Christian Albrechts University at Kiel with support from the Alexander-von-Humboldt Foundation for US Scientists and concluded when the first two authors visited University of California-Santa Barbara.

1 Introduction

Recent empirical studies suggest that economic variables, such as output and unemployment, behave asymmetrically during the stages of a business cycle in the sense that expansions are more persistent but less sharp than recessions. See, for example, Mittnik and Niu (1994) for a discussion of the empirical evidence on asymmetry in economic time series. These observations have been at least partially responsible for the growing interest in nonlinear econometrics, because nonlinear data-generating processes (DGPs) can transform symmetric white noise into asymmetric time series.

Table 1
Classification of Data-Generating Processes

DGP Category	Iid Innovation Process $\{\epsilon_t\}$	Transmission Mechanism $f(\cdot)$	Output Process $\{y_t\}$
1	symmetric	linear	symmetric
2	symmetric	nonlinear	asymmetric
3	asymmetric	linear	asymmetric
4	asymmetric	nonlinear	asymmetric

Nonlinear DGPs are, however, not the only possible explanation for asymmetries in observed variables. Letting the variable of interest, y_t, be generated by the DGP

$$y_t = f(\epsilon_{t-1}, \epsilon_{t-2}, \ldots) + \epsilon_t$$

the characteristics of the *output process*, $\{y_t\}$, depend on the properties of both the *transmission mechanism*, $f(\cdot)$, and the iid *innovation process*, $\{\epsilon_t\}$. Distinguishing, as in Mittnik and Niu (1994), between DGPs with linear and nonlinear transmission mechanisms and symmetric and asymmetric innovation processes, DGPs may be classified into the four categories summarized in Table 1. If the innovation process is symmetric and the transmission mechanism is linear, i.e., category 1 in Table 1, the output of the DGP will be symmetric. In all other cases, the observed output is potentially asymmetric.

Traditionally, econometric modeling has been largely confined to category 1, and has therefore excluded the possibility of asymmetries. Only in recent years have econometricians begun paying more attention to DGPs of category 2, i.e., symmetric innovation processes combined with nonlinear transmission mechanisms. Models that belong to this category are, for example, bilinear models (Granger and Anderson 1978), Markov-switching models (Hamilton 1989), threshold autoregressions (Tong 1990), and smooth-transition autoregressions (Teräsvirta 1994). However, most of the empirical work focuses on testing whether or not the observed data are generated by a nonlinear transmission mechanism.

The purpose of this study is to investigate the power of symmetry tests when the DGP belongs to category 3, i.e., when innovation processes are asymmetric and transmission mechanisms are linear. This includes particular shapes of asymmetries, as defined by some authors.[1] Here, we are not concerned with detecting particular shapes of asymmetries; rather, we are interested in detecting the presence of asymmetry as such. Once an asymmetry has been identified, one can conduct further investigations to specify its particular shape.

Specifically, we consider four symmetry tests: the Neftci (1984) test; a test employed by DeLong and Summers (1986) that is based on the one-dimensional skewness statistic, and includes a multidimensional version; the HRC test, which is based on the characteristic symmetry function proposed by Heathcote, Rachev, and Cheng (1995); and the RMK test, based on the multidimensional spectral measure for stable distributions proposed by Rachev, Mittnik, and Kim (1996). The power of these tests is investigated using Monte Carlo simulations. In these simulations, we use linear autoregressive transmission mechanisms and independently and identically distributed stable Paretian or, in short, iid α-stable innovation processes.

The class of α-stable distributions represents a generalization of the normal distribution in the sense that it allows for both asymmetries and fat tails. The latter is of particular relevance in financial modeling; see, for example, the discussions in (Mittnik and Rachev 1993) and (McCulloch 1994). In addition to a location and scale parameter, the α-stable distribution is characterized by the tail-index or shape parameter $\alpha \in (0, 2]$ and the skewness parameter $\beta \in [-1, 1]$. If $\alpha = 2$, the α-stable distribution coincides with the normal distribution; if $\alpha \in (0, 2)$, it is fat tailed and has only moments of orders less than α. The tail thickness increases as α decreases. If $\alpha < 2$ and $\beta \neq 0$, the distribution is asymmetric and the skewness increases as β moves away from 0 to ± 1.

The paper is organized as follows. Section 2 gives a short summary of the asymmetry tests under consideration. The design and results of Monte Carlo experiments are presented in Section 3, and Section 4 concludes with final remarks.

[1]For example, Sichel (1993) defines "deepness" and "steepness" asymmetries.

2 Review of Symmetry Tests

In this section, we briefly describe the Neftci, skewness-statistic, HRC, and RMK tests.[2] Throughout this section we assume that $\{y_t\}$ is a univariate stationary process, from which we observe the sample $y_1, \ldots, y_n$.

2.1 Neftci Test

Neftci (1984) proposed a symmetry test based on the concepts of finite-state Markov processes. Assuming that $\{y_t\}$ is a stationary zero-mean process, he defines the indicator process

$$I_t = \begin{cases} 1, & \text{for } \Delta y_t > 0 \\ 0, & \text{for } \Delta y_t \leq 0 \end{cases} \tag{1}$$

which can be represented by a Markov process with appropriately defined transition probabilities. Neftci considered a second-order Markov process. Below, we adopt Rothman's (1991) first-order specification with transition probabilities $p_{ij} = \mathrm{P}(I_t = i | I_{t-1} = j)$, $i, j = 0, 1$. In this case, ignoring initial conditions, the approximate log likelihood for a given realization of $\{I_t\}$, S_n, is

$$l(S_n,\ p_{11},\ p_{00}) = n_{11}\ \ln(p_{11}) + n_{01}\ \ln(1 - p_{11}) + n_{00}\ \ln(p_{00}) + n_{10}\ \ln(1 - p_{00})$$

where n_{ij} denotes the number of occurrences of sequences $\{I_t = i | I_{t-1} = j\}$ in the sample. The approximate maximum-likelihood estimator is

$$\hat{p}_{ij} = \frac{n_{ij}}{n_{ij} + n_{ij'}}, \quad i, j = 0, 1, \quad j \neq j' \tag{2}$$

The null hypothesis of symmetry is H_0: $p_{11} = p_{00}$; the alternative against which one tests is either H_1: $p_{11} > p_{00}$ or H_1: $p_{11} > p_{00}$, implying that expansions are more persistent than contractions. The corresponding likelihood-ratio statistic, $-2(l_0 - l_1)$, is asymptotically $\chi^2(1)$-distributed.

2.2 Tests Based on the Skewness Statistic

A common test for asymmetry is based on the skewness statistic

$$\frac{m_3}{\sqrt{m_2^3}} \tag{3}$$

where m_2 and m_3 are the second- and third-centered moments of $\{y_t\}$, respectively. In practice, Equation (3) is estimated by replacing m_2 and m_3 with their sample estimates

$$\hat{m}_2 = n^{-1} \sum_{t=1}^{n} (y_t - \bar{y})^2 \quad \text{and} \quad \hat{m}_3 = n^{-1} \sum_{t=1}^{n} (y_t - \bar{y})^3$$

with $\bar{y} = n^{-1} \sum_{t=1}^{n} y_t$. For a normally distributed data-skewness statistic, Equation (3) is approximately normally distributed with $N(0, 6/n)$ (see Gasser 1975).

A multidimensional version of this test is also based on the third-central moment of the multivariate normal distribution (see, for example, Lütkepohl 1991, p. 153ff). Let Y_t be a K-dimensional Gaussian white-noise process with $Y_t \sim N(\mu, \Sigma)$, and P be a lower triangular matrix with a positive diagonal that satisfies $PP' = \Sigma$. Then, the independent standard-normal random variables $w_t = [w_{1t}, \ldots, w_{Kt}]' := P^{-1}(Y_t - \mu)$ are $N(0, I_K)$ distributed. Hence, $\mathrm{E}[w_{1t}^3, \ldots, w_{Kt}^3]' = 0$. Defining $m_3 := [m_{31}, \ldots, m_{3K}]'$ with $m_{3k} = n^{-1} \sum_{i=1}^{n} w_{ki}^3$, $k = 1, \cdots, K$, statistic $\sqrt{n} m_3$ converges in distribution to $N(0, 6I_K)$ and

$$\frac{n}{6} m_3' m_3 \xrightarrow{d} \chi^2(K)$$

Below, we will apply the multidimensional version of the skewness-statistic test to stacked vectors of a univariate time series. Specifically, from the scalar series y_t we construct the bivariate series $[y_1,\ y_2]$, $[y_3,\ y_4], \ldots$ and compare the powers of the one- and two-dimensional versions of the test.

[2] Several other tests have been suggested in the literature, but will not be examined here. For example, Aki (1987, 1993), Doksum et al. (1977), Antille et al. (1982), Sichel (1993), McQueen and Thorley (1993), Rothman (1993), and Ramsey and Rothman (1996).

2.3 Tests Based on the Characteristic Symmetry Function

Csörgö and Heathcote (1987) proposed a statistic for testing symmetry using the *characteristic symmetry function* (CSF), whose behavior is indicative of the presence of symmetry. The CSF measures the distance between distribution functions. Given the sample $y_1, \ldots, y_n$, the test employs the empirical CSF

$$\theta_n(\tau) = \frac{1}{\tau}\arctan\left\{\frac{V_n(\tau)}{U_n(\tau)}\right\}, \quad \tau \in [a, b] \tag{4}$$

where $U_n(\tau)$ and $V_n(\tau)$ are the real and imaginary parts of the empirical characteristic function, respectively, i.e.,

$$U_n(\tau) = n^{-1}\sum_{j=1}^{n}\cos(\tau y_j) \quad \text{and} \quad V_n(\tau) = n^{-1}\sum_{j=1}^{n}\sin(\tau y_j)$$

and $[a, b]$, $0 < a < b < r_n$ is a *working region* with the smallest positive root of $U_n(\tau)$, denoted by r_n, as the upper bound.

In the univariate case, symmetry of y_t about $\bar{y}$ is given if and only if the imaginary part of the characteristic function of $y_t - \bar{y}$ is zero or, equivalently, if and only if the empirical CSF is constant. Significant deviations of the empirical CSF from a constant value indicate asymmetry. Assuming the finiteness of associating moments, deviations of $\theta_n(\tau)$ from $\theta(\tau)$ are reflected in stochastic process

$$S_n(\tau) = \sqrt{n}[\theta_n(\tau) - \theta(\tau)]$$

which converges weakly to a zero-mean Gaussian process, G. Under symmetry, the maximum distance of $\theta_n(\tau)$ from a constant can be measured by a statistic based on $|\theta_n(\tau_{max}) - \theta_n(\tau_{min})|$, where τ_{max} = $\text{argmax}_{\tau\in[a,b]}S_n(\tau)$ and τ_{min} = $\text{argmin}_{\tau\in[a,b]}S_n(\tau)$. Defining τ_{lmax} and τ_{lmin} as the points at which the variance of $S_n(\tau)$ takes on its first local supremum and its last local infimum over $[a, b]$, respectively, the test statistic

$$S_n = \frac{\sqrt{n}|\theta_n(\tau_{lmax}) - \theta_n(\tau_{lmin})|}{\sqrt{\sigma_n^2(\tau_{lmax}) + \sigma_n^2(\tau_{lmin}) - 2\text{Cov}_n(\tau_{lmax}, \tau_{lmin})}} \tag{5}$$

is asymptotically normally distributed.[3]

Heathcote, Rachev, and Cheng (1995) introduced two procedures for testing the symmetry of a general multivariate distribution. The first is a multivariate generalization of the test of Csörgö and Heathcote (1987); the second considers specifically multivariate α-stable laws. Theoretical and empirical aspects of the latter test are investigated in Rachev, Mittnik, and Kim (1996).

For bivariate, mutually independent Gaussian random variables, $Y_t = [y_{1,t},\ y_{2,t}]$, $t = 1, 2\ldots$, a symmetry test can be performed by examining departures of $\text{E}[\sin(Y_t - \mu, \tau)]$ from zero, where the mean vector μ should be estimated as the median or some trimmed average.[4] Thus, the HRC test procedure is based on the process $n^{-\frac{1}{2}}\sum_{j=1}^{n}\sin(Y_j - \hat{\mu}_n, \tau)$, $\tau \in [a, b]$ (see Heathcote, Rachev, and Cheng [1995] for the choice of the working region). The test of the null hypothesis of symmetry involves the following steps:

1. Select a weakly consistent estimator $\hat{\mu}_n$ of μ; a weakly consistent estimator $\hat{\sigma}_n^2(\tau)$ of $\sigma^2(\tau)$; and the working region.
2. Find the point τ_{max} which maximizes $\sigma_n^2(\tau)$ over the chosen working region.
3. Define $Z_n(\tau) = n^{-\frac{1}{2}}\sum_{j=1}^{n}\sin(Y_j - \hat{\mu}_n, \tau)$ and form statistic $Z_n(\tau)/\hat{\sigma}_n^2(\tau_{max})$.

Statistic $Z_n(\tau)/\hat{\sigma}_n^2(\tau_{max})$ is asymptotically standard normal. The working region will be within the largest region, over which the supremum can be taken. The test is conservative in that sense.

The second symmetry test in Heathcote, Rachev, and Cheng (1995) assumes that the observations are in the domain of attraction of an α-stable law. The procedure is based on the tail estimators of the spectral measure

[3] Here, $\text{Cov}_n(\tau_1, \tau_2)$ is the estimate of the covariance of G, which is obtained by replacing U and V with their sample versions, U_n and V_n. Then, $\sigma_n^2(\tau) := \text{Cov}_n(\tau, \tau)$.

[4] The median or a trimmed mean should be used if $\alpha \geq 1$ cannot be ruled out for the α-stable distribution governing Y_t.

(see also Rachev, Mittnik, and Kim, 1996) and assumes that $X_j = [x_{1j}, x_{2j}]$, $j = 1, \ldots, n$ are two-dimensional, mutually independent α-stable random vectors ($0 < \alpha < 2$). For the bivariate case, the version of this test adopted in Rachev, Mittnik, and Kim (1996) and referred to as the RMK test consists of following steps:

1. For every observation $[x_{1t}, x_{2t}]$ we write polar coordinates $\rho_t := \sqrt{x_{1t}^2 + x_{2t}^2}$ and inverse tangent $\tilde{\delta}_t := \arctan(x_{1t}/x_{2t})$.

2. Let $k = k_n$ be a sequence of integers satisfying $1 \le k \le \frac{n}{2}$, with $\frac{k}{n} \to \infty$ as $k \to \infty$ and $n \to \infty$, and derive the estimator for the normalized spectral measure, $\phi(\delta)_n$, by

$$\phi_n(\delta) = \frac{1}{k} \sum_{t=1}^{n} I_{\{\delta_t \le \delta,\ \rho_t \ge \rho_{n-k+1:n}\}}, \qquad \delta \in (0, 2\pi] \tag{6}$$

   where $I_{\{\}}$ is the usual indicator function and $\rho_{i:n}$ denotes the i^{th}-order statistic. Parameter δ_t in Equation (6) is defined as

$$\delta_t = \begin{cases} \tilde{\delta}_t, & \text{for } x_{1t}, x_{2t} \ge 0 \\ \pi - \tilde{\delta}_t, & \text{for } x_{1t} < 0, x_{2t} \ge 0 \\ \pi + \tilde{\delta}_t, & \text{for } x_{1t}, x_{2t} < 0 \\ 2\pi - \tilde{\delta}_t, & \text{for } x_{1t} \ge 0, x_{2t} < 0 \end{cases}$$

   Rachev and Xin (1993) proved the strong consistency of $\sup\phi_n(\delta)$. In practice, one may take the grid $(\delta_1, \ldots, \delta_d)$, $\delta_1 = \frac{2\pi}{d}$, $\delta_d = 2\pi$, where d is the number of grid points and $\frac{2\pi}{d}$ is the step width.

3. Under some regularity conditions (see Heathcote, Rachev, and Cheng 1995) one can use the sample supremum of $\phi(\delta)$ in the region $0 < \delta \le \pi$, namely

$$\Phi_n := \sup_{0<\delta\le\pi} \sqrt{k} \frac{|\phi_n(\delta) - \phi_n(\delta + \pi) + \phi_n(\pi)|}{\sqrt{2\phi_n(\delta)}} \tag{7}$$

   as test statistic.

From the functional limit theorem for ϕ_n, shown in Heathcote, Rachev, and Cheng (1995), one can easily verify that $\hat{\Phi}_n$ is asymptotically standard normal. Assuming sufficiently large values of n and k, we reject the null hypothesis of multivariate symmetry at the significance level γ, when $\Phi_n > z_{\gamma/2}$, where $z_{\gamma/2}$ is the $100(1 - \gamma/2)$ percentile of the standard normal distribution.[5]

3 Design and Results of Simulation Experiments

In our simulations, we consider the standard normal distribution and nine different α-stable distributions. As transmission mechanisms, we choose autoregressive (AR) processes of orders 1 and 2, namely,

$$y_t = a y_{t-1} + u_t \tag{8}$$

and

$$y_t = a y_{t-1} - a^2 y_{t-2} + u_t \tag{9}$$

where parameter a assumes the values 0, .3, .6, .9, and 1.0. Note that by construction, the two roots of the AR(2) process are identical and equal to a.[6]

The averages and standard deviations of the sample skewness[7] given by Equation (3) from 1,000 replications of samples of size 100 for the levels and first differences of y_t are given in Table 2.[8] The results

[5]Rachev, Mittnik, and Kim (1996) conduct extensive simulations to study suitable choices for k and d.

[6]The α-stable pseudorandom variates were generated with the algorithm of Chambers, Mallows, and Stuck (1976), as implemented in Splus Windows, Version 3.1.

[7]More correctly, we should use the term "pseudo-skewness," because the skewness statistic of Equation (3) does not exist for α–stable distributions with $\alpha < 2$.

[8]Tables 2–9 are at the end of this article.

give rise to the following observations:

1. If innovations are symmetric (i.e., the innovations are normal or $\beta = 0$), levels and first differences of the observed output appear symmetric, independent of the AR parameter a.

2. In the first row (AR(1) with $a = 0$) we have $y_t = u_t$. Thus, it reflects merely the properties of the generated innovations. For $\beta \neq 0$, the sample skewness depends on the tail-index α. For a fixed $\beta \neq 0$, the sample skewness increases as α moves away from 2. Hence, the power of symmetry is expected to depend on α.

3. Both smaller values for the AR parameter a and the lower AR-lag order lead to a higher sample skewness in the level of the output process. In fact, if the AR(1) process is integrated, i.e., $a = 1$, the output becomes practically symmetric.

4. For the AR(1) process, the dependence of the sample skewness on the AR parameter a is reversed when first differences are considered. The sample skewness reduces as we overdifference "more excessively," i.e., the smaller a becomes. For the differenced AR(1) process, exactly the opposite holds. Excess differencing results in a loss of asymmetry.

These observations recommend that we test the symmetry of both the observed and the linearly prewhitened data.

For our simulations, we consider the three sample sizes $n = 50$, 100, and 200, for each of which we generate 1,000 replications. The simulation results are summarized in Tables 3 through 9. Note that the entries in all of these tables represent percentages of acceptance of the null hypothesis of symmetry using the 5% significance level.

The results for the Neftci test of the levels y_t using a first-order specification for the state-indicator sequences, reported in Table 3, indicate the following:

1. By construction, the Neftci test captures the asymmetry in the *differenced* series and works best when the sample skewness is extreme (cf. the sample skewness reported in Table 2). Hence, any empirical results based on this test should be interpreted as the presence or absence of asymmetry in the differences and not in the levels of the data.

2. The test performs relatively better for the AR(1) than for the AR(2) process. For the latter, its power is rather low. The results indicate considerable sensitivity of the power of the Neftci test with respect to the transmission mechanisms.

3. In summary, we find for the Neftci test that the frequency of type-II errors is too high when $\beta \neq 0$ and $\alpha < 2$. Moreover, it is sensitive to the tail-fatness index α—even in the absence of asymmetry, i.e., when $\beta = 0$.

The Neftci test is nonparametric, and thus imposes little structure on the data (see, for example, Sichel 1989). Our simulations also show that the test has proper power only in the AR(1) case, which is the order we assumed for the state-indicator sequence. This suggests that any test result should be interpreted with caution, if the nature of the DGP is unknown.

For the test based on the one-dimensional skewness statistic, we calculated Equation (3) for level, differenced, and prewhitened data, denoted by y_t, Δy_t, and $\hat{u}_t$, respectively. We used Akaike's (1974) information criterion (AIC) to specify the autoregressive prewhitening filter with maximum lag order 5. Knight (1989) showed that the AIC is consistent in the presence of heavy-tailed innovations. As it turned out, the results for the prewhitened $\hat{u}_t$s were not affected by the choices of a and the lag length. Therefore, we only report the results that involved $\hat{u}_t$ derived from AIC-determined AR filters with $a = 0$, keeping in mind that the results for all other a values considered are similar. The results of the test, shown in Tables 4–6, give rise to the following observations:

1. The results in Table 4 show a moderate power but a high frequency of type-I errors, when $\alpha < 2$ and $\beta = 0$. This frequency increases as α decreases.

2. A comparison of Tables 3 and 5 illustrates that the decrease in power of the skewness-statistic test, when using an AR(2) instead of an AR(1) model, is much smaller than for the Neftci test (see, for example, the entries for $a = .9$). Thus, the skewness statistic appears to be less sensitive with respect to the AR specification of the DGP than that of the Neftci test.

3. Table 6 indicates that the test performs better for $\hat{u}_t$ than for y_t and Δy_t, and that the power is practically invariant with respect to the AR coefficient and the lag order.

Both the Neftci test and the test based on the skewness statistic (Equation [3]) are adversely affected by fat tails, regardless of whether or not the distribution is symmetric. Thus, these tests should not be used if one suspects the data to be fat tailed.

For the tests based on the multidimensional version of the skewness statistic, the HRC and the RMK tests, we considered only two-dimensional cases; i.e., we used stacked vectors of $\hat{u}_t$, namely $[\hat{u}_1,\ \hat{u}_2]$, $[\hat{u}_3,\ \hat{u}_4]$, ... (see Section 2.2).[9]

The results of the test based on the multidimensional version of the skewness statistic, summarized in Table 7, indicate a severe tendency to reject the null. For example, for the symmetric case of $\alpha = 1.2$, $\beta = 0$, and $n = 200$, the null of symmetry was rejected in 99.6% of the cases. Even for $\alpha = 1.8$ and $\beta = 0$, it still rejects 91.8% of the cases. The frequency of type-II errors is, however, lower than that of the one-dimensional version of the test.

For the HRC test, the working region was chosen by setting $a = n/10$ and $b = \min\{(r_n - 1), n\}$. The results of the HRC test, reported in Table 8, show that it is very conservative and of low power. Under the null of symmetry, i.e., $\beta = 0$, it is rather sensitive with respect to the tail index, α.[10]

For the RMK test parameter, we set $d = n/10$, and, to account for the relatively small sample size $n = 50$, $k = n/5$, and set $k = n/10$, for $n = 100$ and 200.[11] The results, summarized in Table 9, indicate that the RMK test has moderate power for the small sample sizes and quickly becomes more powerful as the sample size increases. In contrast to the HRC test and the multidimensional skewness-statistic test, it is virtually not affected by the tail-thickness parameter, α.

4 Concluding Remarks

We have investigated the problem of testing for the symmetry of a time series when the DGP has asymmetric, α-stable distributed innovations, and a linear autoregression as a transmission mechanism. Of the tests considered, only the RMK test has a reasonable power and size when innovations are fat tailed. Such non-normalities in the innovations lead to excessive rates of type-I and type-II errors for the other tests.

A modification of the (standard) skewness-test statistic (Equation [3]) for random variables in the domain of attraction of an α-stable law with $\alpha < 2$ is investigated in Mittnik, Kim, and Rachev (1996).

References

Akaike, H. (1974). "A New Look at the Statistical Model Identification." *IEEE Transactions on Automatic Control*, 19:716–723.

Aki, S. (1987). "On Nonparametric Tests for Symmetry." *Annals of the Institute of Statistical Mathematics*, 39:457–472.

Aki S. (1993). "On Nonparametric Tests for Symmetry in R^m." *Annals of the Institute of Statistical Mathematics*, 45:787–800.

Antille A., G. Kersting, and W. Zucchini (1982). "Testing Symmetry." *Journal of the American Statistical Association*, 77:639–646.

Chambers, J. M., C. Mallows, and B. W. Stuck (1976). "A Method for Simulating Stable Random Variables." *Journal of the American Statistical Association*, 71:340–344.

Csörgö, S., and C. R. Heathcote (1987). "Testing Symmetry." *Biometrika*, 74:177–184.

DeLong, J. B., and L. H. Summers (1986). "Are Business Cycles Asymmetric?" In: R. J. Gordon (ed.), *The American Business Cycle: Continuity and Change*. Chicago, IL: Chicago University Press.

Doksum, K. A., G. Fenstad, and R. Aaberge (1977). "Plots and Tests for Symmetry." *Biometrika*, 64:473–487.

Gasser, T. (1975). "Goodness-of-Fit Tests for Correlated Data." *Biometrika*, 62:563–570.

[9] Note that this reduces the number of effective "observations" to $n/2$.

[10] Performing additional simulations, we found that statistical tests based on the CSF require a sample size of 500 or more to yield reasonable results.

[11] On the choice of k, see the discussion in Rachev, Mittnik, and Kim (1996).

Granger, C. W. J., and A. P. Anderson (1978). *An Introduction to Bilinear Time Series Models.* Göttingen: Vandenhoek and Ruprecht.

Granger, C. W. J., and T. Teräsvirta (1993). *Modelling Nonlinear Economic Relationships.* Oxford: Oxford University Press.

Hamilton, J. D. (1989). "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle." *Econometrica*, 57:357–384.

Heathcote, C. R., S. T. Rachev, and B. Cheng (1995). "Testing Multivariate Symmetry." *Journal of Multivariate Analysis*, 54:91–112.

Knight, K. (1989). "Consistency of Akaike's Information Criterion for Infinite Variance Autoregressive Processes." *Annals of Statistics*, 17:824–840.

Lütkepohl, H. (1991). *Introduction to the Multiple Time Series Analysis.* New York: Springer-Verlag.

Mandelbrot, B. (1963). "The Variation of Certain Speculative Prices." *Journal of Business*, 26:394–419.

McCulloch, J. H. (forthcoming). "Financial Applications of Stable Distributions." In: *Handbook of Statistics*, vol. 14. Amsterdam: North-Holland.

McQueen, G., and S. Thorley (1993). "Asymmetric Business Cycle Turning Points." *Journal of Monetary Economics*, 31:341–362.

Mittnik, S., J. -R. Kim, and S. T. Rachev (1996). "A Skewness Test Statistic for Heavy-Tailed Variates." Unpublished manuscript, Institute of Statistics and Econometrics, Christian Albrechts University at Kiel.

Mittnik, S., and Z. Niu (1994). "Asymmetries in Business Cycles: Econometric Techniques and Empirical Evidence." In: W. Semmler (ed.), *Business Cycles: Theory and Empirical Methods*, pp. 331–350.

Mittnik, S., and S. T. Rachev (1993). "Modeling Asset Returns With Alternative Stable Distribution." *Econometric Reviews*, 12:261–330.

Neftci, S. N. (1984). "Are Economic Time Series Asymmetric Over the Business Cycle?" *Journal of Political Economy*, 92:307–328.

Rachev, S. T., S. Mittnik, and J. -R. Kim (1996). "Testing for Symmetry of Multivariate Heavy-Tailed Distributions." Unpublished manuscript, Institute of Statistics and Econometrics, Christian Albrechts University at Kiel.

Rachev, S. T., and H. Xin (1993). "Test on Association of Random Variables in the Domain of Attraction of a Multivariate Stable Law." *Probability and Mathematical Statistics*, 14:125–141.

Ramsey, J. B., and P. Rothman (1996). "Time Irreversibility and Business Cycle Asymmetry." *Journal of Money, Credit and Banking*, 28:1–21.

Rothman, P. (1991). "Further Evidence on the Asymmetric Behavior of Unemployment Rates Over the Business Cycle." *Journal of Macroeconomics*, 13:291–298.

Rothman, P. (1993). "The Comparative Power of the TR Test Against Simple Threshold Models." *Journal of Applied Econometrics*, 8:187–195.

Sichel, D. E. (1989). "Are Business Cycles Asymmetric? A Correction." *Journal of Political Economy*, 97:1255–1260.

Sichel, D. E. (1993). "Business Cycle Asymmetry: A Deeper Look." *Economic Inquiry*, 31:224–236.

Teräsvirta, T. (1994). "Specification, Estimation and Evaluation of Smooth Transition Autoregressive Models." *Journal of the American Statistical Association*, 89:208–218.

Tong, H. (1990). *Non-Linear Time Series.* New York: Oxford University Press.

Table 2
Pseudoskewness of AR Processes with α-Stable Innovations[a]

	$\{\beta, \alpha\}$									
a^b	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
	AR(1)									
0	5.2687	3.7116	1.5981	2.5873	1.6505	0.7769	0.0361	−0.2194	−.0424	0.0126
	(2.4102)	(2.3211)	(1.7444)	(4.6785)	(3.6006)	(2.0272)	(5.1747)	(3.6809)	(2.5641)	(0.2379)
.3	4.6889	3.3103	1.4218	2.3136	1.4720	0.6923	0.0325	−0.1965	−.0410	0.0121
	(2.1503)	(2.0672)	(1.5491)	(4.1749)	(3.2236)	(1.8118)	(4.6191)	(3.2939)	(2.2906)	(0.2428)
.6	3.3768	2.3914	1.0237	1.6847	1.0608	0.4983	0.0280	−0.1431	−.0372	0.0097
	(1.5921)	(1.5170)	(1.1482)	(3.0462)	(2.3556)	(1.3301)	(3.3801)	(2.4085)	(1.6792)	(0.2695)
.9	1.4489	0.9876	0.4214	0.7214	0.4461	0.2039	0.0243	−0.0605	−.0349	0.0099
	(0.8676)	(0.7899)	(0.6347)	(1.4044)	(1.1134)	(0.6923)	(1.6030)	(1.1318)	(.8063)	(0.3808)
1	0.0151	0.0005	−0.0039	0.0130	0.0037	0.0094	0.0132	−0.0054	−.0133	0.0237
	(0.3853)	(0.4147)	(0.4249)	(0.5441)	(0.4998)	(0.4393)	(0.5537)	(0.5303)	(.4405)	(0.3919)
	AR(2)									
.3	3.6854	2.6103	1.1181	1.8353	1.1566	0.5421	0.0288	−0.1576	−0.366	0.0105
	(1.7211)	(1.6432)	(1.2434)	(3.3118)	(2.5609)	(1.4421)	(3.6692)	(2.6171)	(1.8239)	(0.2631)
.6	2.0629	1.4429	0.6135	1.0336	0.6366	0.2954	0.0129	−0.0896	−.0248	0.0088
	(1.0919)	(1.0242)	(0.8003)	(1.9301)	(1.5043)	(0.8937)	(2.1660)	(1.5348)	(1.0975)	(0.3583)
.9	0.6270	0.3728	0.1473	0.2944	0.1652	0.0788	0.0163	−0.0127	−.0220	0.0208
	(0.6212)	(0.5834)	(0.5421)	(0.8076)	(0.6771)	(0.5768)	(0.8251)	(0.6911)	(.5842)	(0.4964)
	ΔAR(1)									
0	−0.0301	0.0150	−0.0019	0.0112	−0.0050	−0.0081	0.0275	−0.0310	−.0047	0.0013
	(0.6727)	(0.5242)	(0.3361)	(0.8153)	(0.5907)	(0.3531)	(0.6739)	(0.6340)	(.4624)	(0.2060)
.3	1.7569	1.2583	0.5376	0.8604	0.5492	0.2539	0.0329	−0.1022	−.0126	0.0049
	(1.0581)	(0.9384)	(0.7012)	(1.7699)	(1.3458)	(0.7751)	(1.8881)	(1.3568)	(.9602)	(0.2193)
.6	3.4438	2.4357	1.0482	1.6664	1.0778	0.5010	0.0339	−0.1714	−.0222	0.0104
	(1.6835)	(1.5845)	(1.1994)	(3.1411)	(2.4163)	(1.3716)	(3.4603)	(2.4340)	(1.7047)	(0.2244)
.9	4.8546	3.4323	1.4783	2.3537	1.5229	0.7105	0.0322	−0.2259	−.0338	0.0146
	(2.2533)	(2.1535)	(1.6312)	(4.3474)	(3.3451)	(1.8908)	(4.8025)	(3.3912)	(2.3649)	(0.2302)
1	5.1862	3.6603	1.5772	2.5271	1.6273	0.7666	0.0311	−0.2360	−.0445	0.0138
	(2.3773)	(2.2835)	(1.7253)	(4.6191)	(3.5498)	(2.0061)	(5.1117)	(3.6103)	(2.5167)	(0.2348)
	ΔAR(2)									
.3	3.1696	2.2355	0.9598	1.5299	0.9891	0.4619	0.0311	−0.1596	−.0152	0.0078
	(1.5710)	(1.4746)	(1.1074)	(2.9002)	(2.2337)	(1.2738)	(3.1959)	(2.2467)	(1.5706)	(0.2233)
.6	3.8459	2.7158	1.1659	1.8599	1.2140	0.5550	0.0312	−0.1917	−.0225	0.0131
	(1.8167)	(1.7295)	(1.2958)	(3.4483)	(2.6877)	(1.5122)	(3.8276)	(2.7006)	(1.8822)	(0.2319)
.9	2.0715	1.4666	0.6287	1.0298	0.6446	0.2994	0.0149	−0.0931	−.0201	0.0011
	(1.0908)	(1.0073)	(0.7861)	(1.9627)	(1.5199)	(0.9032)	(2.1627)	(1.5434)	(1.0798)	(0.3198)

[a]The entries represent the averages of the estimated skewness statistic based on 1,000 replications of samples of size 100. Standard deviations are given in parentheses. SN refers to the standard normal distribution, i.e., $\alpha = 2$.
[b]Value of the autoregressive parameter in Equations (8) and (9).

Table 3
Results for Neftci Test of Output y_t[a]

n	a[b]	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
		$\{\beta, \alpha\}$									
		AR(1)									
50	0	89.8	89.4	90.3	88.7	88.1	91.1	89.5	91.8	89.7	88.1
	.3	59.4	78.8	88.6	79.9	86.1	90.5	88.3	91.6	91.0	91.5
	.6	23.7	55.1	86.3	59.1	78.2	88.1	79.7	88.2	90.2	93.2
	.9	8.5	32.6	78.9	34.6	58.6	85.4	53.6	72.9	86.0	95.6
	1	5.4	28.6	77.4	25.2	54.3	85.2	40.3	68.2	86.8	96.4
100	0	89.1	90.5	91.3	91.0	90.3	90.6	91.2	91.9	91.3	90.1
	.3	40.8	68.9	88.5	73.3	85.5	91.1	90.8	92.4	91.7	92.1
	.6	5.7	35.5	83.1	49.1	73.3	88.0	81.3	89.6	92.5	95.6
	.9	0.4	10.8	71.4	24.0	48.0	83.7	48.6	72.9	87.0	97.2
	1	0.2	8.7	68.1	13.3	38.6	82.7	31.2	65.1	86.3	96.9
200	0	90.4	90.0	92.3	89.1	92.0	89.9	91.1	91.8	89.5	90.9
	.3	14.6	51.5	86.3	61.2	80.4	91.4	91.4	92.0	92.3	93.1
	.6	0.0	10.4	70.5	30.2	58.9	87.3	82.1	89.7	93.1	95.3
	.9	0.0	1.1	52.4	15.5	30.7	87.4	51.3	73.0	88.9	97.3
	1	0.0	0.3	47.4	5.2	22.2	77.9	24.5	57.9	86.4	97.7
		AR(2)									
50	.3	34.8	63.6	88.8	68.7	84.7	91.8	86.5	91.9	92.2	94.6
	.6	36.9	66.2	93.4	67.9	83.2	95.5	85.2	91.6	96.2	98.7
	.9	79.8	88.3	96.9	83.8	90.8	96.8	90.7	93.7	96.0	99.5
100	.3	10.9	45.9	86.4	59.1	80.9	89.3	87.6	93.1	94.1	95.5
	.6	10.7	45.6	90.7	56.2	76.1	96.5	85.3	94.9	96.4	99.0
	.9	57.2	75.7	94.4	69.5	84.7	95.2	81.6	89.6	95.2	99.2
200	.3	0.4	19.1	79.9	41.6	69.4	90.2	89.5	91.8	94.6	95.9
	.6	0.6	15.1	83.4	39.6	68.4	94.6	87.2	94.9	98.5	99.8
	.9	26.9	61.9	91.5	62.0	79.4	95.4	79.5	89.2	96.2	99.7

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level. SN refers to the standard normal distribution, i.e., $\alpha = 2$.
[b]Value of the autoregressive parameter in Equations (8) and (9).

Table 4
Results for Test Based on One-Dimensional Skewness Statistic (Equation [3]) of y_t[a]

n	a[b]	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
		AR(1)									
50	0	0.4	8.9	51.8	9.8	24.3	60.1	15.5	30.6	59.0	97.1
	.3	1.0	10.8	56.1	11.2	28.1	62.8	17.1	34.4	61.3	96.7
	.6	15.2	22.1	64.3	17.1	35.8	67.9	21.3	42.2	68.3	94.8
	.9	43.1	57.6	78.4	43.5	61.8	80.3	45.3	61.5	77.9	90.8
	1	91.6	89.6	89.8	82.2	86.8	91.2	81.0	82.8	87.5	91.5
100	0	0.0	0.3	25.7	4.0	9.4	40.4	8.5	16.8	43.1	96.3
	.3	0.0	0.8	29.5	4.4	11.2	43.6	9.2	18.9	44.3	95.3
	.6	0.0	3.5	39.1	5.5	16.2	48.3	12.7	24.7	50.6	91.0
	.9	11.4	28.5	58.3	17.9	36.0	59.9	23.8	40.9	61.4	79.8
	1	79.9	76.8	75.4	68.3	71.2	72.7	66.7	67.5	74.9	79.1
200	0	0.0	0.0	5.2	1.8	5.7	24.1	4.3	8.2	28.0	95.3
	.3	0.0	0.0	6.6	2.5	6.1	24.0	5.2	10.4	30.8	94.7
	.6	0.0	0.0	14.9	3.1	8.1	29.4	5.7	14.0	36.0	90.2
	.9	1.0	8.7	40.4	8.7	19.0	42.2	11.0	24.4	45.2	71.7
	1	62.6	61.4	58.0	54.4	55.3	59.9	50.4	53.2	57.3	60.4
		AR(2)									
50	.3	3.4	19.0	62.3	15.2	34.0	66.8	20.3	40.0	66.8	95.5
	.6	27.9	43.8	73.6	32.0	51.9	73.6	35.1	55.2	72.6	86.8
	.9	80.9	82.2	82.7	79.6	83.2	86.1	82.7	80.4	84.1	83.7
100	.3	0.0	2.5	36.7	5.6	15.6	48.0	11.8	23.5	47.8	92.3
	.6	4.3	16.5	51.9	12.9	28.3	55.4	17.1	34.9	54.7	82.7
	.9	41.1	51.8	61.2	41.8	52.9	59.3	38.9	51.7	59.3	67.0
200	.3	0.0	0.0	12.7	3.2	7.4	27.1	5.1	13.1	34.4	92.2
	.6	0.0	2.5	30.9	5.4	14.4	36.4	8.4	17.9	42.5	77.5
	.9	14.2	26.7	46.1	17.3	30.9	44.8	21.5	30.4	42.8	52.9

Column headings are $\{\beta, \alpha\}$ pairs.

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level. SN refers to the standard normal distribution, i.e., $\alpha = 2$.
[b]Value of the autoregressive parameter in Equations (8) and (9).

Table 5
Results for Test Based on One-Dimensional Skewness Statistic (Equation [3]) of Δy_t[a]

n	a[b]	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
						{β, α}					
						AR(1)					
50	0	88.9	89.3	94.1	85.1	89.7	93.5	86.9	86.9	91.7	98.5
	.3	25.9	45.5	79.3	32.9	56.4	80.2	36.3	56.3	79.3	97.7
	.6	5.1	20.2	64.8	16.9	37.1	68.3	18.9	39.2	67.0	97.3
	.9	1.1	11.0	52.9	11.2	26.7	61.9	15.7	31.9	59.9	95.2
	1	0.6	10.3	52.3	10.4	24.9	60.5	15.1	31.6	59.5	96.0
100	0	85.0	83.2	87.9	81.0	81.0	86.9	83.4	78.4	86.0	97.5
	.3	5.4	19.9	60.4	15.7	31.4	66.4	17.9	38.7	64.7	96.3
	.6	0.3	3.6	39.5	7.3	18.0	49.3	10.7	23.8	50.8	95.6
	.9	0.0	0.9	27.8	4.8	11.0	42.4	8.6	17.8	43.6	95.5
	1	0.0	0.4	26.3	3.8	9.7	40.7	8.6	17.4	43.3	96.3
200	0	79.7	72.2	81.5	74.9	70.4	79.5	78.8	69.1	76.2	98.0
	.3	0.9	4.2	37.3	7.8	16.5	49.1	11.6	21.9	50.0	97.1
	.6	0.0	0.1	14.6	3.1	8.6	33.9	6.4	12.0	35.0	96.7
	.9	0.0	0.0	6.2	2.5	5.6	25.5	4.4	9.5	29.7	96.5
	1	0.0	0.0	5.4	2.0	5.6	24.1	4.4	8.4	28.1	96.7
						AR(2)					
50	.3	7.7	22.7	67.2	18.5	39.2	69.8	20.9	41.7	68.2	97.1
	.6	3.3	16.4	58.9	15.0	32.8	64.9	19.6	36.8	63.6	94.9
	.9	24.9	39.9	66.5	28.9	45.9	66.7	32.5	51.1	66.3	83.9
100	.3	0.4	5.3	41.9	8.0	19.8	51.8	11.5	25.9	53.0	95.2
	.6	0.1	2.2	35.4	6.8	14.8	47.0	9.8	22.8	46.1	94.4
	.9	4.7	16.9	47.7	12.4	25.7	48.9	16.2	34.0	50.7	77.9
200	.3	0.0	0.1	16.3	3.6	8.7	36.0	7.4	12.7	37.1	95.6
	.6	0.0	0.1	10.4	3.1	8.5	29.8	5.6	12.3	34.5	94.7
	.9	0.0	2.8	30.6	6.0	14.2	38.5	9.9	18.8	41.2	76.4

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level. SN refers to the standard normal distribution, i.e., $\alpha = 2$.
[b]Value of the autoregressive parameter in Equations (8) and (9).

Table 6
Results for Test Based on One-Dimensional Skewness Statistic (Equation [3]) of $\hat{u}_t$[a]

n	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
					{β, α}					
50	1.4	14.0	58.4	13.7	29.1	64.8	17.5	37.3	63.3	96.3
100	0.0	0.6	28.9	4.2	10.5	42.7	9.1	19.0	44.7	96.5
200	0.0	0.0	6.1	2.3	5.7	23.8	4.8	8.9	28.1	96.4

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level.
SN refers to the standard normal distribution, i.e., $\alpha = 2$.

Table 7
Results for Test Based on Multidimensional Skewness Statistic of $\hat{u}_t$[a]

n	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
					{β, α}					
50	0.0	1.0	30.7	1.0	6.1	41.8	3.2	9.1	41.6	94.6
100	0.0	0.0	7.2	0.2	1.6	18.3	0.4	3.0	20.5	96.2
200	0.0	0.0	0.0	0.0	0.4	5.8	0.4	1.3	9.2	95.8

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level.
SN refers to the standard normal distribution, i.e., $\alpha = 2$.

Table 8
Results for Heathcote-Rachev-Cheng Test of $\hat{u}_t$[a]

n	{1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
					{β, α}					
50	30.0	44.2	75.8	52.0	62.7	81.1	60.7	70.3	80.5	90.1
100	4.5	8.4	49.4	31.3	42.7	66.6	47.9	58.9	72.8	89.2
200	0.5	0.7	16.8	17.8	21.5	51.4	32.1	44.2	64.0	88.9

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level.
SN refers to the standard normal distribution, i.e., $\alpha = 2$.

Table 9
Results for Rachev-Mittnik-Kim Test of $\hat{u}_t$[a]

n	$\{\beta,\alpha\}$ {1, 1.2}	{1, 1.5}	{1, 1.8}	{.5, 1.2}	{.5, 1.5}	{.5, 1.8}	{0, 1.2}	{0, 1.5}	{0, 1.8}	SN
50	50.4	78.6	92.3	91.5	92.7	93.2	86.4	87.0	86.8	91.2
100	3.2	35.8	80.0	57.7	76.4	85.8	81.3	82.5	86.0	82.9
200	0.0	1.8	53.1	11.4	43.0	78.0	80.0	77.6	76.5	77.3

[a]The entries represent the percentages of accepting the null of symmetry at the 5% significance level.
SN refers to the standard normal distribution, i.e., $\alpha = 2$.

Editorial Policy

The *SNDE* is formed in recognition that advances in statistics and dynamical systems theory may increase our understanding of economic and financial markets. The journal will seek both theoretical and applied papers that characterize and motivate nonlinear phenomena. Researchers will be encouraged to assist replication of empirical results by providing copies of data and programs online. Algorithms and rapid communications will also be published.

ISSN 1081-1826