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Sectoral Investigation of Asymmetries in the Conditional Mean Dynamics of the Real U.S. GDP

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Abstract. *We investigate asymmetries in the conditional mean dynamics of four sectors of the U.S. GDP data. Since the statistical evidence on nonlinearities in the conditional mean could be influenced by the presence of outliers, or by a failure to model conditional heteroskedasticity, we explicitly account for outliers by assuming that the innovations are drawn from the stable family, and model time-varying volatility by a GARCH(1,1) process. We also allow for the possibility of long memory in the series with fractional differencing. Our results indicate only weak evidence of significant nonlinearities in the conditional mean in some sectors of the GDP.*

Keywords. GDP, symmetric stable distributions, conditional heteroskedasticity, long memory, nonlinearity

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1 Introduction

The possible existence of asymmetries in economic fluctuations is being tested extensively using aggregate macroeconomic data. Studies such as those by Neftci (1984), Brunner (1997), Beaudry and Koop (1993), Ramsey and Rothman (1996), Pesaran and Potter (1997), and Hess and Iwata (1997) conclude that there are significant asymmetries. However, others, such as those by Falk (1986), Sichel (1989), DeLong and Summers (1986), Diebold and Rudebusch (1990), and Elwood (1998), have either failed to confirm these findings or have found only weak evidence supporting them. Moreover, while Tsay (1988) demonstrated that linearity could be rejected by the presence of outliers, Balke and Fomby (1994) and Scheinkman and LeBaron (1989) actually reported weakened evidence against linearity in U.S. real GNP data once outliers were taken into account. The latter study also showed a weakening of the evidence against linearity after accounting for conditional heteroskedasticity in that series.

The existence of outliers in the real GNP was also demonstrated by Blanchard and Watson (1986), who concluded that fluctuations in economic activity are characterized by a mixture of large and small shocks. That homoskedastic models may not accurately portray this time series was also evidenced by French and Sichel (1993) and Brunner (1997). Of related interest is the possible characterization of this series as a fractionally integrated process displaying long memory (Sowell, 1992a).

Bidarkota (1998) investigated whether or not asymmetries exist in the conditional mean dynamics of the U.S. real GNP, taking into account the possibility of the foregoing other features in the data. Results obtained in that paper indicated robustness of such asymmetries to outliers, conditional heteroskedasticity, and long memory.

A fruitful avenue for further research on nonlinearities is to isolate the sources of nonlinearities found in the aggregate data (see Sichel's 1995 work). Neftci and McNevin (1986), as communicated by Rothman (1991), argued that an analysis of disaggregate data may reveal important evidence on nonlinearities that may be masked in the more aggregate data. French and Sichel (1993) found the nonlinearities in the GNP data to be concentrated mainly in the cyclically sensitive sectors, namely durable goods and structures, and not in nondurables and services sectors. Rothman (1991) found, using unemployment data, that the asymmetries were concentrated mainly in the manufacturing sector.

In this paper, we investigate whether the evidence on nonlinear mean dynamics obtained by Bidarkota (1999) is also due in large part to the dynamics governing the cyclically sensitive sectors of the economy as documented by French and Sichel (1993).

In our investigations, we use a standard ARIMA model augmented to include a nonlinear term in the mean, as also used by Beaudry and Koop (1993). We take into account conditional heteroskedasticity using GARCH models, and long memory with the fractionally integrated extensions of standard ARIMA (ARFIMA) models. Outliers are accounted for using leptokurtic distributions. While several candidate distributions exist, following Bidarkota (1999), we use the stable distributions in this study. Stable distributions are the natural generalizations of Gaussian distributions, and possess central limit attributes (Zolotarev 1986, ch. 1). If the total output in any given sector of the economy can be viewed as an outcome of several individually unimportant shocks, then the generalized central limit theorem dictates that the limiting distribution of such a process, if it exists, must belong to the stable class.

In Section 2 we describe the general model used, and in Section 3 we present the estimation results. Section 4 provides additional evidence using alternate models, namely, threshold autoregressions, and Section 5 provides brief conclusions.

2 Nonlinear ARFIMA-GARCH Model with Stable Errors

This section draws heavily from the work of Bidarkota (1999). The most general model we consider can be represented as

$$\Phi(L)(1-L)^d(\Delta y_t - \mu) = \{\Omega(L) - 1\}\, CDR_t + \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t c_t \qquad z_t \sim \text{iid}\, S_\alpha(0,1) \tag{2.1a}$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 |\varepsilon_{t-1}|^\alpha \tag{2.1b}$$

Here, $\Delta y_t \equiv 100\Delta \ln \text{GDP}_t$ is the growth rate of real sectoral GDP, μ is the mean of the process, d is the differencing parameter that takes real values, $\Phi(.)$ and $\Omega(.)$ are polynomials of orders p and r, respectively, in the lag operator L with $\Phi(0) = \Omega(0) = 1$, and CDR is the current depth of recession, defined by Beaudry and Koop (1993) as the gap between the current level of output and the economy's historical maximum level; i.e., $\text{CDR}_t = \max\{y_{t-j}\}_{j\geq 0} - y_t$.[1]

A random variable X is said to have a symmetric stable distribution $S_\alpha(\delta, c)$ if its log-characteristic function can be expressed as

$$\ln \text{E} \exp(iXt) = i\delta t - |ct|^\alpha$$

The parameters $c > 0$ and $\delta \in (-\infty, \infty)$ are measures of scale and location, respectively, and $\alpha \in (0, 2]$ is the characteristic exponent governing the tail behavior, with smaller values of α indicating thicker tails. The normal distribution belongs to the symmetric stable family with $\alpha = 2$, and is the only member with finite variance, equal to $2c^2$.

Equation (2.1b) describes the evolution of the scale of the conditional distribution. Using α-powers in the volatility specification results in the familiar GARCH(1,1)-normal process for the conditional variance when $\alpha = 2$; i.e., when shocks are normal. This specification is similar to the power ARCH process introduced by Ding, Granger, and Engle (1993), with the exception that the distribution of z_t in the latter is not dependent

[1] We do not consider any moving-average (MA) terms in the specification of the model. Maximum-likelihood estimation of mixed ARMA models with stable errors poses a challenge, although the Whittle estimator (Mikosch et al., 1995) and minimum-dispersion estimators (Brockwell and Davis 1991) have been used in this context.

on the exponent α governing the volatility dynamics. This dependence arises here naturally since we are considering disturbances to be drawn from stable distributions. A similar formulation for modeling the volatility of daily foreign-currency returns using stable errors was employed by Liu and Brorsen (1995). McCulloch (1985) fit a GARCH-stable model to bond returns, but used absolute values in place of α-powers.

Consider the case when $\Omega(L) = 1$. Then, Equation (2.1a) reduces to the standard ARIMA model with integer differencing when $d = 0$, i.e., the unit root case, whereas $d = -1$ indicates overdifferencing or trend stationarity.

In the case of Gaussian errors, the existence of a stationary causal and invertible solution to an ARFIMA model requires $|d| < 0.5$ (Brockwell and Davis 1991). With α-stable shocks, Kokoszka and Taqqu (1995) showed that a unique causal MA(∞) representation to an ARFIMA model exists if $\alpha(d-1) < -1$.[2] This implies that d can be positive only when $\alpha > 1$. Further, for such a model to be a solution to an AR(∞) process requires that $\alpha > 1$ and $|d| < 1 - 1/\alpha$. Consequently, we restrict α and d in Equation (2.1) to satisfy these constraints in order to force our estimated models to possess causal and invertible representations.

Abstracting from fractional differencing for a moment, although addition of the nonlinear CDR term into a standard ARMA framework is ad hoc, this model has the virtues of simplicity and parsimony. It nests ARMA models, and enables tests for the significance of the nonlinear terms governing the conditional mean dynamics to be carried out using the likelihood-ratio (LR) test.[3] Moreover, it permits recessions to be less or more persistent than expansions, depending on the parameter estimates. For instance, when $\Phi(.)$ is of order zero and $\Omega(.)$ is of order one, a positive ω_1 implies that negative shocks are less persistent, whereas a negative ω_1 implies the opposite; $\omega_1 = 0$ reduces to a random-walk model with drift.

The presence of asymmetries essentially implies that either the innovations are asymmetric but the impulse-transmission mechanism is linear, or that the innovations are symmetric but the transmission mechanism is nonlinear, or that the innovations are asymmetric and the transmission mechanism is also nonlinear. It would be hard to disentangle the nonlinear effects, if any, of the innovations themselves from the propagation mechanism. Although asymmetric α-stable distributions exist and are well defined, determining whether the asymmetries in the mean real GDP, if they exist at all, are caused by asymmetric impulses being propagated linearly, or by symmetric impulses being propagated nonlinearly, or by a combination of the two, goes beyond the scope of this work. Our objective here is merely to investigate whether asymmetries, regardless of how they can best be characterized, exist at all in the conditional mean dynamics of the sectoral real GDP in a univariate setting.

There is also a technical reason for restricting ourselves to symmetric stable distributions. Although the stable distribution and density may be evaluated by using Zolotarev's (1986, pp. 74, 78) proper integral representations, or by taking the inverse Fourier transform of the characteristic function, McCulloch (1996) has developed a fast numerical approximation to these that only applies in the symmetric case. This approximation has an expected relative density precision of 10^{-6} for $\alpha \in [0.84, 2]$. We therefore restrict α in this range to enable us to use this approximation.

The exact full-information maximum-likelihood (ML) method for estimating ARFIMA models due to Sowell (1992b) is applicable only when the errors are iid normal. However, Baillie and colleagues (1996) noted that implementing Sowell's ML procedure for more complicated models, such as non-normal or conditionally heteroskedastic models or both as is the case here, was likely to be either computationally extremely demanding or completely intractable. Instead, they used the conditional sum of squares (CSS) estimator, originally proposed in the context of ARFIMA processes by Hosking (1984), to estimate their ARFIMA-GARCH models, with normal or Student's t errors. In our empirical work that follows in Sections 3 and 4 below, we too use the CSS estimator.

The CSS estimation of ARFIMA models consists of fitting an ARMA model to the series $(1-L)^d(\Delta y_t - \mu)$, which is obtained by expanding the differencing operator, $(1-L)^d$, with the binomial expansion, and truncating the infinite series at the first available observation. The CSS estimator is discussed in the context of ARMA models by Box and Jenkins (1976). Its asymptotically normal distribution for the ARFIMA case, when

[2] Since long-memory ARFIMA models are frequently defined in terms of the rate of decay of their autocovariances, their extension to infinite-variance stable shocks is not immediate. See the work of Kokoszka and Taqqu (1995) for the theory of fractionally differenced ARMA time series with infinite-variance stable innovations.

[3] However, Hess and Iwata (1997) showed that the asymptotic distribution of the t-test for the significance of the nonlinear CDR term in the model given by Equation (2.1a) was nonstandard, both in the case when the dependent variable was nonstationary, i.e., was integrated of order one, I(1), and when it was stationary, I(0).

Table 1
Summary statistics.[a]

	Durables	Structures	Nondurables	Services
Mean	1.19	0.63	0.60	0.94
	(0.22)	(0.18)	(0.45)	(0.07)
Standard deviation	4.34	2.69	0.80	0.52
Skewness	0.06	−0.31	−0.11	−0.27
	(0.36)	(0.07)	(0.48)	(0.11)
Excess kurtosis	4.85	0.79	1.02	1.07
	(0.00)	(0.02)	(2.7*e*−3)	(1.7*e*−3)
Jarque-Bera test	183.31	7.60	8.28	11.04
	(0.00)	(2.2*e*−2)	(1.5*e*−2)	(3.9*e*−3)
ADF test				
Level (constant + trend)	−3.75	−1.84	−0.84	0.96
First difference (constant only)	−8.89	−7.04	−6.54	−3.94
Orders of significant autocorrelations (at 10% level)	1, 2, 3	1, 2, 3	None	1, 3
Orders of significant partial autocorrelations (at 10% level)	1, 2	1	None	1, 3
Goldfeld-Quandt test	1.03	0.60	0.91	0.65
	(0.45)	(0.02)	(0.34)	(0.04)
LM test for ARCH	22.55	0.10	12.80	10.09
	(0.03)	(0.75)	(1.6*e*−3)	(0.01)

[a] 1. All statistics reported are for growth rates, defined as $100\Delta \ln y_t$, except the ADF test in levels.
2. *P*-values for mean = 0, skewness = 0, excess kurtosis = 0, and all the other tests reported are in parentheses.
3. Optimal lag order in the ADF test is chosen using the minimum SIC criterion. Asymptotic critical values at the 10% significance level are −3.13 for a model with constant and time trend, and −2.57 for a model with constant but no trend.
4. Optimal lag order for the LM test is chosen using the minimum SIC criterion.

the mean is known and the errors are normal, was derived by Li and McLeod (1986). The CSS procedure is asymptotically equivalent to the full MLE. Some properties of the CSS estimator in the context of ARFIMA models, particularly with respect to its bias, are discussed by Baillie and coworkers (1996). They concluded that the CSS estimator performed quite satisfactorily in comparison to Sowell's (1992b) exact MLE, while being computationally feasible for more complex models.

3 Estimation Results

We use the chain-weighted seasonally adjusted quarterly U.S. GDP sectoral data spanning the period 1947:1 through 1996:4.[4] The sectors used are durable goods GDP, structures, nondurable goods, and services. We estimate Equation (2.1), labeled Model 1, and three of its restricted versions univariately using each of the data series. Imposing $d = 0$ on Model 1 gives Model 2. A homoskedastic version of Model 2 is Model 3. Imposing $\alpha = 2$ on Model 3 gives the Gaussian homoskedastic Model 4.

Summary statistics for each of the four data series are given in Table 1. All four series indicate statistically significant excess kurtosis in their growth rates at conventional significance levels, and the Jarque-Bera test easily rejects normality. The augmented Dickey-Fuller test (ADF test) fails to reject a unit root in levels in all series except the durables at the 10% level when a constant and a linear time trend are included in the regression. The number of lags included in the regression is determined by the minimum Schwarz information

[4]The dataset is obtained from Table 2B of the *Survey of Current Business*, May 1997.

criterion (SIC). The same test performed on growth rates rejects a unit root in all the series when only a constant term is included in the regression. The Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH) rejects homoskedasticity in all series except structures.

The orders of statistically significant autocorrelations and partial autocorrelations given in Table 1 provide some guidance in choosing the orders of the polynomials in Equation (2.1a). We conduct an extensive specification search for each of the models defined above for each of the four data series. The orders of the lag polynomials are restricted such that $p \leq 3$ and $r \leq 3$.

Table 2 provides a summary of the specification search, along with the results of some useful hypothesis tests. For each of the quantities listed in the first column, the first row in the remaining four columns is for durable-goods GDP data, the second row is for structures, the third row is for nondurables, and the fourth row is for services. Surprisingly, for all four series and across all four models considered, the minimum SIC picks models where the CDR term does not appear in the conditional mean specification in Equation (2.1a). For structures and services data, the minimum SIC picks a homoskedastic Gaussian autoregression as the best model, similar to the specification by Beaudry and Koop (1993) for the aggregate GNP data. For durable goods, a homoskedastic autoregression with stable errors is chosen; whereas for nondurables data, an autoregression with GARCH and stable errors is selected by the minimum SIC. Given that the minimum SIC picks a linear conditional mean model in all cases, the need for a likelihood-ratio (LR) test for a linear mean simply does not arise.

A test for normality can be performed by testing $\alpha = 2$. The LR test statistic does not have the usual χ^2 distribution in this case, however, since the null hypothesis lies on the boundary of admissible values for α and standard asymptotic distribution theory does not go through. The small-sample critical values obtained from McCulloch (1997) reject normality in all cases at the 10% significance level or better.

A test for homoskedasticity (no GARCH) can be formulated as a test of $b_2 = b_3 = 0$. Under this null, b_1 and c_0 are trivial transforms of one another. Therefore, it is not clear whether the LR statistic has an asymptotic χ^2 distribution with two or three degrees of freedom. Using the conservative χ^2_3 distribution, we can reject homoskedasticity at the 10% level in three series, the exception being structures.

The LR test for a unit root versus the fractional alternative LR ($d = 0$) rejects in three cases at the 10% level, the exception being nondurables. Although the Monte Carlo simulations by Sowell (1992a) suggest that the true p-values may in fact be higher than the asymptotic χ^2_1 p-values reported in Table 2 for this test, Baillie (1996) pointed out that GNP data are not sufficiently informative about the nature of its long-run properties. The likelihood surface for d is relatively flat, and typically includes both $d = 0$ and $d = -1$.

4 Switching Autoregressions

To explore the robustness of our findings to alternate representations of the nonlinear conditional mean dynamics, we now turn to switching- or threshold-autoregressive models. The most general model we consider in this class is the following:

in regime 1:

$$\Phi_1(L)(1-L)^d(\Delta y_t - \mu_1) = \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t c_t \qquad z_t \sim \text{iid } S_\alpha(0, 1) \tag{4.1a}$$

in regime 2:

$$\Phi_2(L)(1-L)^d(\Delta y_t - \mu_2) = \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t \gamma c_t \qquad z_t \sim \text{iid } S_\alpha(0, 1) \tag{4.1b}$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3 |\varepsilon_{t-1}/\gamma|^\alpha \tag{4.1c}$$

We consider two versions of these models for each of the four data series. In the first version, the switch between regimes is governed by whether $\text{CDR}_{t-1} = 0$ or positive. This type of switching-regression model, but a Gaussian homoskedastic version with no fractional differencing, has been estimated by Beaudry and Koop (1993). In the second version, termed self-exciting threshold autoregressions (SETAR models), the switch is determined by whether $\Delta y_{t-2} > 0$ or not.[5] A heteroskedastic version of such a model, without

[5]More generally, the switch is determined by whether $\Delta y_{t-l} > s$, and l and s are estimated along with other parameters of the model.

Table 2
CDR-augmented regression models.[b]

	Model 4	Model 3	Model 2	Model 1
Order of the model selected by minimum SIC	(0, 0) (1, 0) (0, 0) (1, 0)	(0, 0) (1, 0) (0, 0) (1, 0)	(0, 0) (1, 0) (0, 0) (1, 0)	(2, 0) (1, 0) (0, 0) (1, 0)
Number of parameters	2 3 2 3	4 5 4 5	6 7 6 7	7 6 5 6
Log likelihood	−566.19 −458.82 −230.73 −139.71	−544.46 −457.80 −229.28 −137.26	−540.63 −456.39 −219.57 −133.14	−533.93 −454.86 −218.65 −126.63
AIC	1136.37 923.65 465.45 285.42	1094.92 923.59 464.56 282.53	1093.25 926.78 451.13 280.28	1085.86 925.74 451.31 269.57
SIC	1142.93 933.48 472.01 295.26	1104.76 936.70 474.39 295.64	1112.92 949.73 470.80 303.22	1115.54 951.96 474.25 295.80
LR ($\alpha = 2$)		43.46 ($< 3.7e-3$) 2.04 (< 0.05) 2.90 (< 0.05) 4.90 (< 0.01)	18.38 ($< 3.7e-3$) 2.56 (< 0.05) 6.04 (< 0.01) 2.22 (< 0.05)	19.14 ($< 3.7e-3$) 0.58 (< 0.10) 5.44 (< 0.01) 3.38 (< 0.02)
LR (no GARCH)			7.66 (0.05) 2.82 (0.42) 19.42 ($2.2e-4$) 8.24 (0.04)	
LR ($d = 0$)				9.90 ($1.6e-3$) 3.06 (0.08) 1.84 (0.17) 13.02 ($3.0e-3$)

[b] 1. The general form of the estimated models can be written as

$$\Phi(L)(1-L)^d(\Delta y_t - \mu) = \{\Omega(L) - 1\}\,\mathrm{CDR}_t + \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t c_t \qquad z_t \sim \text{iid } S_\alpha(0,1)$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3|\varepsilon_{t-1}|^\alpha$$

2. Model 1 is the most general. Imposing $d = 0$ gives Model 2. A homoskedastic version of Model 2 is Model 3. Imposing $\alpha = 2$ on Model 3 gives the Gaussian Model 4.
3. For each of the quantities listed in the first column, the first row in the remaining four columns is for durable goods GDP data, the second row is for structures, the third row is for nondurables, and the fourth row is for services.
4. LR is the likelihood-ratio statistic; p-values are given in parentheses for all the tests.
5. LR ($d = 0$) gives the χ_1^2 p-values in parentheses.
6. LR (no GARCH) tests for homoskedasticity using a test of $b_2 = b_3 = 0$. Under this null, b_1 and c_0 are trivial transforms of one another. Therefore, it is not clear whether the LR statistic has an asymptotic χ^2 distribution with two or three degrees of freedom. Conservative χ_3^2 p-values are in parentheses.
7. LR ($\alpha = 2$) is a test for normality. The distribution of the test statistic is nonstandard, since the null hypothesis lies on the boundary of admissible values for α. P-values reported are obtained from McCulloch (1997, Table 4, panel b).

GARCH but with variances that differ across regimes but are otherwise constant within each regime and without fractional differencing, has been estimated by Potter (1995) and Brunner (1997).

As in the previous section, we estimate the model given in Equation (4.1), labeled Model 1, and three of its restricted versions. Imposing $d = 0$ on the switching Model 1 in Equation (4.1) gives Model 2. A

Table 3
CDR switching-regression models.[c]

	Model 4	**Model 3**	**Model 2**	**Model 1**
Order of the model selected by minimum SIC	(0, 0) (1, 1) (0, 0) (1, 1)	(0, 0) (1, 1) (0, 0) (1, 1)	(0, 0) (1, 1) (0, 0) NA	(0, 0) (0, 0) (0, 0) (0, 0)
Number of parameters	4 6 4 6	5 7 5 7	8 10 8 NA	9 9 9 9
Log likelihood	−561.03 −455.90 −229.01 −127.77	−541.70 −454.75 −226.20 −127.69	−535.40 −453.47 −211.44 NA	−534.20 −452.76 −211.17 −124.74
AIC	1130.05 923.79 466.03 263.53	1093.40 923.51 462.41 269.37	1086.80 926.94 438.88 NA	1086.41 923.53 440.34 267.48
SIC	1143.14 943.43 479.12 287.17	1109.77 946.42 487.77 292.28	1112.98 959.67 465.07 NA	1115.86 952.99 469.79 296.94
LR (one regime)	5.28 (0.07) 0.76 (0.86) 2.26 (0.32) 17.02 ($7.0e-4$)	0.72 (0.70) 1.82 (0.61) 4.12 (0.13) 12.16 ($6.8e-3$)	5.42 (0.07) 1.56 (0.67) 14.76 ($6.2e-4$) NA	12.68 (1.7e−3) 7.84 (0.02) 12.52 ($1.9e-3$) 0.78 (0.68)
LR($\alpha = 2$)		38.66 ($< 3.7e-3$) 2.30 (< 0.05) 5.62 (< 0.01) 0.16 (> 0.10)	13.92 ($< 3.7e-3$) 2.86 (< 0.05) 1.18 (< 0.10) NA	14.02 ($< 3.7e-3$) 3.10 (< 0.02) 0.00 (0.99) NA
LR (no GARCH)			12.60 ($5.5e-3$) 1.28 (0.73) 29.52 ($1.7e-6$) NA	
LR ($d = 0$)				2.40 (0.12) 23.66 ($1.1e-6$) 0.54 (0.46) 29.70 ($5.0e-8$)

[c] 1. The estimated models can be represented most generally as follows:
in regime 1:

$$(1 - \phi_1 L - \phi_2 L^2)(1 - L)^d(\Delta y_t - \mu_1) = \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t c_t \qquad z_t \sim \text{iid } S_\alpha(0, 1))$$

in regime 2:

$$(1 - \phi_3 L - \phi_4 L^2)(1 - L)^d(\Delta y_t - \mu_2) = \varepsilon_t \qquad \varepsilon_t \mid I_{t-1} \sim z_t \gamma c_t \qquad z_t \sim \text{iid } S_\alpha(0, 1))$$

$$c_t^\alpha = b_1 + b_2 c_{t-1}^\alpha + b_3|\varepsilon_{t-1}/\gamma|^\alpha$$

The switch between regimes is governed by whether $\text{CDR}_{t-1} = 0$ or is positive.
2. LR (one regime) is a test for linear conditional mean dynamics. The null hypothesis is $\mu_1 = \mu_2$, the corresponding coefficients in the lag polynomials $\Phi_1(.)$ and $\Phi_2(.)$ are equal, and $\gamma = 1$. The χ^2 p-values are given in parentheses.
3. NA denotes missing values, indicating that the relevant models could not be estimated, as the numerical algorithm for likelihood maximization failed to converge.
4. See notes 4–7 from Table 2.

homoskedastic version of Model 2 is Model 3. Imposing $\alpha = 2$ on Model 3 gives the Gaussian homoskedastic Model 4.

Within each of the four models, a specification search for each of our data series is conducted, with the autoregressive lag polynomials in the two regimes restricted to be of orders (3, 3), (2, 2), (1, 1), or (0, 0). We were unable to estimate the self-exciting threshold autoregressive models for the nondurables and services data in several cases, as the numerical algorithm for maximizing the log-likelihood function failed to converge. Therefore, in what follows, we only report the estimation results for the CDR switching models.

Table 3 presents a summary of the estimation results for the CDR switching models. Missing numbers for the services data for Model 2 in the table indicate that the relevant models could not be estimated, as the numerical algorithm for likelihood maximization failed to converge. The LR test for $\alpha = 2$ rejects in all cases at the 10% level or better, with only two exceptions. The LR test for homoskedasticity easily rejects in favor of GARCH in durables and nondurables, but fails to reject in structures. The LR test for $d = 0$ rejects for structures and services, but fails to reject for the other two series.

A test for linearity in the conditional mean dynamics can be formulated as a test for a single regime in threshold autoregressions.[6,7] With the Beaudry and Koop homoskedastic Gaussian Model 4, only durables and services reject linearity at the 10% level or better. After accounting for outliers, only services rejects linearity. Introduction of GARCH leaves durables and nondurables with statistically significant nonlinearities in the conditional mean. Finally, the most general model with outliers, GARCH and long memory rejects linear mean dynamics in all series except services.

Contrary to the evidence documented by Balke and Fomby (1994) and Scheinkman and LeBaron (1989), we do not find the evidence against linear mean dynamics uniformly diminishing as we account for outliers and conditional heteroskedasticity in the sectoral GDP data.

5 Conclusions

Overall, we find only weak evidence of nonlinearities in the conditional mean dynamics of sectoral real GDP data. While the minimum SIC criterion picks only linear mean models among the CDR-augmented models considered in Section 3 for all the data series, the LR tests for a single regime in Section 4 only provide weak evidence in favor of nonlinear mean dynamics.

French and Sichel (1993) documented a concentration of evidence on nonlinearities in real GNP data in the cyclically sensitive sectors, namely, durables and structures. Our results do not support such an inference.

One finding that emerges uniformly across all the series is that Gaussian shocks do not adequately characterize any of the four time series. The empirical distributions appear to be leptokurtic, warranting use of some heavy tailed distributions.

We are led to conclude from our investigations reported in this paper that a disaggregated analysis of the real GDP does not appear promising in isolating the sectors which could account for the nonlinearities found in the aggregate data.

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[6] In general SETAR models, as in footnote 5, when the delay and threshold parameters are estimated, testing the null hypothesis of a single regime results in unidentified nuisance parameters, and standard asymptotic distribution theory does not go through (Hansen 1996). Here, since we do not estimate these parameters but instead set the CDR lag to be one and the threshold to be zero, our tests do not suffer from this problem.

[7] The null hypothesis for this test is the joint hypothesis $\mu_1 = \mu_2$, the corresponding coefficients in the lag polynomials $\Phi_1(.)$ and $\Phi_2(.)$ are equal, and $\gamma = 1$. Although the switch between regimes under the alternative hypothesis is governed by the CDR term, this test is still standard for the switching-regression class of models (cf. with footnote 3 above). See the work of Hess and Iwata (1997), footnote 4, for the validity of this test.

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