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Estimating the Fractional Order of Integration of Interest Rates Using a Wavelet OLS Estimator

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Abstract. *The debate on the order of integration of interest rates has long focused on the I(1) versus I(0) distinction. In this paper we instead use the wavelet OLS estimator of Jensen (1999) to estimate the fractional integration parameters of several interest rates for the United States and Canada from 1948 to 1999. We find that most rates are mean-reverting in the very long run, with the fractional order of integration increasing with the term to maturity. The speeds of mean reversion are lower in Canada, likely because of a positive country-specific risk premium. We also demonstrate that interest rate yield spreads involve noticeable persistence, indicating that these are also not strict I(0) processes. One consequence of these findings is that shocks to most interest rates and their spreads are very long lasting, yet not necessarily infinite.*

Keywords. fractional integration, interest rates, wavelets

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1 Introduction

Interest rates play important roles in both macroeconomics and finance. In macroeconomics, for example, they are crucial to the conduct of monetary policy, as policy is primarily implemented through the setting of short-term interest rates in most industrialized countries. Interest rate movements in turn have an impact on spending and saving decisions, thereby affecting macroeconomic activity. The finance literature is also fertile with models of and uses for interest rates, since their movements are crucial to investment and portfolio decisions.

In modern time-series econometrics it has become standard practice to verify the order of integration of each variable entering a model. If variables are found to be integrated of order one, denoted I(1), then the focus shifts toward locating cointegrating relationships among these variables so as to exploit any long-run equilibrium properties of the data. The order of integration of each variable is usually determined using one or more of the countless unit root tests available, where the unit root hypothesis is tested against the alternative of (mean or trend) stationarity, denoted I(0). Dolado, Jenkinson, and Sovilla-Rivero (1990), for example, present a comprehensive survey on unit roots.

The order of integration of nominal interest rates has long remained a contentious issue. In theory it is impossible for interest rates to follow a unit root process without drift, since this would impose no bounds on the movements of such variables, but in practice they cannot be negative. If a drift term is included, it is also difficult to justify interest rates' tending to infinity in the presence of a unit root. This would imply that expected inflation would also follow a random walk, with the consequence that its path could not be influenced by monetary policy. In macroeconomics one would also imagine that shocks to interest rates resulting from, for example, a currency crisis would eventually dissipate once the crisis subsides. Thus, country-specific risk premiums embedded within interest rates would have to be relatively constant, with perturbations being short-lived. Similarly, a stationary interest rate process is also a requirement of financial term structure models of the type of Cox, Ingersoll, and Ross 1985, since the short-term interest rate factor has to be mean-reverting to make the model tractable.

In practice, however, several authors have failed to reject the unit root hypothesis for interest rates. For example, in their tests for the presence of unit roots in several key macroeconomic variables, Nelson and Plosser (1982) find that the interest rate series is an I(1) process. Although reversing most of the unit root conclusions through the introduction of structural breaks in the unit root tests, Perron (1989) is unable to reverse the Nelson and Plosser conclusion for interest rates.

Because of the inability of empirical studies to reject the unit root hypothesis for interest rates, it is quite likely that the I(0) alternative is too stringent for the unit root tests used. Instead, some authors have suggested that interest rates may in fact be fractionally integrated, or I(d), where $0 < d < 1$. When d is estimated to lie between 0 and 0.5, the process is said to exhibit long memory in the sense that the autocorrelation function (ACF) decays at a much slower rate than the exponential decay of the ACF exhibited by stationary autoregressive moving average (ARMA) processes. In other words, the rate of interest at time t is correlated to the rate at $t - k$ for some $k > 0$, and this correlation diminishes, but is non-negligible, as k increases. When d equals 0.5 or greater but is less than 1, the process will still return to its equilibrium in the long run but will also possess an infinite variance.

In previous work on fractional integration and interest rates, Backus and Zin (1993) show that there is some evidence of long memory in the 3-month zero-coupon rate and that allowing for long memory in the short-term rate improves the fitted mean and volatility yield curves. Pfann, Schotman, and Tschernig (1996) also find that allowing for long memory in the short-term process improves the fit of term structure models.

Intuitively, interest rates may be slowly mean-reverting even if few shocks tend to have long-lasting effects. Parke (1999) introduces error duration models that can generate series displaying long memory. The basic idea is that shocks can be of a stochastic magnitude and of stochastic duration; an observed value of a variable at a point in time is essentially a sum of all shocks that survive up to that point. If only a few shocks are long-lasting, then a long-memory process can ensue. In the case of nominal interest rates, significant shocks to inflation expectations, such as those resulting from shifts in policy regimes, may indeed take a long time to dissipate, and as such nominal interest rates may take a long time prior to reverting to their respective means. If a number of notable long-lasting shocks occur within a short period, interest rates may indeed behave like nonstationary processes.

If interest rates are I(d), then this may help explain why the unit root hypothesis cannot be rejected for such variables. For example, Diebold and Rudebusch (1991) conclude that the standard Dickey-Fuller test has low power against I(d) alternatives. As such, outright estimation of the order of integration d may prove more useful than the use of low-power tests in determining whether interest rates possess a unit root.

Several methods have been proposed for estimating the fractional integration parameter d. Among the most popular is the frequency domain method of Geweke and Porter-Hudak (1983) (henceforth GPH). Unfortunately, this estimator possesses no satisfactory asymptotic properties. Sowell (1990) proposes a maximum-likelihood method to estimate d and the ARMA(p, q) parameters jointly, but as with all maximum-likelihood estimators, it can perform poorly if the model is misspecified. Backus and Zin (1993) use

Sowell's estimator in their empirical work, but the divergent estimates of d that they obtain for each fractionally integrated ARMA (ARFIMA) specification reduces the usefulness of their results.

Jensen (1999) proposes a new estimator of d that is constructed using wavelets. Wavelets can be most simply described as functional transforms in the same spirit as Fourier transforms, but with properties that allow them to more effectively identify either long-rhythmic behavior or short-run phenomena. Jensen's wavelet ordinary least squares (WOLS) estimator of d is derived from the smooth decay of long-memory processes. From Jensen's simulations the mean-square errors (MSEs) of the estimates of d are roughly four to six times smaller than the MSEs of the GPH estimator at the sample sizes that are of interest to us in this study (sample sizes $T = 256$ or 512 observations). This estimator and its properties are discussed more fully below.

Once a proper estimate of the fractional order of integration is obtained, more robust models can be constructed that exploit the properties of the data. For example, Cheung and Lai (1993) show that one can utilize the information content of fractionally integrated processes in an analysis of purchasing power parity. More generally, if we assume that two processes x_1 and x_2 are fractionally integrated such that $x_1 \sim \mathrm{I}(d)$ and $x_2 \sim \mathrm{I}(d)$ with $0 < d < 1$, then a long-run fractional cointegrating relationship may exist for $y = f(x_1, x_2) \sim I(d - b)$, where $b < d$, which may be short-run stationary if $(d - b) = 0$ or may follow a long-memory process if $(d - b) < d$. If the latter, then x_1 and x_2 are said to be fractionally cointegrated, and an error correction term would be most useful in determining the equilibrium relationship to which the series will revert in the very long run. Our discussion on fractional cointegration is expanded in Section 4 within the context of yield spreads and risk premiums.

To motivate the important implications of fractional cointegration, consider, for example, the debate surrounding the empirical work of Baillie and Bollerslev (1989). These authors estimated a cointegrating vector for a group of seven exchange rates from industrial countries, assuming that each exchange rate was I(1). Diebold, Gardeazabal, and Yilmaz (1994) subsequently argued that the error correction term arising from the Baillie and Bollerslev work failed to improve over a simple martingale in an ex ante forecasting experiment, casting doubt on whether the exchange rates were cointegrated at all. Reconsidering their original findings, Baillie and Bollerslev (1994) find that the error correction term arising from their original model is not I(0), as originally believed, but rather I(0.89). That is, their original variables turn out to be fractionally cointegrated. This leads Baillie and Bollerslev to conclude that adjustments to equilibrium are likely to take several years to complete and therefore that their estimated error correction term will yield improvements in forecast performance only several years into the future.

Estimates of the fractional order of integration of interest rates should be of interest to practitioners for at least two reasons. First, knowledge of d will enable them to determine whether shocks to interest rates are short lived, long lived, or infinitely lived. Second, if $d < 1$, then one may suspect that cointegrating relationships involving interest rates may not be precisely I(0), with the consequence that adjustments to re-establish an equilibrium state may follow long-memory processes. As Baillie and Bollerslev (1994) have discovered, this implies that fractionally cointegrated relationships may yield noticeable gains in forecast accuracy only within the context of longer-term forecasts.

In the next section we motivate the intuition underlying wavelets and discuss the WOLS estimator more fully. In Section 3 we use the WOLS estimator to obtain the fractional integration parameters for the zero-coupon term structure data of McCulloch and Kwon (1993) for the United States and standard market rates for Canada. In Section 4 we investigate whether yield spreads and risk premiums are fractionally integrated. The final section concludes.

2 Methodology

2.1 Basic wavelet theory

In mathematics it is often possible to approximate a complicated function as a linear combination of several simple expressions. One of the better known examples is that of spectral, or Fourier, analysis, in which by the spectral representation theorem any covariance-stationary process x_t can be expressed as a linear combination of sine and cosine functions in the frequency domain. For example, the Fourier series of any real-valued function $f(x)$ on the [0,1] interval is expressed as

$$f(x) = b_0 + \sum_{k=1}^{\infty}[b_k \cos 2\pi kx + a_k \sin 2\pi kx] \tag{2.1}$$

where the parameters a_k, b_0, and b_k, for $\forall k$, can be solved using least squares.

Few economic series, however, follow the smooth rhythmic cycles suggested by sine and cosine functions, thereby making Fourier analysis less appealing for economists. A recently developed alternative to Fourier transforms is wavelet transforms, in which the same function $f(x)$ can be expressed in the wavelet domain as

$$f(x) = c_0 + \sum_{j=0}^{\infty}\sum_{k=0}^{2^j-1} c_{jk}\Psi(2^j x - k) \tag{2.2}$$

where $\Psi(x)$ is a so-called mother wavelet, which is *mother* to all dilations and translations of Ψ in Equation (2.2). A simple example of a mother wavelet is

$$\Psi(x) = \begin{cases} 1, & \text{if } 0 \leq x < \frac{1}{2} \\ -1, & \text{if } \frac{1}{2} \leq x < 1 \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

The group of functions $\Psi_{jk}(x) = \Psi(2^j x - k)$ for $j \geq 0$ and $0 \leq k < 2^j$ is orthogonal, and collectively they form a basis in the space of all square-integrable functions $\mathbf{L}_2$ along the [0,1] interval. The index j is the dilation (or scaling) index, which compresses the function $\Psi(x)$, and the index k is the translation index, which shifts the function $\Psi(x)$. More generally, any such basis in $\mathbf{L}_2(\mathbf{R})$ is known as a wavelet, and Equation (2.3) in particular is more commonly known as the Haar wavelet.

Several different wavelets have been proposed in the mathematics literature. For example, Daubechies (1988) presents a system of compactly supported wavelets whereby each wavelet represents different degrees of smoothing of the step function (2.3). In our work we choose to use three representative Daubechies wavelets in addition to the Haar wavelet to verify the robustness of our results to different degrees of smoothing. The Daubechies-20 wavelet is the smoothest wavelet used, followed by the Daubechies-12 and the Daubechies-4, with the Haar wavelet being the least smooth of those used here.

Our choice of the Daubechies system of wavelets is motivated by two factors: first, by its common usage in many applications outside economics (especially signal processing), and second, the fact that the desirable properties of the WOLS estimator were demonstrated with the use of this wavelet. Several alternative wavelets are presented in, for example, Vidakovic 1999.

As noted by Jensen (1999), the strengths of wavelets lie in their ability to simultaneously localize a process in time and scale. They can zoom in on a process's behavior at a point in time, which is a distinct advantage over Fourier analysis. Alternatively, wavelets can also zoom out to reveal any long and smooth features of a series. (The interested reader is referred to Strang 1993 or Strichartz 1993 for more extensive expositions on wavelets.)

2.2 Fractional integration and the wavelet OLS estimator

Consider the random process x_t,

$$(1 - L)^d x_t = \varepsilon_t \tag{2.4}$$

where L is the lag operator, ε_t is i.i.d. normal with zero mean and constant variance σ^2, and d is a differencing parameter. When $d = 0$, the process x_t is simply equal to ε_t,so $x_t \sim N(0, \sigma^2)$, or $x_t \sim \mathrm{I}(0)$. When $d = 1$, however, x_t follows a unit root process (without drift), implying it has a zero mean with infinite variance.

Table 1
Summary of fractional integration parameter values

d	Variance	Shock Duration	Stationarity
$d = 0$	Finite	Short-lived	Stationary
$0 < d < 0.5$	Finite	Long-lived	Stationary
$0.5 \leq d < 1$	Infinite	Long-lived	Nonstationary
$d = 1$	Infinite	Infinite	Nonstationary
$d > 1$	Infinite	Infinite	Nonstationary

More generally, if we allow d to take noninteger values, the process x_t is said to be fractionally integrated, making Equation (2.4) an ARFIMA process. As shown by Hosking (1981), when $0 < d < \frac{1}{2}$, the autocovariance function of x_t declines hyperbolically to zero, making x_t a long-memory process. If $\frac{1}{2} \leq d < 1$, x_t has an infinite variance; it will still, however, revert to its mean (or trend) in the very long run. Table 1 summarizes the different values of d and the corresponding consequences for the mean (or trend) of a process x_t, its variance, the duration of a shock, and the stationarity of the process.

Following Tewfik and Kim (1992) and McCoy and Walden (1996), Jensen (1999) demonstrates that, for an I(d) process x_t with $|d| < \frac{1}{2}$, use of the autocovariance function implies that the wavelet coefficients c_{jk} in Equation (2.2) are distributed as $N(0, \sigma^2 2^{-2jd})$. If $R(j)$ denotes the wavelet coefficient's variance at scale j, then after taking logarithms, an estimate of d can be obtained using ordinary least squares from

$$\ln R(j) = \ln \sigma^2 - d \ln 2^{2j} \tag{2.5}$$

Thus, the wavelet transform is applied to the autocovariance function of a particular interest rate, not to the interest rate itself. The wavelets are used only in the estimation of the d consistent with the observed autocovariance function. Furthermore, because of the form of the wavelet expansion (2.2), it should be noted that the number of observations for the underlying process x_t must be a factor of two. Jensen (1999) demonstrates through Monte Carlo experiments that the WOLS estimator of d in Equation (2.5) has lower MSE than the familiar GPH estimator, while being computationally simpler than the wavelet maximum likelihood estimator of McCoy and Walden (1996).

3 Estimates of *d* for Interest Rate Levels

We consider several nominal interest rates for the United States and Canada. Given the sample size restrictions for the implementation of the estimation, we use monthly data such that we have common sample sizes of $T = 2^8 = 256$ and $T = 2^9 = 512$ observations. We use 1-, 3-, 6-, and 9-month T-bill rates and 1-, 3-, 5-, and 10-year zero-coupon bond rates from 1948:7 to 1991:2, all obtained from McCulloch and Kwon 1993 for the United States. Canadian Socioeconomic Information Management (CANSIM) data for Commercial Paper and Government bond rates are used for Canada from 1956:11 to 1999:6. The computations are performed using the Matlab toolbox *Wavekit* of Ojanen (1998).

3.1 United States

In Table 2 we present the estimates of the fractional integration parameters for U.S. interest rates over the full sample of observations. Three important findings emerge. First, all estimated parameters are at least two standard errors above 0.5, indicating that all rates have an infinite variance. Second, all rates on securities with maturities of 1 year or less are at least two standard errors below 1.0, implying that they are not strict unit root processes. The implication is that they have a tendency to revert to their means in the very long run. Finally, the fractional integration parameters increase as the term to maturity increases. We therefore find that the longest rates, the 5- and 10-year rates, are the rates that display properties that most closely resemble unit root processes. This evidence is strengthened by an examination of the autocorrelation functions plotted in

Table 2
Estimated fractional integration parameters (d), United States, 1948:7 to 1991:2 (512 observations)

	Wavelet			
Rate	Haar	Daubechies-4	Daubechies-12	Daubechies-20
1-Month	0.8127 (0.0334)	0.8479 (0.0346)	0.8608 (0.0318)	0.8580 (0.0324)
3-Month	0.8444 (0.0329)	0.8806 (0.0387)	0.8891 (0.0318)	0.8866 (0.0331)
6-Month	0.8533 (0.0325)	0.8909 (0.0402)	0.8950 (0.0285)	0.8928 (0.0281)
9-Month	0.8570 (0.0324)	0.8930 (0.0387)	0.8976 (0.0258)	0.8930 (0.0246)
1-Year	0.8701 (0.0319)	0.9010 (0.0365)	0.9089 (0.0231)	0.9007 (0.0214)
3-Year	0.9267 (0.0336)	0.9414* (0.0337)	0.9547 (0.0185)	0.9424 (0.0152)
5-Year	0.9550* (0.0395)	0.9640* (0.0346)	0.9791* (0.0202)	0.9672* (0.0185)
10-Year	0.9885* (0.0462)	0.9951* (0.0327)	1.0098* (0.0215)	0.9948* (0.0228)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.

Figure 1. We see that the decay of the autocorrelations of the 1-month rate is more pronounced than it is for the 10-year rate. Both series, however, display very slow decay overall, with the autocorrelations remaining positive even after 5 years.

The finding of increasing nonstationarity with increases in the term to maturity is probably one of the more interesting results of this analysis. Since we are using zero-coupon rates, any differences we uncover are likely due to the effects of term premia. Thus, because the order of integration increases with the term, we suspect that term premia are nonconstant and fluctuate enough to induce the longest rates to follow unit root processes.

In Table 3 we trim our sample to $2^8 = 256$ observations to examine whether the estimated parameters change in any notable manner. This exercise is useful because researchers often do not use data from the 1940s and 1950s, since the thin bond markets in existence at the time did not cause interest rates to fluctuate much in response to market conditions as they do today. The results are now less straightforward. We find that the standard errors of the estimates are noticeably larger, as we would expect given the smaller sample size. The implication is that there are now several estimated parameters within two standard errors of 0.5, indicating that some interest rates may exhibit properties of long-run mean reversion with finite variance. This feature emerges for all rates using the Daubechies wavelets. The second result is that the longer-term rates are still within two standard errors of the unit root scenario. The significant uncertainty surrounding the estimates prevents us, however, from making any firm conclusions other than that we can still exclude zero as a possible order of integration.

Examining the autocorrelation functions for this sample (Figure 2), we now find that both short- and long-term rates decay more quickly, with autocorrelations reaching zero after 3.5 years for the 1-month rate and 4.5 years for the 10-year rate. Based on this evidence we find that there is significant improvement in the rate of mean reversions for all interest rates over the shorter sample, and this is reflected in the estimated

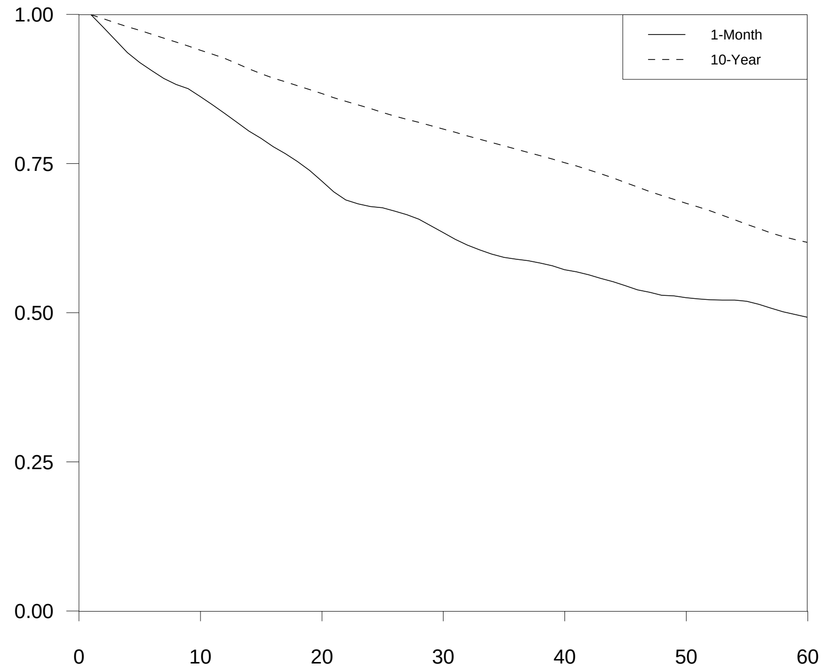

Figure 1
Autocorrelation functions, U.S. 1-month and 10-year rates, 1948:7 to 1991:2.

Table 3
Estimated fractional integration parameters (d), United States, 1969:11 to 1991:2 (256 observations)

	Wavelet			
Rate	Haar	Daubechies-4	Daubechies-12	Daubechies-20
1-Month	0.7168 (0.0491)	0.7075† (0.1066)	0.6756† (0.1335)	0.4706† (0.2474)
3-Month	0.7604 (0.0521)	0.7491 (0.1151)	0.7007† (0.1425)	0.5667*† (0.2186)
6-Month	0.7654 (0.0545)	0.7681 (0.1098)	0.7147† (0.1325)	0.5244*† (0.2404)
9-Month	0.7681 (0.0524)	0.7648 (0.1103)	0.7086† (0.1338)	0.5527† (0.2205)
1-Year	0.7845 (0.0492)	0.7609 (0.1167)	0.7025† (0.1441)	0.6176† (0.1880)
3-Year	0.8464 (0.0415)	0.7118*† (0.1713)	0.5351*† (0.2692)	0.7800* (0.1181)
5-Year	0.8737 (0.0480)	0.6820*† (0.2087)	0.5456*† (0.2854)	0.8389* (0.1059)
10-Year	0.9024* (0.0558)	0.6327*† (0.2630)	0.7217*† (0.2115)	0.8998* (0.0974)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.
†d is within two standard errors of 0.5, the threshold below which the series can have a finite variance.

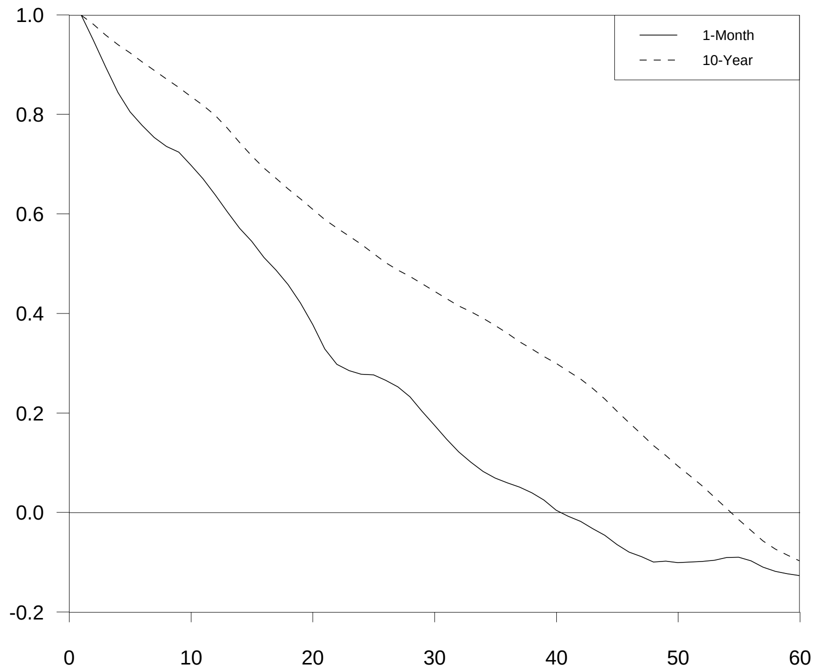

Figure 2
Autocorrelation functions, U.S. 1-month and 10-year rates, 1969:11 to 1991:2.

parameters. We can also state that the shorter rates are less nonstationary than the longer rates, again presumably because of the effects of term premia.

3.2 Canada

In Table 4 we present the fractional integration parameters for the Canadian interest rates from 1956 to 1999. Using the Haar and Daubechies-20 wavelets, we find that the order of integration tends to rise as the term to maturity increases from 1 month to 10 years. As with the U.S. results, we find that the long-term rate is most likely to follow a unit root process. Results using the two other wavelets are largely inconclusive, given the large standard errors associated with the estimates. The autocorrelation functions in Figure 3 demonstrate that short-term rates revert to their means more quickly, consistent with lower orders of integration.

When focusing on the second half of the sample (Table 5), we find that all wavelets yield very similar estimates. Unlike in our previous results, we find here that the orders of integration on the two shortest rates are somewhat higher than for the longer-term rates. Recalling that these are rates on Commercial Paper, which carries a greater default risk over Government bonds, we can surmise that the relatively higher estimates on the short rates are due to the changing effects of such risks (T-bill data are not available far enough back for our purposes). The fractional integration parameters for bond rates from 1 year onward display a similar increasing pattern that we have previously explained is likely due to varying term premia.

The orders of integration on these variables estimated for data covering the last 21 years suggest that all rates follow near unit root processes and clearly are not I(0) and would appear to be long-run mean-reverting. Figure 4 suggests that after 5 years the autocorrelations of both short and long rates remain positive, thereby providing visual evidence that Canadian rates have longer memory than their U.S. counterparts.

4 Estimates of *d* for Spreads and Real Rates

The purpose of this section is to estimate the orders of integration of standard transformations that are often applied to nominal interest rates. We consider both long-short yield spreads and adjustments for inflation.

Table 4
Estimated fractional integration parameters (d), Canada, 1956:11 to 1999:6 (512 observations)

Rate	Wavelet			
	Haar	Daubechies-4	Daubechies-12	Daubechies-20
1-Month	0.8157	0.7042†	0.3892*†	0.7852
	(0.0281)	(0.1397)	(0.3201)	(0.0955)
3-Month	0.8206	0.7241†	0.5765*†	0.7887
	(0.0326)	(0.1301)	(0.2190)	(0.1024)
1- to 3-Year	0.8611	0.4918*†	0.7488*†	0.8559
	(0.0214)	(0.2653)	(0.1295)	(0.0703)
3- to 5-Year	0.8762	0.4588*†	0.7616*†	0.8671*
	(0.0246)	(0.2914)	(0.1309)	(0.0687)
5- to 10-Year	0.9019	0.5226*†	0.7889*	0.8944*
	(0.0286)	(0.2713)	(0.1294)	(0.0688)
10-Year and Over	0.9414	0.4036*†	0.8242*	0.9267*
	(0.0274)	(0.3624)	(0.1288)	(0.0686)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.
†d is within two standard errors of 0.5, the threshold below which the series can have a finite variance.

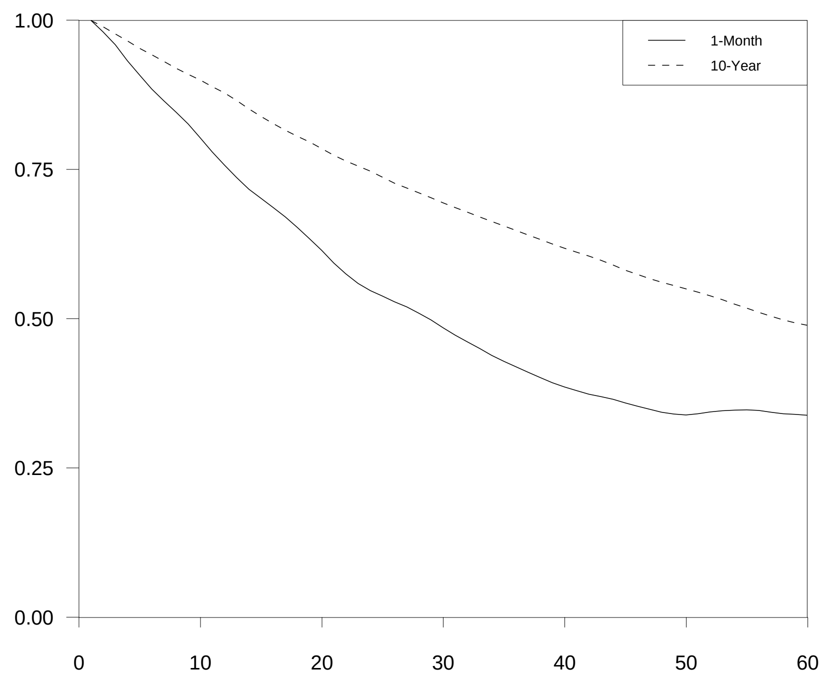

Figure 3
Autocorrelation functions, Canadian 1-month and 10-year rates, 1956:11 to 1999:6.

Table 5
Estimated fractional integration parameters (d), Canada, 1978:3 to 1999:6 (256 observations)

	Wavelet			
Rate	Haar	Daubechies-4	Daubechies-12	Daubechies-20
1-Month	0.8598 (0.0272)	0.9123* (0.0444)	0.9412* (0.0385)	0.9326 (0.0243)
3-Month	0.8752 (0.0285)	0.9202* (0.0454)	0.9529* (0.0362)	0.9520* (0.0266)
1- to 3-Year	0.8751 (0.0267)	0.8983 (0.0426)	0.9378 (0.0259)	0.9299 (0.0116)
3- to 5-Year	0.8841 (0.0307)	0.8997* (0.0515)	0.9438* (0.0310)	0.9303 (0.0147)
5- to 10-Year	0.9008 (0.0359)	0.9059* (0.0644)	0.9511* (0.0337)	0.9438 (0.0170)
10-Year and Over	0.9226 (0.0384)	0.9184* (0.0705)	0.9603* (0.0332)	0.9572 (0.0209)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.

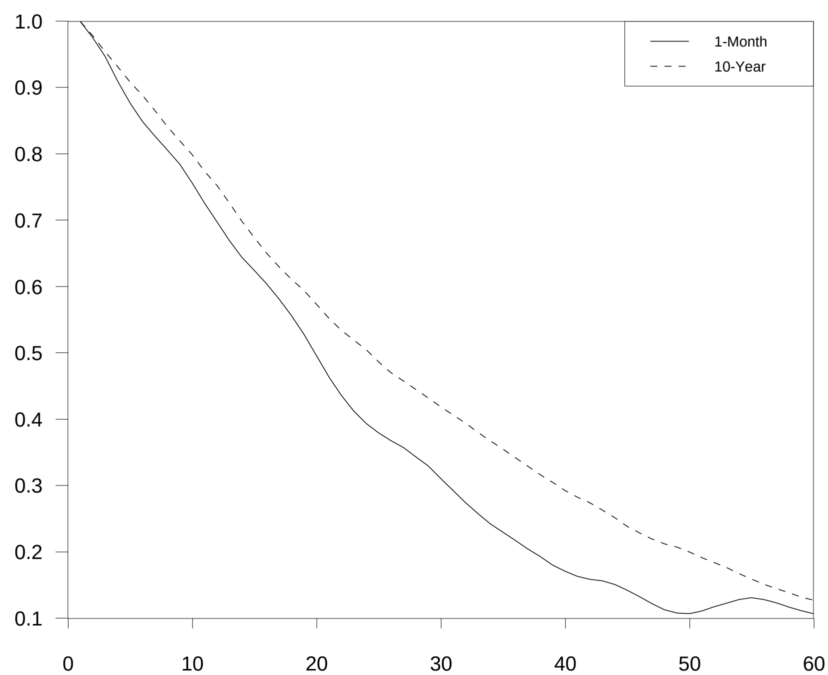

Figure 4
Autocorrelation functions, Canadian 1-month and 10-year rates, 1978:3 to 1999:6.

We can decompose the long (10-year) and short (3-month) nominal interest rates in the following manner:

$$i^{10} = r^{10} + \pi^{e} + rpcan + term \qquad (4.1)$$

$$i^{3} = r^{3} + \pi^{e} + rpcan \qquad (4.2)$$

where r is the real rate, π^e is expected inflation, $rpcan$ is a country-specific risk premium (which is assumed to equal zero for the United States and to be greater than zero for Canada), and $term$ is a positive term risk premium. Subtracting Equation (4.2) from Equation (4.1), we find that the yield spread can be expressed in two different manners:

$$(i^{10} - i^3) = (r^{10} + rpcan + term) - (r^3 + rpcan) \qquad (4.3)$$

or

$$(i^{10} - i^3) = (i^{10} - \pi^e) - (i^3 - \pi^e) \qquad (4.4)$$

Equation (4.3) states that the nominal yield spread equals the difference in risk premium–adjusted real rates, whereas Equation (4.4) states that it equals the inflation-adjusted nominal rates. The real rates, the country-specific risk premium, and the term premium are all unobservable. As such, we cannot formally estimate the orders of integration of the components in Equation (4.3). By using actual inflation as a proxy for expected inflation, however, we can at least estimate the fractional orders of integration of all terms entering Equation (4.4), allowing us to determine whether our proxies for the real rates follow long-memory processes.

Recall that two I(d) variables are said to be fractionally cointegrated if a linear combination of these variables yields an I($d - b$) series, where $d - b < d$. If the two variables are of different orders of integration, say d_1 and d_2, they are fractionally cointegrated if the resulting linear combination yields an I($d - b$) series, where $d = \min(d_1, d_2)$. We therefore are interested in knowing whether the fractional order of integration of the yield spread is lower than the orders of integration of both nominal and real rates, which is evidence of fractional cointegration between the rates.

4.1 United States

In Table 6 we present for the United States the orders of integration of (expected) inflation, nominal rates, nominal rates less expected inflation, and the yield spread. The rates used here are obtained from the St. Louis Fed, and expected inflation is the year-over-year growth of the total consumer price index. Beginning with the nominal rates, we find that the orders of integration are all above 0.86, whereas the order of integration of the spread is 0.66 or less. This implies that the simple difference of the rates, which is the equivalent of a [1, −1]

Table 6

Estimated fractional integration parameters (d), United States: Spread, real and nominal rates, 1978:3 to 1999:6 (256 observations)

Rate or Spread	Wavelet			
	Haar	Daubechies-4	Daubechies-12	Daubechies-20
π^e	1.0142*	0.9282*	0.9421*	1.0131*
	(0.0682)	(0.1567)	(0.1608)	(0.1135)
i^3	0.8645	0.8814*	0.9394*	0.9423*
	(0.0410)	(0.0634)	(0.0592)	(0.0414)
i^{10}	0.9226*	0.9502*	1.0099*	1.0122*
	(0.0611)	(0.0871)	(0.0542)	(0.0275)
$(i^3 - \pi^e)$	0.6402†	0.7104	0.7348	0.7530
	(0.1112)	(0.0809)	(0.0966)	(0.0716)
$(i^{10} - \pi^e)$	0.7560*†	0.6807*†	0.8830*	0.9383*
	(0.1378)	(0.2555)	(0.1234)	(0.0906)
$(i^{10} - i^3)$	0.3457†	0.6549†	0.6625†	0.5948†
	(0.2433)	(0.0969)	(0.1054)	(0.1536)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.
†d is within two standard errors of 0.5, the threshold below which the series can have a finite variance.

cointegration vector, yields a process of lower order of integration. The order of integration of the yield spread, however, is noticeably above 0.0, implying that it follows a long-memory process.

In the same table we also present the orders of integration of the nominal rates less expected inflation, as denoted in Equation (4.4). For each individual wavelet we find that the orders of integration of the "expected-inflation-adjusted" rates are again greater than the order of integration of the yield spread. For example, for the Daubechies-12 wavelet we find the order of the long and short rates to be 0.8830 and 0.7348 respectively, both larger than the spread order of 0.6625. This implies that both of these series are also fractionally cointegrated. That is, the difference between the estimates of long and short *real* rates yields a long-term relationship that is mean-reverting.

4.2 Canada

In Table 7 we present the Canadian results. The short rate used here is a 90-day Treasury bill rate. As with the United States, we find that the nominal rates are fractionally cointegrated, since the simple yield spread has an order of integration lower than the individual nominal rates. In fact, the order of integration is around 0.65, a similar order as for the United States.

The finding that the long-short yield spread is a long-memory process is somewhat surprising, since it has always been believed to be a stationary I(0) process. It is consistent, however, with empirical work that has found it to be a good indicator of economic activity, peaking in explanatory power at the 4- to 6-quarter forecast horizon (for example, see Estrella and Hardouvelis 1991). As explained by Baillie and Bollerslev (1994), long-memory error correction terms should possess adequate forecasting power only at longer horizons. The spread is, in fact, a [1, −1] cointegration vector, and therefore if it is a long-memory process, it should be most useful at longer horizons. Short-memory variables, such as the quarterly growth rate of money, should be superior short-run predictors.

Returning to Table 7, subtracting expected inflation from the nominal rates we find that the orders of integration of either the short or long rate are below the order of integration of the yield spread. For example, with the Daubechies-12 wavelet, the order of integration of the inflation-adjusted short rate is 0.6798; for the long rate it is 0.6904. The simple yield spread, however, has an order of integration of 0.6822. This implies

Table 7

Estimated fractional integration parameters (d), Canada: Spread, real, and nominal rates, 1978:3 to 1999:6 (256 observations)

Rate or Spread	Wavelet: Haar	Daubechies-4	Daubechies-12	Daubechies-20
π^e	1.0085* (0.0563)	0.9693* (0.0716)	1.0095* (0.0714)	1.0180* (0.0565)
i^3	0.8843 (0.0305)	0.9367* (0.0480)	0.9714* (0.0364)	0.9732* (0.0329)
i^{10}	0.9226 (0.0384)	0.9184* (0.0705)	0.9603* (0.0332)	0.9572 (0.0209)
$(i^3 - \pi^e)$	0.4514† (0.1642)	0.6378† (0.0823)	0.6798 (0.0845)	0.6688† (0.0900)
$(i^{10} - \pi^e)$	0.7333 (0.0816)	0.6984† (0.1052)	0.6904† (0.1311)	0.3536† (0.3288)
$(i^{10} - i^3)$	0.6259† (0.0879)	0.6394† (0.1202)	0.6822† (0.1120)	0.7142 (0.0875)

Note: Standard errors in parentheses.
*Unit root is within two standard errors of the estimated d.
† d is within two standard errors of 0.5, the threshold below which the series can have a finite variance.

either that these rates are not cointegrated or that a [1, −1] cointegration vector is not appropriate. We may conjecture that the country-specific risk premium terms in Equation (4.3) are the cause of the additional nonstationarity, consistent with our earlier observation that nominal Canadian rates have larger fractional orders of integration than their U.S. counterparts. The economic implication is that a world interest rate shock will be more rapidly absorbed by U.S. rates than Canadian rates, since the Canadian rates are also affected by a nonstationary risk premium.

5 Conclusion

This paper estimates the fractional integration parameters for several interest rates using a recent estimator with desirable properties. The purpose is to contribute to the debate on the order of integration of nominal interest rates. The estimated orders of integration may be of use to macroeconomic and financial modelers who seek more robust results.

Our findings differ somewhat over U.S. and Canadian rates. For the United States from 1948 to 1991, all short-term interest rates are long-run mean-reverting, whereas longer-term rates are most likely to follow unit root processes, as the estimated fractional integration parameters increase with the term to maturity. Nonconstant term premia may be the cause of this last result. When we restrict our attention to the latter half of the sample, we find that the evidence in favor of nonstationarity is diminished, leaving open the possibility that some rates, especially at the shorter horizons, may be following stationary long-memory processes.

Overall, we conclude that the unit root hypothesis is unduly harsh for the United States except for the longer-term interest rates. Furthermore, the assumption of short-run mean reversion is also strongly rejected by the data. The hypothesis that nominal interest rates follow fractionally integrated processes seems the most plausible.

Canadian rates also exhibit strong persistence over the full sample. Unlike the U.S. rates, however, this persistence remains even over the second half of the sample. This indicates that shocks to interest rates take longer to dissipate in Canada than the United States. As we noted above, long-term bonds may be more nonstationary than short-term bonds because of the additional risk captured in the term premia. Canadian bonds are usually riskier than their American counterparts because of, for example, political uncertainty and exchange rate movements, and as such this additional element of risk may be reflected in the larger order of integration of Canadian bond rates in the last 20 years.

For the applied researcher the following conclusions emerge from our findings. First, the rate of mean reversion decreases with the term to maturity, with longer-term rates reverting more slowly to their means than short-term rates, if they revert at all. Second, if these interest rates are used in cointegration analysis, then the underlying vectors may not be strict I(0) processes. In other words, if the cointegrating relationship is fractionally integrated, then adjustments to shocks may take a long time to be finalized. Finally, the fractional order of integration may indicate whether a given variable would be most adequate as a short-run or long-run indicator. An I(d) variable may be preferable for longer-run forecasts, whereas an I(0) variable would be most appropriate for the short run.

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