Investigating Cyclical Asymmetries

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Abstract. This paper accomplishes two goals. First, it introduces a powerful nonparametric test of asymmetry to the economics literature, namely, the triples test of Randles et al. (1980). Second, it documents the presence of two specific kinds of asymmetry in U.S. macroeconomic time series. Depth, or asymmetry in the distribution of a (detrended) series, is a feature of numerous economic time series; and steepness, or asymmetry in the distribution of first differences, is a feature of hours, employment, and the unemployment rate, but is absent from real GDP and aggregate industrial production. The pattern of asymmetries found provides guidelines for restricting the set of alternative nonlinear models from which to select in modeling these time series.

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1 Introduction

A basic issue facing econometricians investigating a time series for which linearity has been rejected is the selection of an appropriate nonlinear model from among the enormous number of potential choices. A limitation of many tests for nonlinearity is that they cannot clearly indicate any particular form of nonlinearity.2 Cyclical asymmetries form an important class of possible nonlinearities, as many popular theories suggest their existence in aggregate data. (See Verbrugge [1997a] for a long list.) While asymmetry has been noted in a number of economic time series, there are many distinct forms of asymmetry, and most asymmetry tests cannot readily distinguish between them or have low power; thus, there has been little categorization of asymmetry.3 The present paper is a step toward a systematic investigation of two particular forms of asymmetry: depth, defined as asymmetry in the distribution of a (detrended) series; and steepness,

1This paper is a drastically shortened version of Verbrugge (1997a).
2There are numerous nonlinearity tests for which a rejection suggests the particular alternative embedded in the test. However, these parametric tests may have low power against other nonlinear alternatives (see Granger and Teräsvirta [1993]). Below, I suggest that model selection is best begun with nonparametric tests.
3There is some controversy as to which particular form (or forms) of asymmetry are being detected by the procedures of McNevin and Neftçi (1992), Ramsey and Rothman (1996), Kim, Mittnik, and Rachev (1996), and Hinich and Rothman (1997); see Verbrugge (1997a) for a brief discussion.
defined as asymmetry in the distribution of first differences. A second contribution of this paper is the introduction of a nonparametric asymmetry test, called the triples test, to the economics literature. This test has two advantages: it has considerably more power than common tests based upon the skewness coefficient; and since it is not a moment-based test, it cannot be dominated by outliers.

I find clear evidence for depth in numerous economic time series. Steepness is identified in all labor-market series investigated; it is not a characteristic of real GDP or of aggregate industrial production, although it is a feature of government spending.

Though this work has several sharp implications, for brevity, only two will be discussed. First, the asymmetries provide clues about underlying economic mechanisms, and may provide a means of differentiating among competing theories. Second, this study is the first step in a nonlinear model-selection procedure, which is best begun using nonparametric tests such as these, since they do not select a model a priori. Such tests highlight the particular nonlinear features of the data that are important, which aids in selecting an appropriate set of nonlinear models to investigate further. Model selection should include subsequent competition between alternatives (see Granger, King, and White 1995).

Section 1 forms an introduction to this paper. Section 2 reviews studies closely associated with this one. Section 3 presents the methodology and results of this paper, and Section 4 contains the conclusion.

2 Related Literature

The number of studies investigating different aspects of asymmetry is growing rapidly. The present survey will confine itself to investigations of the specific asymmetries of depth and steepness. The seminal study of time-series asymmetry is Nefci’s (1984) study of the U.S. unemployment rate. His procedure, applied to an unfiltered time series, will identify steepness. It was critiqued and corrected by Sichel (1989) and Rothman (1991). Falk (1986) applied Nefci’s procedure to detrended cross-country data, and found little evidence of asymmetry.

Other studies that have used this method on detrended data include Westlund and Ohlén’s (1991) and McNevin and Nefci’s (1992); the later study utilized one-sided moving average filters, making their interpretation especially problematic.

DeLong and Summers (1986) introduced the use of a test based upon the coefficient of skewness, and studied production and unemployment data in six OECD countries. Unlike Nefci’s procedure, it does detect steepness in trending data. The only series found to exhibit steepness was the U.S. unemployment rate. A

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4This study restricts itself to the investigation of key quarterly U.S. macroeconomic time series; Verbrugge (1997a) studies numerous other U.S. quarterly and monthly time series, and the effect of aggregation and temporal aggregation on asymmetry. Verbrugge (1997b) is a related cross-country investigation. The scope of this paper is limited in that, though I document these asymmetries and sketch the outline of an appropriate procedure, no attempt is made to select and estimate an appropriate nonlinear model for any of the series investigated.

5See Randles et al. (1980) and Eubank, LaRiccia, and Rosenstein (1992) for Monte Carlo studies, and Verbrugge (1997a) for a power comparison. This outlier property showed up during this investigation (see Verbrugge [1997a]).

6Steepness is found in more disaggregated output series, such as in durables and in several two-digit sectors, and in nonresidential investment and inventories (see Verbrugge [1997a]).

7Though stylized facts such as these can rule out linear models, they can not immediately discriminate between all possible asymmetric models. In that sense, work like this is but the first step in moving toward a more complete and accurate picture of the economy. However, see Hooker (1997) for an example of a reduced-form parametric study utilizing asymmetries in the data to discriminate between different theories.

8Upon estimation of a particular nonlinear model, the economic significance of the asymmetry may be more readily determined (see Potter [1994, 1995]. A thorough analysis may require knowledge of a loss function.

9For example, the detection of depth in a series would suggest that a three-state model-growth-switching model, or perhaps some sort of asymmetric bilinear model, should be in the set of models considered. Similarly, the detection of steepness in a series should lead the researcher to search among specifications that can capture that form of asymmetry, such as Threshold Autoregressive (TAR), Self-Exciting Threshold Autoregressive (SETAR), or Smooth Threshold Autoregressive (STAR) models. Linearity rejection combined with a failure to detect asymmetry might suggest that a (standard) bilinear model should be tested for. Granger and Teräsvirta (1993) discuss numerous nonlinear time-series models and associated tests. (See also Mittnik and Niu [1994]). A model-selection procedure that begins with nonparametric tests like these, which appropriately narrow the list of possibilities by noting the major forms of nonlinearity that characterize the series in question, is likely to be more efficient, and less judgmental, than selecting the set of alternatives a priori. Similar general points have been made previously by Granger and Teräsvirta (1995) and by Teräsvirta (1996) (see conclusion).

10Verbrugge (1997a) contains a much more complete survey of the literature. In addition, the author has a (undoubtedly incomplete) bibliography of work on asymmetries done through 1996, including parametric studies. See also Mittnik and Niu (1994) for a survey and discussion of the literature.

11“Nefci asymmetry” in detrended data is neither necessary nor sufficient for steepness. Thus, it is not a good test for steepness in trending series, although it may be of independent interest.
similar test was used by Westlund and Öhlén (1991), who failed to find steepness in Sweden’s unemployment rate. Sichel (1993) improved upon this test and confirmed earlier studies, and his procedure has been applied by Sensier (1996) to U.K. data.

Sichel (1993) was the first study to investigate depth. He detected depth in U.S. GNP, industrial production, and the unemployment rate. Sensier (1996) found evidence for depth in two inventories series, but not in real U.K. GDP. Several other studies not explicitly investigating depth nonetheless bear on the question (see Verbrugge 1997a). \footnote{An argument can be made that the TR test procedure (e.g., Ramsey and Rothman [1996]), the REVERSE test procedure (Hinich and Rothman [1997]), and some of the procedures used by McNevin and Neftci (1992) might be indicating the presence of depth rather than steepness, since asymmetry is being detected in levels of a detrended or first-differenced series. (Personal communication between the author and Phil Rothman.)}

What, then, is the evidence to date? Two U.S. unemployment rates are steep. No indisputable evidence has been found supporting steepness in any production or income series, or in the unemployment rate of any other country. U.S. industrial production and U.S. real GDP are deep, as are the U.S. unemployment rate and two U.K. inventory series. About these asymmetries, not much more can be said with confidence, and much work remains to be done.

3 Methodology and Results

3.1 Methodology

A time series is steep if it possesses asymmetry in first differences (loosely, if its contractions are “steeper” than its expansions, or vice versa). A time series is deep if it possesses asymmetry in levels around a trend (loosely, if its troughs are deeper than its peaks are tall, or vice versa). \footnote{Ramsey and Rothman (1996) note that depth is a form of longitudinal asymmetry, that steepness is a form of transversal asymmetry, and that other forms of longitudinal and transversal symmetry are possible.} These two types of asymmetries are illustrated in Figure 1 for a trendless time series.

![A deep time series (a transversal asymmetry)](image1)

![A steep time series (a longitudinal asymmetry)](image2)

Figure 1

Depth and steepness of trendless times series.

Note: Illustrated above are two distinct kinds of asymmetries which a trendless time series might exhibit.

The asymmetry test utilized is the nonparametric triples test of Randles et al. (1980). Its intuitive basis is the following. Take all possible triples from the sample of size $N$ (i.e., $\binom{N}{3}$ combinations). If “most” of these triplets are right-skewed, infer that this is true of the underlying distribution. More formally, a triple of observations $(X_i, X_j, X_k)$ is a right triple (is skewed to the right) if the middle observation is closer to the smaller observation than it is to the larger. An example of a right triple is $XXX$. 

$\text{XX}$ ________ $X$. 

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Let

\[ f^*(X_i, X_j, X_k) := \frac{1}{3} \{ \text{sign}(X_i + X_j - 2X_k) + \text{sign}(X_i + X_k - 2X_j) + \text{sign}(X_j + X_k - 2X_i) \} \]  

(1)

The range of this function is \([-\frac{1}{3}, 0, \frac{1}{3}]\); a right triple is a triple that maps into \(\frac{1}{3}\), and a left triple is defined analogously. The triples test statistic is given by

\[ \frac{\hat{\eta} - \eta}{\sqrt{\hat{\sigma}^2 / N}} \]  

(2)

where

\[ \hat{\eta} := \frac{1}{\binom{N}{3}} \sum_{i<j<k} f^*(X_i, X_j, X_k), \]  

(3)

and

\[ \hat{\sigma}^2 / N := \frac{1}{\binom{N}{3}} \sum_{c=1}^{3} \binom{3}{c} \left( \frac{N-3}{3-c} \right) \hat{\varsigma}_c \]  

(4)

where

\[ \hat{\varsigma}_1 := \frac{1}{N} \sum_{i=1}^{N} (f_1^*(X_i) - \hat{\eta})^2 \quad \text{with} \quad f_1^*(X_i) := \frac{1}{\binom{N-1}{2}} \sum_{j<k} f^*(X_i, X_j, X_k), \]  

(5)

\[ \hat{\varsigma}_2 := \frac{1}{\binom{N}{2}} \sum_{j<k} (f_2^*(X_j, X_k) - \hat{\eta})^2 \quad \text{with} \quad f_2^*(X_j, X_k) := \frac{1}{N-2} \sum_{i=1}^{N} f^*(X_i, X_j, X_k), \]  

(6)

and

\[ \hat{\varsigma}_3 := \frac{1}{9} - \hat{\eta}^2. \]  

(7)

The null hypothesis is \(\eta = 0\). The asymptotic distribution of the test statistic is standard normal, so conventional critical values may be used.

The triples test is highly regarded in the statistics literature. Eubank, LaRiccia, and Rosenstein (1992) suggest that the triples test is the test of choice against unimodal alternatives to symmetry. The test’s insensitivity-to-outliers advantage has been noted. The disadvantage of this test is that data are assumed to be independent draws from a common distribution, which is clearly not the case for the series investigated herein. This necessitates an appropriate adjustment of the critical values, done here as follows. The null hypothesis of the test (in this context) is: the data-generating process is a (linear) ARMA process with well-behaved iid symmetric errors. A Monte Carlo procedure is performed to estimate the finite-sample \(p\)-value of the test statistic on each individual series. One begins by selecting and estimating an appropriate ARMA model for the series, and then performing 1,000 replications of the estimated process by drawing with replacement from a symmetrized distribution of the estimated residuals, calculating the test statistic for each replication.\(^{14}\)

Since this test applies only to stationary series, nonstationary series must be rendered stationary by filtering. The filter used must extract the component appropriate for the particular asymmetry being tested for. First differencing is the obvious filter to use in testing for steepness, since it arguably induces stationarity in all of the economic time series investigated here, and asymmetry in first differences is precisely the definition of steepness. However, to test for depth, series which are trending or are otherwise nonstationary must be detrended by some means. As Westlund and Öhlén (1991) point out, trend elimination is always at least partly

\(^{14}\)The symmetrized distribution of the residuals is composed of the estimated residuals and their additive inverses.
judgmental, and it may remove part of the asymmetry dimension of the series. Any detrending procedure must satisfy two criteria: first, it must succeed in rendering the series stationary; and second, it must be a linear filter, otherwise the filter itself may induce asymmetry in the original series. I consider three candidates: Hodrick-Prescott (HP) filtering, linear detrending, and Beveridge-Nelson (BN) detrending. The HP filter is appropriate to use in all cases, as it is a linear filter (hence it cannot induce any form of nonlinearity in the series), and as it will render stationary any time series that is integrated of order four or less. Simple linear detrending is inappropriate for processes that are not trend-stationary, as it will not render such processes stationary. Below, if the null of a unit root is rejected at the 5% level of significance by ADF tests for a given series, linear detrending is utilized. BN detrending is inappropriate for processes that do not possess unit roots; hence, it is utilized only on series for which ADF tests fail to reject a unit root at the 5% level of significance. In cases where results differ across unit-root tests, both methods are utilized. Note that this test cannot distinguish between a (asymmetric) nonlinear data-generating process and a linear process with asymmetric disturbances. Thus, once asymmetry is identified, a natural next step in the model selection process is to apply a general test of nonlinearity. This task is not attempted here; it is left for future work.

3.2 Data and results
The following quarterly post-war U.S. data are analyzed: the unemployment rate; the index of total good-producing hours; logs of total private employment, the industrial production index, real GDP, real consumption, real investment, and real government spending; “log” of real net exports; the index of coincident indicators; and the CPI. Data are from the Bureau of Economic Analysis. I utilize the standard setting of the smoothing parameter—1,600—for the HP filter. The BN filter necessitates losing observations near the beginning of the series; generally, three years are dropped. The p-value reported is the two-sided significance level at which the null of the coefficient equaling 0 can be rejected.

Results in Table 1 indicate that depth is pervasive, and steepness is surprisingly common. Significant depth is detected in most of the HP-filtered series investigated, aside from real consumption, real government spending, and the CPI. Note that while real GDP is negatively deep, one of its component series, net exports, is positively deep. If HP or linear detrending is taken to be appropriate, it follows that depth is a systematic characteristic of U.S. economic time series.

As the results of Sichel (1993) suggest, BN-detrended series are much less likely to exhibit asymmetry than HP-filtered series. What is the intuition? As noted in footnote 15, an ARIMA trend may track the asymmetry in the underlying series; in such cases, BN detrending can remove part of the asymmetry in the series. This suggests that BN detrending is inappropriate for the purpose of detecting depth asymmetry.

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15The appropriate test procedure is not to test the residuals from an (appropriate) ARIMA regression. Why? It frequently turns out that the fitted series closely mimics the original series, including its asymmetry, leaving the residuals insignificantly asymmetric and leading to type-II error, as the Monte-Carlo procedure makes clear. Clark (1987) and Westlund and Öhlén (1991) offer a similar criticism: when detrending, the flexibility of ARMA representations can yield a trend fit that is too successful (having a high R^2), leaving the residual cycle noisy and insignificant. Failure to detect much asymmetry in residuals has been noted previously (e.g., Potter 1994).

16This filter is of interest if for no other reason than that it is widely applied in the macro literature, and its properties have been extensively studied; see, for example, Harvey and Jaeger (1993), King and Rebelo (1993), and Cogley and Nason (1995). Applied to quarterly data with the standard parameter setting, the filter’s properties mimic that of a band-pass filter that retains those components of the data with periodicity between 6 and 32 quarters. Most importantly, the HP filter is linear; therefore it cannot induce asymmetry into the data.

17The BN decomposition is only defined on unit-root processes (see Hamilton [1994]). In this context, it has another potentially serious drawback, which is alluded to above: the ARIMA fit itself may be asymmetric.

18As net exports are frequently negative, but might conceivably grow in absolute value along with GDP, the data are transformed as follows: \( \log(X_t) - \log(X_{t-1}) \). What is going on? In that case, the BN trend has greater amplitude than the investment series itself. More colorfully, the BN trend appears to be fluctuating around the BN cycle.

19Data are from the Bureau of Economic Analysis. I utilize the standard parameter setting, the filter’s properties mimic that of a band-pass filter that retains those components of the data with periodicity between 10 and 100,000 (see Verbrugge [1997a]). In short, depth statistics are quite insensitive to the choice of this parameter, and there is no straightforward relationship between the parameter value and detected asymmetry.

20These data may be obtained from EconData at the University of Maryland by anonymous FTP.

21Upon the suggestion of H. Pesaran, I investigated the sensitivity of the depth statistics to choosing the smoothing parameter between 10 and 32 quarters. Most importantly, the HP filter is linear; therefore it cannot induce asymmetry into the data.

22A RATS procedure is kindly provided by Philip Meguire, based on the algorithm of Paul Newbold (1990).

23In the case of real investment, detrending by either method yields a significantly asymmetric series, but the sign of the asymmetry is actually reversed. What is going on? In that case, the BN trend has greater amplitude than the investment series itself. More colorfully, the BN trend appears to be fluctuating around the BN cycle.
Table 1
Depth and Steepness Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Depth</th>
<th>Steepness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Filter</td>
<td>$\hat{\eta}$</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>HP</td>
<td>.062</td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>.059</td>
</tr>
<tr>
<td></td>
<td>none</td>
<td>.010</td>
</tr>
<tr>
<td>Total goods-producing hrs.</td>
<td>HP</td>
<td>-.058</td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>.001</td>
</tr>
<tr>
<td>Log total private employment</td>
<td>HP</td>
<td>-.046</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.088</td>
</tr>
<tr>
<td>Log real GDP</td>
<td>HP</td>
<td>-.038</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.004</td>
</tr>
<tr>
<td>Log real consumption</td>
<td>HP</td>
<td>-.011</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.019</td>
</tr>
<tr>
<td>Log real investment</td>
<td>HP</td>
<td>-.057</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.043</td>
</tr>
<tr>
<td></td>
<td>LD</td>
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<tr>
<td>Log real government spending</td>
<td>HP</td>
<td>.017</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.025</td>
</tr>
<tr>
<td>“Log” real net exports</td>
<td>HP</td>
<td>.072</td>
</tr>
<tr>
<td></td>
<td>LD</td>
<td>.112</td>
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<tr>
<td>Log industrial production</td>
<td>HP</td>
<td>-.050</td>
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<tr>
<td></td>
<td>BN</td>
<td>.006</td>
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<tr>
<td>Log index of coincident indicators</td>
<td>HP</td>
<td>-.045</td>
</tr>
<tr>
<td></td>
<td>BN</td>
<td>.025</td>
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<td>Log CPI</td>
<td>HP</td>
<td>.010</td>
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<tr>
<td></td>
<td>BN</td>
<td>.010</td>
</tr>
</tbody>
</table>

Notes: This table reports tests of depth and steepness of various economic time series. Depth refers to asymmetry in levels of a detrended series; steepness refers to asymmetry in first differences. The detrending filters used for depth test are Hodrick-Prescott filtering (HP), linear detrending (LD), and Beveridge-Nelson detrending (BN); the filter used for the steepness test is first differencing (FD). First differences of a logged series are growth-rate approximations. The test used is the triples test of Randles et al. (1980).

1 $Z$ is the test statistic, which is asymptotically distributed standard normal.
2 (two-sided) $p$-values for standard normal variable.
3 (two-sided) $p$-values calculated using Monte Carlo methods described in the paper.

Each labor-market series tested exhibits steepness, while there is no evidence of steepness in “output” variables.24 (Results in Verbrugge [1997a] indicate that steepness is found in several components of investment, and in several components of industrial production. Further, temporal aggregation generally masks underlying asymmetry to a greater or lesser extent.) Notably, there is no evidence of steepness in (aggregate) investment growth rates, while there is strong evidence of steepness in the quarterly CPI.25 26 The next generation of macromodels will need to come to grips with this asymmetry in the labor market, and the lack of such asymmetry in production data.

24 This is aside from real government spending and (not surprisingly) in the index of coincident indicators. This finding does not support a model with growth-regime switching (for example).
25 Both nonresidential investment and inventory levels exhibit significant steepness (see Verbrugge [1997a]).
26 This finding confirms results in Ramsey and Rothman (1996) and Barnhart and Dwyer (1996); price level steepness is nearly ubiquitous in cross-country data (see Verbrugge [1997b]).
Small sample $p$-values of the triples test are by and large encouraging; a rejection of symmetry at the 5% level of significance using standard $p$-values would rarely be misleading. Thus, serial correlation does not appear to overly distort the size of the test for this sample size.

4 Conclusion

This paper accomplished two goals. First, a powerful nonparametric asymmetry test, the triples test of Randles et al. (1980), was introduced and applied to test for the presence of two distinct forms of asymmetry: steepness and depth. Given its relative power and specificity, the test procedure outlined herein may well be the procedure of choice in ascertaining the existence of these asymmetries. It is complementary to extant tests in the literature, in that it may perform the important role of categorizing the exact form of asymmetry, in addition to detecting it. Second, the pattern of two distinct types of asymmetry in key U.S. macroeconomic time series was documented. Asymmetry is prevalent in the U.S. economy. Depth is found in many labor-market and output variables; steepness is found in many labor-market variables, but not in aggregate production or output series.27 This analysis is extended in Verbrugge (1997a, 1997b).

Two important implications were suggested. First, the asymmetries provide clues about underlying economic mechanisms, and may help differentiate between competing theories. Second, this type of study is the first step in a model-selection procedure, which is best begun using nonparametric tests such as these that do not select a model a priori. Teräsvirta (1996) states, “... it seems to me that an important precondition for a successful model-selection technique is that the set of alternative models from which to choose be sufficiently restricted.” Knowing the precise pattern of asymmetry present in the data can be a significant aid in suitably restricting this set.

References


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27 The implications for the existence of a *Business Week* free-fall month are unclear (see Verbrugge [1997a]).


The SNDE is formed in recognition that advances in statistics and dynamical systems theory may increase our understanding of economic and financial markets. The journal will seek both theoretical and applied papers that characterize and motivate nonlinear phenomena. Researchers will be encouraged to assist replication of empirical results by providing copies of data and programs online. Algorithms and rapid communications will also be published.