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The Current Depth-of-Recession and Unemployment-Rate Forecasts

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Abstract. *Building upon Beaudry and Koop's (1993) analysis, we consider a "current depth of the recession" (CDR) variable in modeling the time-series behavior of the postwar quarterly U.S. unemployment rate. The CDR approach is consistent with the state-dependent behavior in the unemployment rate documented in the business-cycle asymmetry literature. We show that while the CDR effect is significant in-sample, no statistically significant out-of-sample forecast improvement is obtained relative to the linear alternative. Augmenting an AR(2) model by inclusion of the CDR term, however, does not significantly worsen the out-of-sample forecast performance.*

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Keywords. nonlinearity, business cycle asymmetry, depth of recession, forecasting

1 Introduction

Since the publication of Neftci's (1984) seminal paper, many researchers have documented significant nonlinearity in the movement of the postwar quarterly U.S. unemployment rate over the course of the business cycle. Such nonlinear dynamic behavior is said to be asymmetric in the sense that the unemployment rate increases quickly in recessions, but declines relatively slowly during expansions. Cyclical asymmetry of this type is inconsistent with the conventional linear time-series tools used in the analysis of macroeconomic data.

The importance of this issue for time-series modeling of the unemployment rate is (at least) twofold. First, the standard second-order approach for producing in-sample diagnostics may be misleading. For example, Potter (1995) shows how a state-dependent impulse response function differs rather strongly from one based on an estimated autoregressive moving average (ARMA) model. Second, it may be possible to reduce out-of-sample mean squared prediction error (MSPE) with a nonlinear forecast relative to a conventional ARMA forecast. Rothman (1998), for example, conducts an extensive out-of-sample forecasting competition between a linear model and many nonlinear alternatives for the postwar U.S. unemployment rate. In several cases, the nonlinear forecasts produce statistically significant MSPE reductions.

Our primary interest in this paper is in out-of-sample forecasting for the U.S. unemployment rate. In particular, we focus on the out-of-sample forecasting performance of a class of models introduced by Beaudry and Koop (1993). These authors augment an autoregressive model of U.S. GNP growth rates by adding a term they label *CDR*, a variable that measures the "current depth of the recession" in the following sense: for each

Table 1
Tests for Integration of Unemployment Rates: 1948:Q1–1995:Q1

Statistic[a]	Observed Value of Statistic (Significant at the 5% level)
Stock-Watson q_f^τ	−23.02
Stock-Watson q_f^μ	−16.78
Dickey-Fuller $\hat{\tau}_\tau$	−3.84
Dickey-Fuller $\hat{\tau}_\mu$	−3.11

[a]The Stock-Watson and Dickey-Fuller statistics for a unit root are described in Stock and Watson (1988) and Fuller (1995). Statistics were calculated with an AR(2) correction.

observation, CDR equals the difference between (1) the maximum value of GNP (measured as the natural logarithm of the level of GNP) from the start of the sample to the current period; and (2) the current value of GNP. CDR, then, takes on positive values during a recession and the early periods of a business-cycle expansion. Once GNP has reached a new peak and continues growing, CDR equals zero. Our definition of CDR for the unemployment rate differs slightly from that used by Beaudry and Koop (1993), since the unemployment rate: (1) is a counter-cyclical series; and (2) does not exhibit a strong secular trend (so that, for example, a local minimum of the unemployment-rate series at the onset of a recession does not necessarily represent a global minimum up to that observation in the sample).

Our interest in the CDR model for the unemployment rate reflects recent work which has documented the significance of CDR-type dynamics for many important business-cycle indicators; see, for example, Bradley and Jansen (1996) and Van Dijk and Franses (1997). In running an out-of-sample forecasting competition with a CDR model, our paper complements the analysis carried out by Jansen and Oh (1996), who examine both the in-sample and out-of-sample performance of CDR models for U.S. GNP and industrial production growth rates. Our paper is also an extension of Rothman's (1998) work, since CDR models are not considered in that study.

The paper proceeds as follows. Section 2 compares the relative in-sample fits of the linear and CDR models. In Section 3, the out-of-sample forecasts from the two models are evaluated. Section 4 concludes.

2 The Time-Series Models and In-Sample Fits

A time-series plot of the quarterly U.S. unemployment rate, from 1949:Q3 to 1995:Q1, appears in Figure 1.[1] The models discussed in this section were estimated directly on this time series, with no detrending or first differencing. The lack of first differencing was justified by the Stock-Watson and Dickey-Fuller unit-root results reported in Table 1, which show that both sets of test statistics reject the unit-root and unit-root-plus-drift null hypotheses at the 5% significance level. Given the well-known low power of such tests against near-unit roots, the evidence presented in Table 1 constitutes rather strong rejection of the respective unit-root null hypotheses. The decision to eschew detrending was based on the finding that inclusion of a trend variable as a regressor increased the value of Aikaike information criterion (AIC) relative to the models estimated with no trend.

Letting X_t denote the natural logarithm of GNP, Beaudry and Koop (1993) define CDR as follows:

$$\mathrm{CDR}_t = \max\{X_{t-s}\}_{s\geq 0} - X_t. \tag{1}$$

Construction of a CDR series for the unemployment rate, U_t, first requires replacing "max" with "min" in

[1]We constructed our quarterly series by taking the quarterly average of the Citibase LHUR monthly series from 1948:01–1995:04.

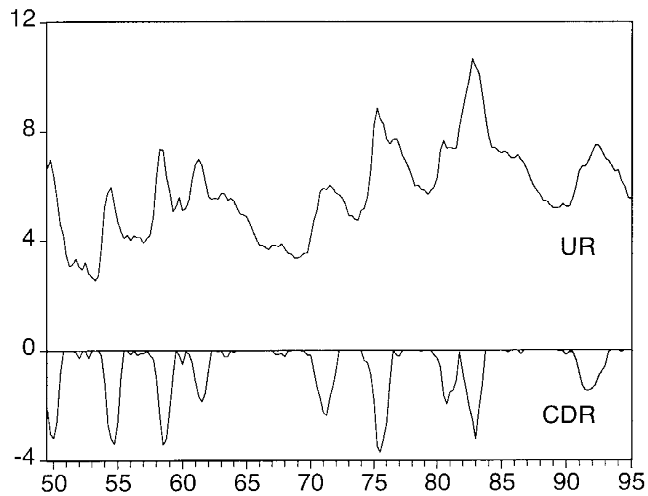

Figure 1
Time-series plots of U.S. quarterly unemployment rate and CDR_{t-1}, 1949:Q3–1995:Q1. UR is the quarterly U.S. unemployment rate. CDR equals the minimum of the unemployment rate over the previous six quarters minus last quarter's unemployment rate; see Equation 2.

Equation 1, since U_t is a counter-cyclical series. The lack of a strong secular trend in U_t also requires use of a local, rather than a global, minimum. For example, the U.S. unemployment rate reached its lowest value in the postwar period in the second quarter of 1953. Use of a global minimum in the definition of $\{\mathrm{CDR}_t\}$ for the unemployment rate would imply that the U.S. economy has been in recession since the second half of 1953. To constrain the construction of our unemployment CDR_t series to the use of local minima, we restricted the search for the minimum of U_t as follows:

$$\mathrm{CDR}_t = \min\{U_{t-s}\}_{s=0,\ldots 5} - U_t. \tag{2}$$

According to Equation 2, CDR_{t-1} takes on a negative value if last quarter's unemployment rate was higher than the minimum value of U_t over the past one and a half years.[2] A graph of CDR_{t-1} for the 1949:Q3–1995:Q1 sample period also appears in Figure 1.

To set up the forecasting experiment, we decided to identify an ARMA and a CDR model using the AIC over the sample period 1949:Q3–1989:Q4, leaving aside 21 out-of-sample observations.[3] The following AR(2) model yielded the lowest value of the AIC out of the set of ARMA models considered:

$$\begin{aligned} U_t = \underset{(3.35)}{0.309} + \underset{(27.04)}{1.5385}\,U_{t-1} - \underset{(-10.93)}{0.640}\,U_{t-2} + \hat{\epsilon}_{1t}, \end{aligned} \tag{3}$$

with $\hat{\sigma}^2_{\epsilon_1} = 0.106$, Ljung-Box Q-statistic at 24 lags = 30.21 (p-value = 0.18), AIC = −350.2, and t-statistics in parentheses. The p-value of Ramsey's RESET test statistic for Equation 3 is 0.93, suggesting no model misspecification. These results imply that there is very little structure in the series left unexplained by the standard linear-modeling approach.

[2]Truncation of s in Equation 2 at lag length 5 is optimal in the sense that this value of s minimized the AIC across a large set of estimated CDR models.
[3]We show that our results are robust to use of a longer simulated out-of-sample period.

In identifying a CDR model, we augmented an AR(2) model for the unemployment rate by allowing for various lag combinations of CDR_t to be included. The following CDR model yielded the lowest AIC value:

$$\begin{array}{lllll} U_t = 0.210 & + \; 1.740\,U_{t-1} & - \; 0.764\,U_{t-2} & + \; 0.124\,\text{CDR}_{t-1} & + \; \hat{\epsilon}_{2t}, \\ \ (2.28) & \quad (25.28) & \quad (-11.85) & \quad (3.90) & \end{array} \tag{4}$$

with $\hat{\sigma}^2_{\epsilon_2} = 0.098$, Ljung-Box Q-statistic at 24 lags = 26.66 (p-value = 0.32), AIC = -359.5, and t-statistics are in parentheses. The p-value for the RESET test of model misspecification is 0.96. Using the AIC to discriminate between alternative models, the CDR model (Equation 4) dominates the AR(2) model given by Equation 3; it achieves a moderate 8.2% reduction in residual variance. Also, the p-value for an F-test of the AR(2) model against the CDR model is extremely low, 0.0001. These results suggest that Equation 4 is an arguably superior characterization of the unemployment rate's dynamic behavior.[4]

To further assess the significance of the t-statistic and F-statistic for adding CDR_{t-1} to an AR(2) model of the unemployment rate, we ran the following Monte Carlo simulation. In each iteration, we first generated a Gaussian AR(2) process using the parameter estimates and residual variance given in Equation 3. We next constructed a CDR_t sequence and estimated a CDR model by including CDR_{t-1} in an AR(2) model of the artificial series, storing the t-statistic for the coefficient on CDR_{t-1} and the F-statistic obtained by dropping CDR_{t-1} and examining the decrease in the residual sum of squares. We allowed for 10,000 iterations in the simulation to estimate the probability of obtaining a t-statistic (absolute value of) and an F-statistic greater than those observed for Equation 4. The estimated probability of both events was 0.0001, suggesting that the relatively high t-statistic and low p-value for the F-statistic reported in Equation 4 are not spurious. Augmenting the AR(2) model by inclusion of CDR_{t-1}, then, captures an important state-dependent cyclical phenomenon.[5]

3 The Out-of-Sample Forecasts

Twenty-one observations were available for simulated out-of-sample forecasting, 1990:Q1–1995:Q1. This is a useful span of data over which to carry out the forecasting experiment, because it covers the most recent U.S. recession and subsequent still-continuing expansion. For the forecasting experiment, each model was recursively estimated; i.e., k-step-ahead forecasts made in period t are based on models estimated using data through period t.

These alternative forecasts were evaluated at short- and medium-term horizons, i.e., one-quarter through eight-quarters ahead. As a first check, the bias of the models' forecasts was computed. To calculate the bias at each forecast step, a regression of the forecast error on a constant was run; the forecast error was calculated as the actual value minus the predicted value for each out-of-sample observation. The estimated constant term in this regression is the estimated bias. The second and third columns of Table 2 report the estimated bias for the AR(2) and CDR forecasts over the 1990:Q1–1995:Q1 period. The results show that at most, forecast horizons considered both models produce statistically unbiased forecasts; the CDR model's eight-step-ahead forecast bias has a p-value equal to 0.06, suggesting a slight forecast bias. The AR(2) model tends to overpredict at almost all forecast horizons, while the CDR model underpredicts and then overpredicts at the shorter and longer forecast steps, respectively.

The MSPE ratios for the two sets of forecasts are reported in the fourth column of Table 2. At all forecast steps, the AR(2) forecasts generated a lower MSPE relative to the CDR forecasts; the CDR MSPEs are all at least

[4] They also suggest that the RESET test does not have power against the type of state-dependence/nonlinearity introduced by inclusion of CDR_{t-1}.

[5] To check whether our evidence in favor of the CDR model given by Equation 4 is driven by the decision not to detrend the data, we ran another simulation in which each iteration's artificial series was generated by an AR(2) plus linear-trend model fitted to the U.S. quarterly unemployment rate. We obtained an estimated probability in this case equal to 0.0021.

Table 2
Estimated Bias and MSPE Ratios for AR(2) and CDR Out-of-Sample Forecasts of U.S. Unemployment Rate: 1990:Q1–1995:Q1[b]

	Estimated Bias		MSPE Ratios
Forecast Step	AR(2)	CDR	CDR/AR(2)
1	0.039 (0.35)	0.002 (0.97)	1.154 (0.35)
2	0.114 (0.23)	0.019 (0.87)	1.259 (0.30)
3	0.166 (0.26)	0.007 (0.97)	1.288 (0.36)
4	0.140 (0.43)	−0.086 (0.76)	1.356 (0.38)
5	0.167 (0.39)	−0.128 (0.70)	1.331 (0.46)
6	0.114 (0.52)	−0.281 (0.37)	1.345 (0.52)
7	0.052 (0.71)	−0.443 (0.10)	1.295 (0.62)
8	−0.004 (0.96)	−0.534 (0.06)	1.191 (0.74)

[b]The bias reported in the left side of the table is calculated as the estimated constant term in a regression of the forecast error on a constant. The *p*-values for the null hypothesis that the bias equals zero appear in parentheses. The right side of the table reports the MSPE ratios (CDR MSPE/AR(2) MSPE) for the out-of-sample forecasting exercise. The *p*-values for the Mizrach (1995) robust forecast-comparison statistic, testing the null hypothesis that the MSPE ratio equals one, appear in parentheses.

15% higher than the corresponding MSPEs for the AR(2) model. It is important, however, to assess the statistical significance of these MSPE ratios.

This was done through the use of Mizrach's (1995) robust forecast-comparison statistic.[6] The *p*-values for the Mizrach forecast-comparison test appear in parentheses in the fourth column of Table 2. At all forecast horizons, the *p*-values for the Mizrach test are greater than or equal to 30%, showing that there is no statistically significant difference in the MSPEs produced by the two sets of forecasts over the 1990:Q1–1995:Q1 out-of-sample forecast period.

To investigate the robustness of our results, we repeated our analysis for a different set of simulated in-sample and out-of-sample periods, 1949:Q3–1979:Q4 and 1980:Q1–1995:Q1, respectively. This experimental design yields a significantly longer out-of-sample forecast period, one with 61 observations, and includes the sharp 1981–1982 recession and subsequent long expansion. We first estimated the AR(2) and CDR models for the 1949:Q3–1979:Q4 in-sample period. Our results were very similar to those reported for Equations 3 and 4 above. An analogous Monte Carlo simulation once again showed that the high statistical significance of the estimated coefficient for CDR_{t-1} was inconsistent with an AR(2) data-generating process.[7]

The forecasting results for the 1980:Q1–1995:Q1 out-of-sample forecasting period are reported in Table 3. At all forecast horizons, the AR(2) model underpredicts the true value of the unemployment rate. In seven out

[6]Details on computing the Mizrach (1995) robust forecast-comparison statistic are reported in the Appendix.
[7]Full details on the 1949:Q3–1979:Q4 in-sample results are available from the authors. We also established that these results were not driven by our decision not to include a trend in the estimation of the AR(2) model over this in-sample period.

Table 3
Estimated Bias and MSPE Ratios for AR(2) and CDR Out-of-Sample Forecasts of U.S. Unemployment Rate: 1980:Q1–1995:Q1[c]

Forecast Step	Estimated Bias AR(2)	Estimated Bias CDR	MSPE Ratios CDR/AR(2)
1	0.084	0.012	0.928
	(0.00)	(0.20)	(0.43)
2	0.186	0.001	0.902
	(0.00)	(0.97)	(0.51)
3	0.304	−0.029	0.891
	(0.01)	(0.80)	(0.59)
4	0.452	−0.053	0.890
	(0.04)	(0.82)	(0.64)
5	0.572	−0.119	0.876
	(0.09)	(0.75)	(0.63)
6	0.600	−0.271	0.881
	(0.11)	(0.52)	(0.67)
7	0.568	−0.455	0.885
	(0.06)	(0.24)	(0.69)
8	0.513	−0.630	0.894
	(0.02)	(0.09)	(0.72)

[c]The bias reported in the left side of the table is calculated as the estimated constant term in a regression of the forecast error on a constant. The *p*-values for the null hypothesis that the bias equals zero appear in parentheses. The right side of the table reports the MSPE ratios (CDR MSPE/AR(2) MSPE) for the out-of-sample forecasting exercise. The *p*-values for the Mizrach (1995) robust forecast-comparison statistic, testing the null hypothesis that the MSPE ratio equals one, appear in parentheses.

of eight cases for the AR(2) model, the *p*-values for the null hypothesis of no forecast bias are less than or equal to 9%; they are especially low at the first three forecast steps. Over this longer simulated out-of-sample forecasting period, then, the AR(2) forecasts show strong evidence of bias in most cases. In contrast, there is at most moderate evidence of forecast bias for the CDR model in only one case, i.e., the *p*-value for the no-forecast null hypothesis is 9% at the eight-quarter-ahead forecast for the CDR model. In all other cases, the forecast bias *p*-values are greater than or equal to 20%. The CDR model tends to underpredict at the one-step- and two-step-ahead horizons and overpredict at the longer forecast horizons.

In this simulated out-of-sample forecasting period, the CDR model generates lower MSPEs relative to the AR(2) model at all forecast steps. These MSPE reductions range from approximately 7–12%. However, the Mizrach forecast-comparison test results suggest that the MSPE ratios are not significantly different from one at any forecast horizon.

Comparison of Tables 2 and 3, then, shows the following. First, the degree of bias exhibited by the AR(2) forecast changes significantly between the two simulated out-of-sample periods. Second, the AR(2) model generates lower MSPEs over the 1990:Q1–1995:Q1 forecast period, while the reverse occurs over the 1980:Q1–1995:Q1 forecast period. Third, in all cases from both simulated out-of-sample forecasting periods, none of the MSPE reductions are statistically significant at the conventional levels. In this sense, our forecasting results are rather robust to a significant change in specification of the simulated out-of-sample period.

4 Conclusions

The two main findings of this paper are as follows. First, there is a statistically significant current depth of the recession effect in the univariate time-series representation of the postwar quarterly U.S. unemployment rate. Our paper is the first to report such a phenomenon in the literature. The evidence we present in favor of CDR-type dynamics for the unemployment rate complements similar results obtained for postwar U.S. GNP and industrial production growth rates. Second, there is no statistically significant difference with respect to MSPE in the out-of-sample forecasting performances of the linear and CDR unemployment-rate models.

There are a couple of ways to interpret this result. One view might be to simply ignore the CDR effect in characterizing the time-series properties of the U.S. unemployment rate, since it "makes no difference" for forecasting.

We, however, favor a different conclusion in light of a standard critique of nonlinear time-series methods, especially when applied to macroeconomic data, i.e., that the greater flexibility for in-sample fits offered by nonlinear time-series models can be costly (in the MSPE sense) in out-of-sample forecasting; this is the case with some of the nonlinear models that Rothman (1998) employs. Our results show that the CDR model certainly did no worse relative to the AR(2) model in out-of-sample forecasting; so use of the model generated no such extra MSPE cost. Accordingly, we suggest that our in-sample evidence in favor of a CDR effect be considered in future modeling of this important business-cycle indicator's time-series properties. For example, the standard linear approach to univariate and multivariate impulse-response analysis for the U.S. unemployment rate is likely to be biased and misleading; so that calculation of nonlinear impulse-response functions along the lines of Potter (1995) and Koop, Pesaran, and Potter (1996) would appear to be appropriate.

Appendix: Testing Equality of MSPEs

To start, let $e_{1,j}$ and $e_{2,j}$ denote the period-j prediction errors from models 1 and 2, respectively. Mizrach's (1995) robust forecast-comparison test assumes that the two prediction errors are stationary (up to the fourth order) draws from a bivariate population (E_1, E_2). Let $U = E_1 - E_2$, and $V = E_1 + E_2$. If the MSPEs in the original population are equal, it is straightforward to see that the covariance in the transformed series is zero.

Accordingly, the null hypothesis of Mizrach's (1995) robust forecast-comparison test is $\mathrm{cov}(U, V) = 0$; the two-sided alternative hypothesis is $\mathrm{cov}(U, V) \neq 0$. Let $u_j = e_{1,j} - e_{2,j}$, and $v_j = e_{1,j} + e_{2,j}$. The robust test statistic is formed as follows:

$$\mathrm{COVSTAT} \equiv \frac{(1/n)\sum_{j=1}^{n} u_j v_j}{\left[\sum_{t=-k}^{k}(1 - [|t|/(k+1)])s'_{UVUV}(t)\right]^{1/2}}, \tag{A1}$$

where k is the step of the forecasts, e.g., $k = 1$ for one-step-ahead forecasts, n is the sample size of both $\{u_j\}$ and $\{v_j\}$, and:

$$s'_{UVUV}(t) = (1/n)\sum_{j=t+1}^{n} u_j v_j u_{j-t} v_{j-t}. \tag{A2}$$

The asymptotic distribution of COVSTAT is standard normal. Through Monte Carlo simulations, Mizrach (1995) showed that COVSTAT is properly sized in both normal and non-normal populations.

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