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Tests for Nonlinearity in EMS Exchange Rates

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Abstract. *This paper tests for nonlinearity in EMS exchange rates using the bispectrum. The early experience of the ERM witnessed numerous realignments. We find that exchange rates follow a linear process over the period 1979–1987, consistent with the predictions of the realignment target zone model, where a stabilizing nonlinearity is absent. But from 1987–1992, no realignments occurred, and many currencies conformed to a nonlinear process, consistent with the credible target zone model where an inherent nonlinearity stabilizes exchange rates. However, the Italian lira and the Irish pound follow a linear process, which suggests that a target zone has not proven effective in stabilizing exchange rates.*

Keywords. bispectrum, spectral analysis

1 Introduction

The Exchange Rate Mechanism (ERM) of the EMS imposes policy commitments where members agree to undertake corrective actions to limit exchange rate movements. The ERM seeks to improve exchange rate stability by specifying a currency band or target zone that imposes upper and lower boundary values for the spot exchange rate. While safely inside the target zone, policy makers can pursue domestic policy agendas. But should the exchange rate approach a boundary value, policy makers must intervene in the foreign exchange market to defend the target zone. In a seminal paper, Krugman (1991) demonstrated that if the target zone arrangement is credible, then forward-looking agents anticipate foreign exchange intervention, making the spot rate less responsive to fundamentals than is the case under a flexible exchange rate regime. An important result of Krugman's work is that if the target zone is credible, there is a nonlinear relationship between the spot rate and fundamentals that improves exchange rate stability. If, however, there is uncertainty surrounding whether or not policy makers defend the target zone, exchange rate stability is compromised. Bertola and Caballero (1992) showed that under plausible conditions, the nonlinearity governing the spot rate and the improved stability will disappear altogether if there is a possibility that the current target zone will be realigned rather than defended.

The presence of a nonlinearity in the data generating process of ERM spot exchange rates has proven difficult to detect.[1] Diebold and Nason (1990), for example, dismissed the importance of nonlinearities based

[1]The presence of a nonlinearity in the second moments of spot exchange rates is well documented. Domowitz and Hakkio (1985) and Engle,

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on comparisons of forecasts produced from linear and nonlinear models. Diebold and Nason found that out-of-sample forecasts allowing for nonlinearity did not outperform forecasts generated from a simple random walk specification. Similarly, Meese and Rose (1990, 1991) concluded that incorporating nonlinearities into structural exchange rate models that included fundamentals such as interest rates, money supplies, and output did not appear promising.[2]

In this paper, we test for nonlinearity in bilateral EMS exchange rates using the estimated bispectrum. The estimated bispectrum is a nonparametric frequency domain technique that first tests for Gaussianity, and if rejected, then tests for linearity.[3] One appeal of this methodology is that it is univariate and fundamentals do not have to be identified. That is, the bispectrum does not depend on specifying a particular nonlinear function that links the exchange rate and fundamentals;[4] and perhaps more importantly, this methodology avoids the problem of specifying an explicit process that generates fundamentals.

Frankel and Phillips (1992) noted that the emergence of the ERM was greeted with considerable skepticism. Although the ERM experienced improved exchange rate stability in comparison to the prior Snake regime, its first years were characterized by numerous and often times large realignments of the target zone boundary values. Frankel and Phillips reported that from its inception until the last general realignment in January 1987, ERM target zones did not enjoy a high degree of credibility. As a result, the realignment target zone model of Bertola and Caballero (1992) offers a more accurate description of exchange rate movements than does the credible target zone model of Krugman (1991). Consequently, it is unlikely that a stabilizing nonlinearity in EMS exchange rates exists. The results of linearity tests based on the bispectrum support this claim, finding that, in general, exchange rates are governed by a linear process. In other words, linearity tests applied over the 1979–1987 period offer empirical evidence in support of the realignment target zone model, in that low credibility serves to negate the stabilizing effect of a target zone arrangement.

The experience of the ERM from January 1987 to the fall of 1992 was markedly different: no realignments occurred. Since 1987, institutional developments along with greater policy coordination have improved credibility (Frankel and Phillips 1992).[5] Although less than perfect, increased credibility supports the improved exchange rate stability hypothesis that is associated with the credible target zone model, where an inherent nonlinear process stabilizes currency values. We offer empirical evidence in support of this claim, finding that for many currencies, linearity is rejected over the period 1987–1992. Specifically, linearity is rejected for the Belgian franc, Danish krone, French franc, and the Dutch guilder. These results are broadly consistent with studies by De Jong (1994)[6] and Pesaran and Samiei (1992).[7] We found, however, that the Italian lira and the Irish pound conformed to a linear model, thereby dismissing the improved stability hypothesis for these currencies.

The remainder of the paper is organized as follows. Section 2 presents a brief review of the theoretical target zone literature when the target zone is credible, and when there is a possibility of realignment. Section 3 presents a review of the bispectrum, and outlines the Gaussianity and nonlinearity testing procedures. In Section 4, we examine the behavior of EMS bilateral exchange rates and test for Gaussianity and nonlinearity. Concluding comments are presented in Section 5.

2 Exchange Rate Target Zone Models

An exchange rate target zone is a compromise between fixed and flexible exchange rate regimes. A target zone arrangement stipulates upper and lower boundary values, within which exchange rates are allowed to

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Ito, and Lin (1990), among others, report conditional heteroskedasticity in residuals from both time series and structural exchange rate models. In this paper, we focus on the detection of an intrinsic nonlinearity in the data generating process that governs the level of spot exchange rates.

[2]Meese and Rose (1991) examined flexible and sticky price monetary models (Dornbusch 1976; Frenkel 1976; Mussa 1976) and explicit optimization models (Lucas 1982; Hodrick 1988).

[3]Hinich and Patterson (1985) applied the bispectrum to successfully detect a nonlinearity in stock returns, and Ashley and Patterson (1989) documented a nonlinearity in the industrial production index using the bispectrum.

[4]The credible target zone model of Krugman (1991) offers an explicit functional form that links exchange rates and fundamentals. Flood, Rose, and Mathieson (1991) and Smith and Spencer (1992), however, reported little empirical evidence in support of the credible target zone model.

[5]Frankel and Phillips (1992) provided a chronology of institutional changes.

[6]In contrast, De Jong (1994) reported that the Dutch guilder follows a linear process.

[7]Pesaran and Samiei (1992) reached a similar conclusion. Pesaran and Samiei estimated a rational expectations target zone model, where the exchange rate is treated as a limited dependent variable. They found that the target zone model fits the French franc exchange rate better than a model that ignores a currency band.

fluctuate freely. While the exchange rate is safely inside the boundaries, policy can be directed toward objectives other than the exchange rate. Should the exchange rate approach a boundary value, however, policy makers are committed to undertake actions that preserve the target zone.

The flexible price monetary model has served as a simple means of analyzing the behavior of the (spot) exchange rate under a target zone arrangement.[8] The law of motion for the exchange rate is given by

$$x(t) = k(t) + \alpha E[dx(t)]/dt \tag{1}$$

where $x(t)$ is the logarithm of the exchange rate, α is the interest semi-elasticity of money demand, and E is an expectational operator conditioned on current information. The variable $k(t)$ represents fundamentals, and includes foreign and domestic money supplies that are assumed to be under the direct control of the monetary authorities. Thus, the spot rate depends on fundamentals and its own expected rate of change, weighted by α.

The target zone arrangement is characterized by a central parity rate, $c(t)$, which in turn defines the boundary values. For simplicity, let the boundaries be determined as a multiple of the central parity rate. That is, the boundary values are equal to the central parity rate, plus or minus a fraction of the central parity. Let $\underline{x}$ and $\bar{x}$ and represent the lower and upper boundaries, respectively, where $\underline{x} = c(t)(1-\theta)$ and $\bar{x} = c(t)(1+\theta)$ for some fraction $0 < \theta < 1$. Policy makers are thus committed to manipulate $k(t)$, by altering money supplies, to ensure $\underline{x} < x(t) < \bar{x}$.

While the exchange rate is safely within its boundaries, fundamentals are assumed to follow the continuous-time analog of a random walk with drift. That is,

$$dk(t) = \eta dt + \sigma dz(t) \tag{2}$$

where η is the predictable change in the money supply, σ is the instantaneous variance parameter, and $dz(t)$ is the increment of a standard Wiener process.

Froot and Obstfeld (1991a, b) suggested a two-step procedure for determining the exchange rate solution path. In the first step, a family of functions $x(t) = G[k(t)]$ is identified that satisfies Equation (1), while the driving process is given by Equation (2). The second step involves identifying the particular function from the family of functions that is consistent with the policy regime in place.

Froot and Obstfeld (1991a, b) demonstrated that the family of functions in step one is represented by

$$G(k) = k + \alpha\eta + A_1 \exp(\lambda_1 k) + A_2 \exp(\lambda_2 k) \tag{3}$$

where $\lambda_1 = (-\eta + \sqrt{\eta^2 + 2\sigma^2/\alpha})/\sigma^2$ and $\lambda_2 = (-\eta - \sqrt{\eta^2 + 2\sigma^2/\alpha})/\sigma^2$.[9] The constants of integration are given by A_1 and A_2. The general solution, represented by Equation (3), contains a linear part $k + \alpha\eta$ and a nonlinear part $A_1 \exp(\lambda_1 k) + A_2 \exp(\lambda_2 k)$.

The second step identifies the constants of integration, which are determined by the boundary conditions that describe the exchange rate regime and the conduct of the monetary authorities. We examine three cases: (1) a flexible exchange rate regime, (2) a credible target zone, and (3) a target zone regime where there exists a possibility of a central parity rate realignment.

Under a flexible exchange rate regime, fundamentals follow Equation (2) forever. If expectations are formed rationally and speculative bubbles are ruled out, then the exchange rate is a linear function of the underlying fundamental value.[10] That is, the appropriate boundary conditions under a pure float are $A_1 = A_2 = 0$. The exchange rate is then described by

$$x = k + \alpha\eta. \tag{4}$$

Thus, under a flexible exchange rate regime, the exchange rate is governed by a linear data generating process.

Under a credible target zone, policy makers intervene in the foreign exchange markets to ensure that the exchange rate does not move outside the target zone. The target zone may be equivalently viewed as imposing

[8]Part of the popularity of this model stems from the ability to derive closed-form solutions for the exchange rate. Miller and Weller (1991a, b) examined the behavior of exchange rates in Dornbusch's (1976) model of price inertia; however, a closed-form solution is not available.

[9]The time notation is suppressed where it is clearly understood.

[10]Intrinsic bubbles can give rise to nonlinearities under a flexible exchange rate regime (Froot and Obstfeld 1991b).

limits on the values of fundamentals. For instance, suppose $G(\bar{k}) = \bar{x}$ and $G(\underline{k}) = \underline{x}$ where $\bar{k}$ and $\underline{k}$ are the maximum and minimum values of the fundamental, respectively, that preserve the target zone. The appropriate boundary conditions are known as "smooth pasting conditions" that require $G'(\bar{k}) = G'(\underline{k}) = 0$.[11] This condition is sufficient to identify both A_1 and A_2. If the target zone is credible, then as the exchange rate approaches one of its boundaries, agents anticipate activist policy. In other words, the exchange rate becomes less responsive to fundamentals, because policy is anticipated—even though no actual intervention takes place. This increased stability in the absence of policy is coined the "honeymoon effect" by Krugman (1991). Therefore, the constants of integration for a credible target zone are $A_1 < 0$ and $A_2 > 0$, and according to Equation (3), *a credible target zone induces a nonlinear relationship between the exchange rate and fundamentals.*

Once the possibility of central parity realignment is introduced, however, the importance of a nonlinear relationship becomes weakened. Following Bertola and Caballero (1992), assume that intervention only occurs at preannounced values of the fundamental, $c(t) + k^*$ and $c(t) - k^*$. Intervention can take two forms: (1) policy makers intervene and defend the current target zone, or (2) the target zone is realigned with no accompanying change in fundamentals. The current target zone is defended when monetary authorities adjust the money supply, $\Delta m = \pm k^*$, such that $k = c(t) \pm k^*$ and the spot exchange rate is driven to the center of the target zone. In contrast, the current target zone is realigned when the central parity rate is redefined as $c(t) \pm 2k^*$, and there is no accompanying change in the money supply. If p represents the probability of realignment and $(1 - p)$ the probability that the current target zone is defended, Bertola and Caballero showed that the general solution is given by

$$G(k; c) = k + \alpha\eta + A_1 \exp[\lambda_1(k - c)] + A_2 \exp[\lambda_2(k - c)] \tag{5}$$

where λ_1 and λ_2 are defined as in Equation (3). At a preannounced value, assuming that the exchange rate is not expected to change once the fundamental attains its critical value, it follows that

$$G(c + k^*; c) = pG(c + 2k^*; c + 2k^*) + (1 - p)G(c; c). \tag{6}$$

Combining Equations (5) and (6) identifies the constants of integration. If $p = 0$, then the exchange rate solution is identical to the credible target zone. As p rises from 0, however, the exchange rate becomes more responsive to fundamentals; and if $p = \frac{1}{2}$, the exchange rate is described by the linear solution shown in Equation (4).[12] *If there exists a possibility that the target zone will be realigned rather than defended, the nonlinearity becomes less prominent and disappears completely if a realignment is equally likely as a defended target zone.*

The importance of a nonlinear relationship between the exchange rate and underlying fundamentals depends on the exchange rate regime and the anticipated policy response. That is, under a flexible exchange rate regime, the exchange rate follows a linear solution path. For a credible target zone, the exchange rate is a nonlinear function of fundamentals, but if there is some likelihood that the current target zone is not defended and is instead realigned, the nonlinear component of the solution path becomes less important.[13]

3 The Bispectrum and Tests for Nonlinearity

Hinich (1982) developed tests based on the bispectrum to determine whether a time series is Gaussian and linear.[14] In this section, we present a brief review of the bispectrum[15] of a time series, and describe the testing procedures presented by Hinich.

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[11] Flood and Garber (1991) link the "smooth pasting" conditions to the theory of speculative attack on asset price-fixing regimes, thus providing an intuitive account of the boundary conditions for a credible target zone. Also see Dixit (1993) for a discussion on "smooth pasting" conditions.

[12] If $p > \frac{1}{2}$, then the exchange rate solution path follows a nonlinear process. However, in this case the exchange rate is more responsive to changes in fundamentals as the exchange rate approaches a boundary value than it is under a flexible exchange rate regime. Eichengreen and Wyplosz (1993) noted that a high probability of realignment could be destabilizing, the result of which replaces "the target zone honeymoon with a target zone divorce" (p. 120).

[13] The distribution for exchange rates under a credible target zone is U-shaped, which suggests that the exchange rate is more likely observed near one of its boundary values. Once the possibility of realignment is introduced, the long-run distribution is symmetrical and triangular, with more weight in the center than at boundary values (Bertola and Caballero 1992).

[14] Subba Rao and Gabr (1980) also presented tests for Gaussianity and linearity based on the bispectrum for small samples. Hinich's (1982) work, in contrast, was aimed at the asymptotic properties of the bispectrum. Because the samples used in this paper are large, we concentrate on Hinich's procedures.

[15] Brillinger and Rosenblatt (1967) and Priestley (1981) present thorough discussions of the bispectrum.

The bispectrum is defined as follows. Let $\{x(t)\}$ represent a zero mean, stationary time series. The third order cumulant function is given by

$$c(m, n) = E[x(t)x(t+m)x(t+n)]. \tag{7}$$

For a frequency pair (ω_1, ω_2), the *bispectrum* is the (double) Fourier transform of $c(m, n)$:

$$B(\omega_1, \omega_2) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c(m, n) \exp[-i(\omega_1 m + \omega_2 n)]. \tag{8}$$

The principal domain of the bispectrum is the triangular set $\Omega = \{0 \le \omega_1 \le \pi, \omega_2 \le \omega_1, 2\omega_1 + \omega_2 \le 2\pi\}$, due to the symmetries of the third order cumulant function (Brillinger and Rosenblatt 1976).[16]

To illustrate the usefulness of the bispectrum, consider the case where $\{x(t)\}$ is linear and Gaussian. Consequently, the power spectrum contains all the useful information in the time series. The bispectrum is then zero for all frequency pairs, because third order cumulants vanish. If $\{x(t)\}$ is linear, but not Gaussian, then the bispectrum is nonzero, but it is a constant independent of frequency. If $\{x(t)\}$ is nonlinear, then the bispectrum is not constant, and is instead a function of frequency.

A consistent estimator of the bispectrum for a finite sample $x(0), x(1), \ldots, x(N-1)$ is constructed as follows. Let $\omega_n = 2\pi n/N$ for $n = 0, 1, \ldots, N-1$. For each pair of integers j and k, define

$$F(j, k) = X(\omega_j)X(\omega_k)X^*(\omega_{j+k}). \tag{9}$$

where $X(\omega_j) = \sum x(t) \exp(-i\omega_j t)$ and $X^*(\cdot)$ is the complex conjugate. To obtain a consistent estimator of the bispectrum, $F(j, k)$ must be smoothed. A consistent estimator of the bispectrum is formed by averaging a square of M^2 estimates whose centers are defined by the lattice $L = \{(2m-1)M/2, (2n-1)M/2$: $m = 1, 2, \ldots, n$ and $m \le N/2M - n/2 + \frac{3}{4}\}$. For squares that lie completely inside the principle domain, a consistent estimator of the bispectrum is

$$\hat{B}(m, n) = M^{-2} \sum_{j=(m-1)M}^{mM-1} \sum_{k=(n-1)M}^{nM-1} F(j, k). \tag{10}$$

The choice of M governs the trade-off between bias and variance, and Ashley, Patterson, and Hinich (1986) suggest $M \approx .7\sqrt{N}$ for finite samples. Instead of estimating the double Fourier transform of the third order cumulant function, Equation (10) demonstrates that the bispectrum can be equivalently estimated as the product of the Fourier transform at frequency j and k times the conjugate at frequency $j + k$, and then smoothed.

The *estimated standardized bispectrum* is given by $2|\hat{X}_{m,n}|^2$ where

$$\hat{X}_{m,n} = \frac{\hat{B}(m, n)}{[N/M^2]^{\frac{1}{2}}[S(g_m)s(g_n)S(g_{m+n})]^{\frac{1}{2}}} \tag{11}$$

and $S(\cdot)$ is the (smoothed) estimated power spectrum for $g_i = (2i-1)M/(2N)$. Hinich (1982) demonstrated that under the null hypothesis of Gaussianity and linearity, $2|\hat{X}_{m,n}|^2$ is approximately chi-squared with 2 degrees of freedom for frequency pairs (f_m, f_n). The *test statistic for Gaussianity and linearity* is then

$$\hat{S} = 2 \sum_m \sum_n |\hat{X}(f_m, f_n)|^2 \tag{12}$$

which is chi-squared with $2P$ degrees of freedom where P is the number of squares with centers in the principle domain.

If $\{x(t)\}$ is linear but not Gaussian, then $2|\hat{X}_{m,n}|^2$ is distributed independent, noncentral chi-squared with 2 degrees of freedom. The noncentrality parameter is estimated by

$$\hat{\lambda} = \left\{ 2 \sum_m \sum_n |\hat{X}(f_m, f_n)|^2 / P \right\} - 2. \tag{13}$$

[16]The bispectrum is a complex function that measures the multiplicative nonlinear interaction for the frequency pairs (ω_1, ω_2) over its principle domain.

Thus, if the null is true, then the sample dispersion of $2|\hat{X}(f_m, f_n)|^2$ should be consistent with the population dispersion of $\chi^2(2, \hat{\lambda})$. If the time series under investigation is nonlinear, then the sample dispersion exceeds the dispersion of $\chi^2(2, \hat{\lambda})$. David (1970) showed that the sample 80% quantile, $\hat{\xi}_{.8}$, is asymptotically distributed as $N(\xi_{.8}, \sigma^2)$ where σ^2 is estimated by

$$\hat{\sigma}^2 = .8(1 - .8)f^{-1}(\hat{\xi}_{.8})P^{-1}, \tag{14}$$

$\xi_{.8}$ is the population 80% quantile of $\chi^2(2, \hat{\lambda})$, and $f(\cdot)$ is the corresponding density function. Ashley and Patterson (1989) defined the following *test statistic for linearity*

$$Z \equiv \hat{\xi}_{.8}/\hat{\sigma} \tag{15}$$

which is distributed $N(0, 1)$. The statistic (15) examines whether the sample dispersion of the standardized bispectrum estimates is significantly greater than the population dispersion.

An overview of the testing procedures based on the bispectrum is as follows:

1. Estimate the bispectrum using Equation (10) and then standardize the estimates as shown in Equation (11).
2. Test for Gaussianity using $\hat{S}$, as defined in Equation (12).
3. If Gaussianity is rejected, test for linearity using the sample dispersion of $2|\hat{X}(f_m, f_n)|^2$. That is, compare the sample dispersion with the population dispersion of $\chi^2(2, \hat{\lambda})$.

4 EMS Bilateral Exchange Rates

In this section, we examine the time series properties of EMS bilateral exchange rates. The data set consists of currency values of countries that have been members of the ERM since its inception in 1979: Belgium, Denmark, France, Ireland, Italy, Germany, and The Netherlands. Because of the dominant role of Germany in the European community (see, e.g., Herz and Roger 1992), currency values are expressed in terms of the deutschemark. For each exchange rate series, there are 706 observations representing the Friday spot market close price over the period March 1979–August 1992.[17] This section is divided into three parts. In Section 4.1, we examine the Martingale hypothesis, which holds that if exchange rates have a stochastic trend, then their first differences should be a white noise series. Gaussianity and linearity tests based on the bispectrum are reported in Sections 4.2 and 4.3, respectively.

4.1 Tests of the Martingale Hypothesis

In this section, we investigate the Martingale hypothesis for EMS bilateral exchange rates. The Martingale version of the efficient markets hypothesis maintains that asset returns are temporally uncorrelated. Tests of the Martingale hypothesis then center on whether exchange rates possess "both a unit root and uncorrelated increments" (Fong and Ouliaris 1995, p. 255). Meese and Singleton (1982) and Baille and Bollerslev (1989), among others, have examined the first condition finding that exchange rates contain a unit root. The second condition—uncorrelated increments—has been recently investigated by Fong and Ouliaris (1995), who employed the spectral shape tests proposed by Durlauf (1991). Fong and Ouliaris reported that many of the holding period returns of major exchange rates expressed in terms of the US dollar exhibit significant autocorrelations, thereby violating the Martingale hypothesis.

The unit root hypothesis is examined by performing augmented Dickey-Fuller (ADF) unit root tests on each bilateral exchange rate.[18] Unit root test results are summarized in Table 1. For each exchange rate measured in levels, we fail to reject the unit root hypothesis, as indicated in the first two columns of Table 1. After first-differencing the data, the unit root hypothesis is rejected, as indicated in the last two columns. Consequently, each exchange rate appears to contain a unit root.[19]

[17] Data is from the *World Currency Monitor* and various issues of the *Wall Street Journal*.

[18] We also used the unit root test developed by Phillips and Perron (1988), and conclusions based on this test are the same as those for the augmented Dickey-Fuller test.

[19] Although there is some controversy regarding whether or not bilateral EMS rates contain a unit root, the first-difference operator is a linear filter, and therefore does not affect nonlinearity testing.

Table 1
Unit Root Test Statistics

	(x_t) In Levels		(x_t) In First Differences	
Series (x_t)	**ADF(τ)**	**ADF(ϕ)**	**ADF(τ)**	**ADF(ϕ)**
Belgian Franc	−0.76	2.34	−6.44	20.75
Danish Krone	−1.79	5.78	−9.81	48.16
French Franc	−0.58	1.71	−6.60	21.80
Irish Pound	−1.06	1.07	−8.41	35.35
Italian Lira	−0.65	2.37	−8.39	35.22
Dutch Guilder	−2.88	4.30	−7.94	31.62

Notes: Unit root tests are based on the empirical model:

$$\Delta x_t = \alpha + \beta t + \gamma x_{t-1} + \sum_{i=1}^{12} \Delta x_{t-1} + e_t$$

where t is a linear time trend. The null hypothesis associated with ADF(τ) holds that $\gamma = 0$, and the 5% critical value is −3.42. The null hypothesis associated with ADF(ϕ) is $\gamma = 0$ and $\beta = 0$. The 5% critical value is 8.30.

Tests based on the normalized cumulated periodogram investigate whether exchange rate increments are temporally uncorrelated. The cumulated periodogram is a frequency domain technique that addresses whether each frequency component contributes equally to the variance of the series. The normalized cumulated periodogram is computed in two steps. First, the mean is removed from the time series, the Fourier transform is applied, and the sum of squares of the real and imaginary parts are calculated to arrive at the periodogram. Second, each periodogram component is normalized by the variance and cumulated. The normalized cumulated periodograms for the Belgian franc, Danish krone, French franc, Irish pound, Italian lira, and Dutch guilder are shown in Figure 1.

The null hypothesis associated with the Kolmogorov-Smirnov test holds that changes in the exchange rate are a white noise series, and have temporally uncorrelated increments. The Kolmogorov-Smirnov test specifies critical upper and lower values at each frequency component. If the cumulated periodogram exceeds the upper critical value or falls below the lower critical value at a frequency component, then the null is rejected.[20] The dotted lines in Figure 1 represent the 5% critical values.

The null hypothesis is accepted for the French franc and the Italian lira, indicating that returns are temporally uncorrelated and thereby supporting the Martingale hypothesis. Or, explained differently, returns for the French franc and the Italian lira are white noise series. This finding, however, does not imply that the time series is an independent random variable.[21] Only when the time series is Gaussian does whiteness imply independence.[22] Interestingly, whiteness is rejected for the Belgian franc, Danish krone, Irish pound, and Dutch guilder. Rejections of the null indicate that returns are temporally correlated, thus violating the Martingale hypothesis.[23]

4.2 Tests for Gaussianity

The statistical tests for Gaussianity are based on the estimated bispectrum given by Equation (10). To arrive at consistent estimates, the estimated bispectrum is smoothed by averaging estimates over 120 squares, as

[20]The critical values of the Kolmogorov-Smirnov test are given by the lines $f/n \pm 1.36(n-1)^{-\frac{1}{2}}$ where f is the frequency and n is the sample size (Fuller 1976, p. 287).

[21]Priestley (1981) presented a simple example where $\{x(t)\}$ is represented by

$$x(t) = \beta e(t-1)e(t-2) + e(t)$$

and $\{e(t)\}$ is an independent random process. In this example, $\{x(t)\}$ is a white-noise series, but it is clearly not independent.

[22]This point is forcefully made by Hinich and Patterson (1985).

[23]Fong and Ouliaris (1995) report that the Anderson-Darling and Cramer-von Mises statistics are more reliable than is the Kolmogorov-Smirnov test when heteroskedasticity is present. We also compute the Anderson-Darling and Cramer-von Mises statistics. Conclusions based on these statistics are consistent with the test results reported in Figure 1.

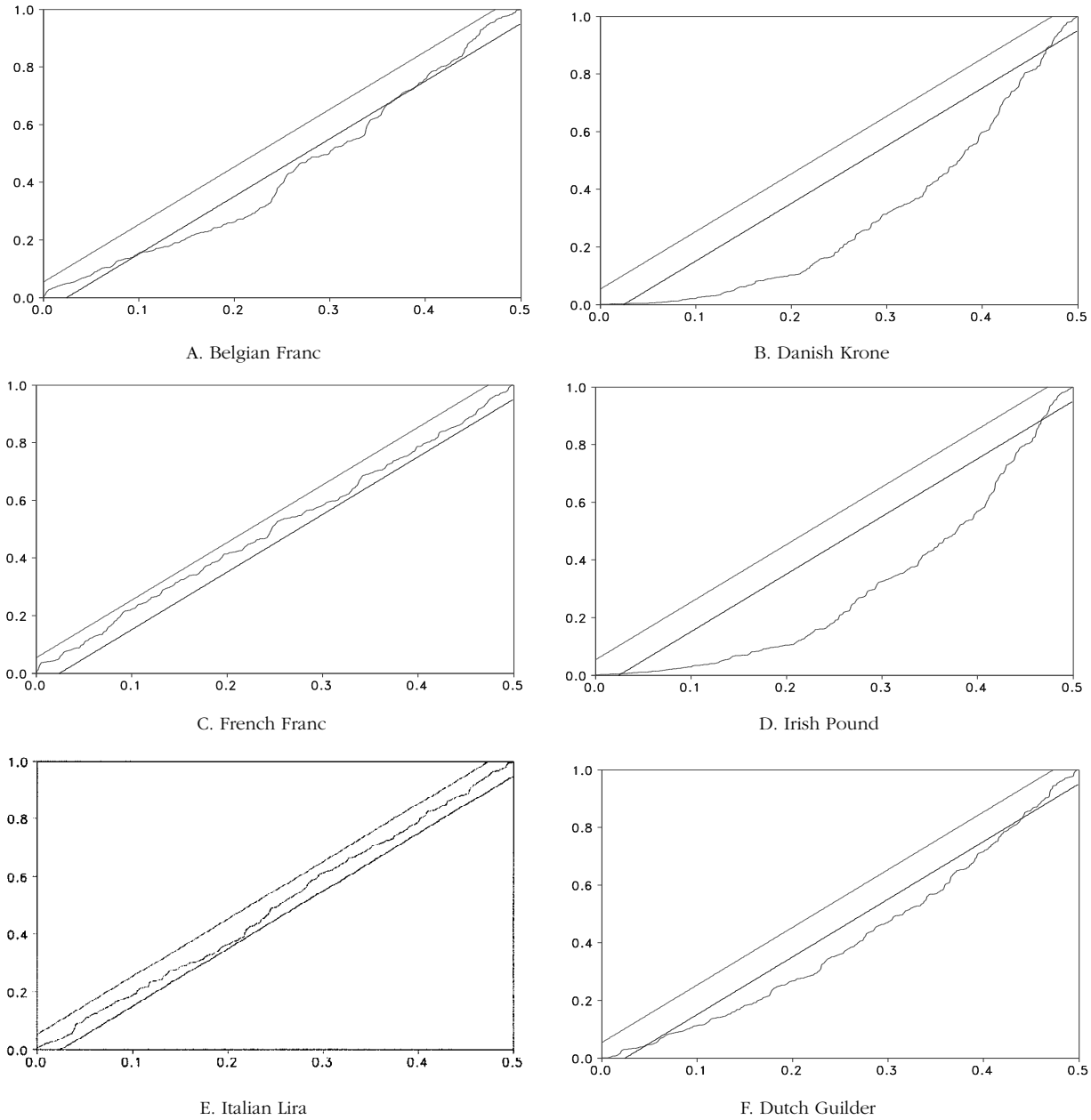


Figure 1
Normalized cumulative periodograms for EMS bilateral exchange rates. The horizontal axes measure frequency in cycles per week. Dotted lines represent 5% critical values.

suggested by Ashley, Hinich, and Patterson (1986).[24] The test statistic for Gaussianity, $\hat{S}$, sums the standardized smoothed bispectrum estimates. Under the null hypothesis of Gaussianity, $\hat{S}$ is distributed chi-squared with 2P degrees of freedom, where P is the number of squares, i.e., P = 120.[25] Thus, $\hat{S}$ is well approximated by the standard normal (Hinich 1982).

[24]Before the bispectrum is computed, the time series are prewhitened using an AR(4) filter. Ashley, Patterson, and Hinich (1986) present an equivalence theorem which proves Gaussianity and linearity tests are invariant to linear filtering.

[25]Gaussianity and linearity test statistics reported in this paper are based on bispectral estimates whose squares lie completely inside the principle domain. In addition, we compute test statistics that include squares in which bispectral estimates that lie outside the principle domain are included. Conclusions reached from Gaussianity and linearity tests are the same for each specification.

Table 2
Gaussianity Test Statistics

Series	Gaussianity Test Statistic
Belgian Franc	6.00
Danish Krone	6.01
French Franc	5.05
Irish Pound	5.83
Italian Lira	4.65
Dutch Guilder	4.92

Notes: The test statistic is distributed N(0,1). The critical values are 1.65 and 2.33 for the 1% and 5% levels, respectively.

The test statistic for Gaussianity for each bilateral exchange rate is shown in Table 2. For each exchange rate, Gaussianity is rejected at the 5% significance level.[26] Rejection of Gaussianity is consistent with numerous previous studies (see, e.g., Boothe and Glassman 1987; Wasserfallen 1989). Table 3 lends further insight to the Gaussianity tests. The five largest estimates of the standardized bispectrum are shown in the second column, along with corresponding frequency pairs shown in the first column. Under the null of Gaussianity, the individual standardized bispectral estimates are chi-squared with 2 degrees of freedom. The 5% critical value is 5.99. For each exchange rate series, the five largest values easily exceed 5.99. Moreover, if the null is true, then we expect 5% of the estimates to exceed the critical value. The last column, however, indicates that more than 8% of the standardized bispectrum estimates for the French franc, Irish pound, and Dutch guilder exceed the critical value. More than 9% of the standardized bispectrum estimates for the Italian lira exceed the critical value, while more than 10% of the estimates for the Belgian franc and the Danish krone exceed the critical value.

4.3 Tests for Linearity

Since its inception in March 1979, members of the ERM have maintained a system of fixed but flexible exchange rates while limiting exchange rate movements to a target zone. The early experience of the ERM witnessed numerous realignments. From its inception until the beginning of 1987, there were 11 realignments, which seriously compromised the credibility of the target zone system. But from 1987 until the currency crisis in the fall of 1992,[27] no further realignments occurred and the ERM experienced a significant gain in credibility (Frankel and Phillips 1992). The behavior of the French franc over the entire period is plotted in Figure 2. From March 1979 to January 1987, there were five realignments, but from 1987 to the fall of 1992, no realignments occurred. Thus, the early part of the sample period (1979–1987) is more closely associated with the realignment target zone model of Bertola and Caballero (1992), where the exchange rate is unlikely to follow a nonlinear process. The more recent experience, however, relates closely to the credible target zone model of Krugman (1991), where an inherent nonlinearity improves exchange rate stability. As a result, linearity tests examine two nonoverlapping subperiods, one of which more closely corresponds to the credible target zone model, while the other corresponds to the realignment target zone model.

Although Gaussianity and linearity tests are linked, a rejection of Gaussianity as reported in the previous part does not necessarily rule out linearity.[28] If bilateral exchange rates are linear, but not Gaussian, then the sample estimates of $2|\hat{X}_{m,n}|^2$ are distributed as a noncentral chi-squared random variable with 2 degrees of freedom, and the noncentrality parameter is given by Equation (13). That is, the sample dispersion of $2|\hat{X}_{m,n}|^2$ should not differ significantly from the population dispersion of $\chi_2^2(2, \hat{\lambda})$ if the time series is generated by a linear process.

Linearity test statistics are collected in Table 4. The first two columns correspond to the 1979–1987 subperiod, while the last two columns correspond to the more recent experience of the ERM, 1987–1992. For

[26] For each exchange rate, we find that the coefficient of kurtosis is significantly different from three.

[27] In September 1992, the ERM came increasingly under attack and yielded by allowing exchange rates to fluctuate 15% around a central parity rate, rather than the customary (for most countries) 2.25% range.

[28] We also compute Gaussianity test statistics for each subperiod. For each currency and subperiod, Gaussianity is rejected at the 1% significance level.

Table 3
Summary of Bispectral Estimates

Series	Frequency Pair (f_m, f_n)	Five Largest Values	Percentage Exceeding 5% Level
Belgian Franc	(.211, .023)	11.72	13.19
	(.180, .023)	11.59	
	(.054, .023)	11.26	
	(.086, .086)	8.89	
	(.133, .023)	8.52	
Danish Krone	(.054, .023)	42.36	14.18
	(.117, .117)	12.48	
	(.054, .008)	11.33	
	(.070, .008)	11.17	
	(.133, .054)	8.72	
French Franc	(.133, .008)	27.35	8.79
	(.008, .008)	14.83	
	(.133, .039)	9.29	
	(.133, .102)	8.66	
	(.086, .039)	7.85	
Irish Pound	(.054, .054)	40.24	8.79
	(.117, .117)	12.87	
	(.008, .008)	11.78	
	(.195, .023)	11.75	
	(.133, .086)	10.94	
Italian Lira	(.008, .008)	17.98	9.89
	(.117, .039)	16.02	
	(.086, .070)	13.27	
	(.148, .008)	11.69	
	(.086, .008)	10.30	
Dutch Guilder	(.195, .039)	26.34	8.79
	(.133, .070)	16.03	
	(.164, .054)	12.52	
	(.023, .023)	10.37	
	(.117, .117)	8.53	

Notes: Frequency is measured in cycles per week. The five largest values of $|\hat{X}(f_m, f_n)|^2$ are distributed chi-squared with 2 degrees of freedom under the null of Gaussianity. The 5% critical value is 5.99. Under the null of Gaussianity, only 5% of $|\hat{X}(f_m, f_n)|^2$ should exceed the critical value.

the 1979–1987 subperiod, the *sample* 80% quantile for each bilateral rate and the *population* 80% quantile for $\chi_2^2(2, \hat{\lambda})$, which is in parentheses, are collected in the first column. For each rate, the sample dispersion exceeds that of $\chi_2^2(2, \hat{\lambda})$. Linearity test statistics then examine whether the sample dispersion is significantly greater than that of $\chi_2^2(2, \hat{\lambda})$. The second column shows linearity test statistics for the 1979–1987 subperiod, where the test statistic is distributed N(0,1). We fail to reject linearity at the 5% significance level for each currency, with the exception of the Italian lira.[29] Rejection of linearity supports the realignment target zone model, where the presence of a target zone does not improve exchange rate stability.

[29]The Italian lira is generally regarded as the most volatile currency. If the probability of realignment exceeds 50%, then according to the realignment model summarized in Equations (5) and (6), there may exist a nonlinear relationship between the spot exchange rate and fundamentals. In fact, the exchange rate is more responsive to changes in fundamentals than is the case under linearity.

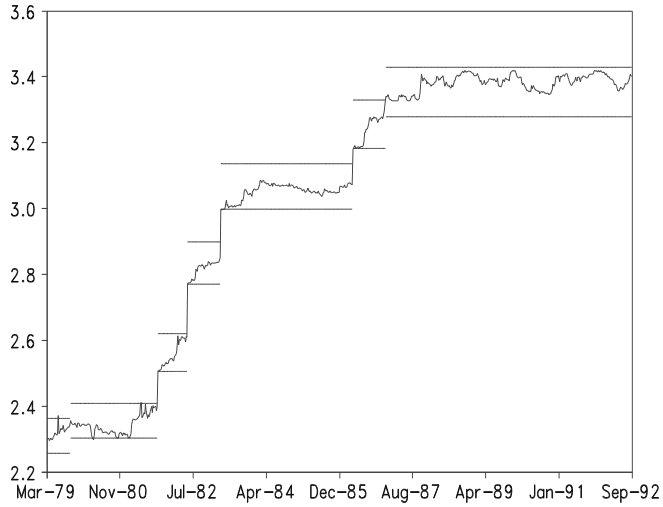


Figure 2
The French franc/deutschemark exchange rate time path. Lines represent ERM target zones. Discontinuities correspond to realignments of the central parity rate.

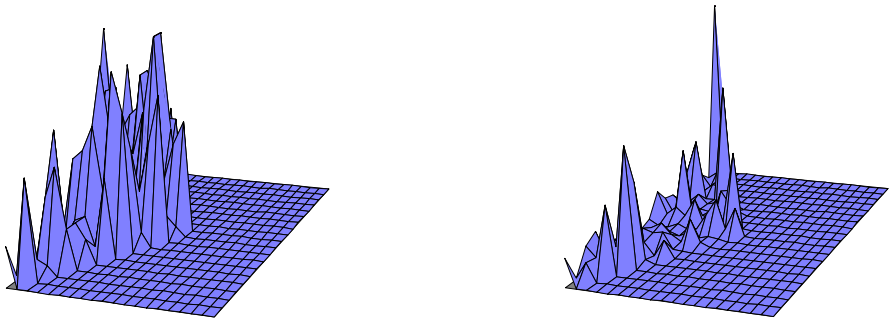
Table 4
Linearity Test Statistics

	1979–1987		1987–1992	
Series	**Sample 80% Quantile Range**	**Linearity Test Statistic**	**Sample 80% Quantile Range**	**Linearity Test Statistic**
Belgian Franc	3.38 (3.35)	0.90	4.43 (3.38)	9.54
Danish Krone	3.41 (3.35)	0.69	3.94 (3.37)	5.79
French Franc	3.42 (3.27)	1.58	4.07 (3.35)	7.31
Irish Pound	3.35 (3.24)	1.26	3.48 (3.34)	1.49
Italian Lira	3.85 (3.29)	5.69	3.47 (3.34)	1.56
Dutch Guilder	3.37 (3.30)	0.45	3.64 (3.27)	4.45

Notes: The population 80% quantile range for $\chi^2_{.8}(\hat{\lambda})$ is shown in parentheses. The test statistic is distributed N(0,1). The critical values are 1.65 and 2.33 for the 1% and 5% levels, respectively.

The sample dispersions of the standardized bispectral estimates over the period 1987–1992 exceed their population values for each exchange rate, as shown in the third column of Table 4. The last column examines whether the sample dispersion is significantly greater than the population value. Linearity is rejected for the Belgian franc, the Danish krone, the French franc, and the Dutch guilder. For these currencies, a target zone arrangement may have proven stabilizing, as stressed by the credible target zone model. A rejection of linearity implies that returns are not independent, and are therefore potentially forecastable (Hinich and Patterson 1985). This holds even if the estimated power spectrum is flat, as suggested by the Kolmogorov-Smirnov test, as is the case for the French franc. Thus, the existence of a nonlinearity and uncorrelated exchange rate returns are consistent with one another. We fail to reject linearity for the Italian lira and the Irish pound. For these currencies, a target zone arrangement does not appear to have improved exchange rate stability even over the more recent experience of the ERM.

Figure 3 illustrates standardized bispectrum estimates for the French franc, which offer an intuitive account of the Gaussianity and linearity testing procedures. Figure 3A corresponds to the 1979–1987 subperiod, and Figure 3B corresponds to the 1987–1992 subperiod. Recall that the bispectrum is independent of frequency and is constant if the series conforms to a linear model, and is zero if the series is Gaussian. Clearly, the standardized bispectral estimates are nonzero over its triangular principle domain (observations outside the principle domain are set equal to zero) for both subperiods. This finding reflects the rejection of Gaussianity. Because Gaussianity is rejected, linearity tests then examine whether the standardized bispectrum estimates



A. 1979–1987 subperiod B.1987–1992 subperiod

Figure 3
Standardized bispectral estimates for the French franc/deutschemark.

are constant over their principle domain. For the early subperiod, linearity is not rejected, which indicates that the bispectral estimates shown in Figure 3A do not differ significantly. In contrast, linearity is rejected over the later subperiod, which suggests that the bispectral estimates plotted in Figure 3B do in fact differ significantly over the principle domain.

5 Conclusion

In this paper, we test for nonlinearity in the data generating process of EMS bilateral exchange rates operating under the ERM using the bispectrum. The EMS system has undergone dramatic change from one of frequent realignment to one where realignments are the exception rather than the rule. Many studies identify the beginning of 1987—the date of the last general realignment—as separating the experience of the ERM into two distinct periods (see, e.g., Frankel and Phillips 1992; Eichengreen and Wyplosz 1993). The early history of the ERM stands in stark contrast to the credible target zone model. Instead, exchange rate movements more closely correspond to the predictions of the realignment target zone model, where the existence of a stabilizing exchange rate nonlinearity is unlikely. Results of linearity tests based on the bispectrum support this claim in that EMS bilateral exchange rates follow a linear process, suggesting that the ERM has not proven entirely effective in limiting exchange rate movements. One exception is the Italian lira. In the case of the lira, linearity is rejected; however, it is not evident that this finding supports improved stability. Instead, it is likely that a target zone has made the lira more responsive to changes in fundamentals.

From 1987 until the currency crisis in the fall of 1992, no realignments occurred, and the ERM enjoyed enhanced credibility. Exchange rate volatility dropped significantly, and the tendency for weaker currencies to depreciate against the deutschemark decreased substantially. An important implication is that this period is more closely associated with the credible target zone model that stresses improved stability traced to a nonlinear relationship between exchange rates and fundamentals. Linearity tests support this claim. Linearity is rejected for the Belgian franc, the Danish krone, the French franc, and the Dutch guilder. In these cases, the ERM appears to have limited exchange rate movements, thereby stabilizing currency values. On the other hand, we fail to reject linearity for the Irish pound and the Italian lira. For these currencies, increased stability is suspect, and it is unlikely that a target zone arrangement has proven effective in stabilizing currency values.

While the bispectrum dismisses the need for specifying and modeling fundamentals, it is not equipped to identify the form of a detected nonlinearity. Consequently, the bispectrum does not offer a direct test of the specific nonlinear relationship stressed by the credible target zone model. The absence of a nonlinearity, however, does indicate a rejection of the credible target zone model. In future work, we plan to extend the analysis to jointly consider interest rate differentials and exchange rate movements.

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