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# Detecting Equilibrium Correction with Smoothly Time-Varying Strength

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***Abstract.*** *Simulations are used to check the probability of detecting a time-varying equilibrium correction by applying the existing tests of no cointegration and parameter constancy. Smooth-transition regressions are chosen to describe the nonlinearity, and the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test are applied. It turns out that both tests perform well separately, but the joint power is quite low. The most notable result of this study is the high power when dealing with unrestricted cointegration, that is, when no cointegrating vector is estimated and the cointegrated variables freely enter the model in levels. The power of the parameter constancy test for the unrestricted cointegration is close to the power when the cointegrating vector is assumed to be known.*

***Keywords.*** time-varying equilibrium correction, cointegration, parameter constancy, smooth-transition regression

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## 1 Introduction

The concept of cointegration is based on the assumption that a pair of integrated economic variables are linked by a long-run stationary equilibrium relationship (see Engle and Granger 1987). It is implicitly assumed that the variables are cointegrated at all time periods and that the rate of adjustment toward the long-run equilibrium is constant over the sample. Recently, there have been several empirical macroeconometric studies in which the equilibrium correction term enters the model nonlinearly (see, for example, Michael, Nobay, and Taylor 1997; Ericsson, Hendry, and Prestwich 1998; Teräsvirta and Eliasson 2001 modeling money demand; and van Dijk and Franses 2000 and Ripatti and Saikkonen forthcoming describing interest rates). This raises the question of the reliability of the existing tests for detecting cointegration and nonlinearities in a nonlinear equilibrium correction model. Two recent articles have studied similar questions. Balke and Fomby (1997) compare the performance of the Engle-Granger and the Johansen cointegration tests when the adjustment follows a threshold cointegration model, and van Dijk and Franses (2000) perform a study where the adjustment is of the smooth-transition regression (STR) type. In the former study the cointegrating relationship is locally cointegrating; that is, the system may not be equilibrium correcting in all time periods.

 

In the latter study the simulations are equilibrium correcting in all periods, although the strength of the equilibrium correction varies. These studies conclude that both the Engle-Granger and the Johansen cointegration tests work well when the equilibrium correction is nonlinear.

This article will consider the probability of detecting time-varying equilibrium correction by applying the existing tests of no cointegration and parameter constancy. The simulated series are locally cointegrated, and the time-varying equilibrium correction is characterized by an STR model. There are no tests for simultaneously testing the joint hypotheses of no cointegration and constant parameters, and a two-step procedure will be applied. The investigator is assumed to follow a standard modeling procedure. First, the null of no cointegration is tested. The series are generated according to a two-dimensional equation system, and the Johansen cointegration test (Johansen 1995) is applied to test the hypothesis of no cointegration. If this hypothesis is rejected, the cointegrating relationship is estimated. A lagged estimated relationship forms the equilibrium correction (EC) term, and from there on a single equation is used. The constancy of the coefficient of the EC term is tested as in Lin and Teräsvirta 1994, so that the alternative to parameter constancy is a smoothly changing parameter. The purpose of the article is to find out how often an investigator correctly reaches the conclusion that there exists a cointegrating relationship whose strength varies over time. Furthermore, the aim is to find out which factors affect the probability of arriving at such a conclusion.

The results of the simulations show that an investigator rarely reaches the conclusion of a time-varying equilibrium correction when such a conclusion is correct. Considering the case in which the sample size is large and the coefficient of the equilibrium correction term is high (both of which have positive effects on the power of both tests), the joint power of the two tests is still low and varies a great deal, depending on the other parameter values. The power is at its lowest when the short-run dynamics are very strong, since this makes it difficult to estimate the cointegrating relationship, and the estimated equilibrium correction term often seems redundant. The low significance level of the cointegrating term makes it difficult for the parameter constancy test to detect the time variation. By not estimating the cointegrating relationship, however, and instead including the cointegrated variables in levels, the power of the equilibrium correction equation without a priori restrictions strongly increases and becomes almost as high as it is when the cointegrating relationship is assumed to be completely known a priori.

The outline of this article is as follows. Section 2 reviews the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The simulation setup is also presented. In Section 3 the testing procedure and the results of the Monte Carlo simulations appear, and Section 4 concludes.

## 2 Methodology

### *2.1 Johansen cointegration test*

A vector time series $\{\boldsymbol{y}_t\}$ is said to be cointegrated of order $b$ if each of the series taken individually is integrated of order $d$, $I(d)$, $d \geqslant 1$, whereas a linear combination of the series, $\boldsymbol{\beta}' y$, is $I(d-b)$, $b > 0$, where $\boldsymbol{\beta}$ is the cointegrating vector. The vector is not unique, since it can be multiplied by any nonzero scalar and still satisfy the cointegration condition.

There are several tests for no cointegration. The Johansen cointegration test (see Johansen 1995, chap. 6) is one of the most popular tests in empirical studies, and it is therefore chosen for this study. The testing procedure will be reviewed briefly here. Start by considering the unrestricted VAR($p$) model,

$$\boldsymbol{y}_t = \sum_{i=1}^{p} \boldsymbol{\Pi}_i \boldsymbol{y}_{t-i} + \varepsilon_t, \quad t = 1, \ldots, T \tag{2.1}$$

where $\boldsymbol{y}_t$ is an $(n \times 1)$ vector, $\boldsymbol{\Pi}_i$ are $(n \times n)$ matrices and $\varepsilon_t \sim$ n.i.d.$(0, \Omega)$. Each individual variable in $\boldsymbol{y}_t$ is assumed to be $I(1)$ . The reduced form of the equilibrium correction model becomes

$$\Delta \boldsymbol{y}_t = \sum_{i=1}^{p-1} \Gamma_i \Delta \boldsymbol{y}_{t-i} + \boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \varepsilon_t, \quad t = 1, \ldots, T \tag{2.2}$$

where $\Gamma_1 = \boldsymbol{\Pi}_1 - I_n$, $\Gamma_2 = \boldsymbol{\Pi}_2 + \Gamma_1, \ldots,$ and $\boldsymbol{\Pi} = I - \sum_{i=1}^{p-1} \boldsymbol{\Pi}_i$. Under the null hypothesis $H(h)$, there are assumed to be exactly $h$ linear combinations of $\boldsymbol{y}$ that are $I(0)$. Consequently, $\Pi$ can be rewritten as $\boldsymbol{\alpha\beta}'$ where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $(n \times h)$. The alternative hypothesis is that there are $n$ cointegrating relations, where $n$ is the number of elements of $\boldsymbol{y}_t$, which would imply that every linear combination of $\boldsymbol{y}_t$ is stationary, and no restrictions would be imposed on $\Pi$. The likelihood ratio test of $H(h)$ against $H(n)$ is given by

$$\mathcal{L}_n - \mathcal{L}_h = -\frac{\mathcal{T}}{2} \sum_{i=h+1}^{n} \ln\left(1 - \hat{\lambda}_i\right) \tag{2.3}$$

The eigenvalues $\hat{\lambda}_i$ of $\boldsymbol{\Pi}$ can be found by performing the first steps of Johansen's algorithm for the maximum-likelihood estimator of $\boldsymbol{\beta}$ in $H(h)$: $\boldsymbol{\Pi} = \boldsymbol{\alpha\beta}'$. If Equation (2.3) consisted only of stationary variables, the test statistic would be distributed as a chi-square asymptotically. Under the null hypothesis, however, the test statistic will depend on $(n - h)$ random walks, and the critical values are not the standard ones but can be found in Johansen 1995, chapter 15.

### 2.2 Lin and Teräsvirta parameter constancy test

An STR model is a flexible nonlinear model with a continuum of regimes. It is locally linear, and transitions from one extreme regime to another are determined by the so-called transition variable. Certain discrete regime models, such as switching regression models with two regimes and an observable switching variable, are nested in the STR class of models.

Consider the following single-equation STR model:

$$\boldsymbol{y}_t = \boldsymbol{x}_t' \varphi + \boldsymbol{x}_t' \theta F(s_t) + u_t, \qquad t = 1, \ldots, T \tag{2.4}$$

where $\varphi = \left(\varphi_0, \varphi_1, \ldots, \varphi_m\right)'$ is the parameter vector for the linear component and $\boldsymbol{x}_t = (1, x_{1t}, \ldots, x_{mt})' = \left(1, y_{t-1}, \ldots, y_{t-p}, z_{1t}, \ldots, z_{qt}\right)'$ is the corresponding vector of stationary and ergodic explanatory variables. The nonlinear component is specified as a linear component multiplied by a nonlinear function, $\boldsymbol{x}_t' \theta F(s_t)$ and $u_t \sim$ n.i.d.$\left(0, \sigma^2\right)$ for simplicity. The nonlinear function $F(s_t)$ is the transition function, which is continuous and bounded. It is customary to bound $F$ between zero and unity. Hence, the model will change locally from $E\left(\boldsymbol{y}_t | \boldsymbol{x}_t\right) = \boldsymbol{x}_t' \varphi$ for $F = 0$ to $E\left(\boldsymbol{y}_t | \boldsymbol{x}_t\right) = \boldsymbol{x}_t' (\varphi + \theta)$ for $F = 1$ with the transition variable $s_t$. When $s_t = t$ the STR model can be interpreted as a linear model with time-varying parameters, which is the case that will be considered here. (For more elaborate discussions on STR models, see Granger and Teräsvirta 1993 and Teräsvirta 1998.)

The transition function of a $k$th-order logistic smooth transition regression model [LSTR($k$)] has the form

$$F(t) = F(\gamma, \boldsymbol{c}; t) = \left(1 + \exp\left\{-\gamma \prod_{i=1}^{k} (t - c_i)\right\}\right)^{-1}, \ \gamma > 0 \tag{2.5}$$

when a time trend $(t)$ is used as the transition variable. The slope parameter $(\gamma)$ determines how rapid the transition is, and the vector of location parameters $\boldsymbol{c} = (c_1, \ldots, c_k)'$ decides where the transitions occur. The STR model (2.4) becomes linear when $\gamma = 0$, so that the transition function $F(t) \equiv \frac{1}{2}$. For notational simplicity the transition function in (2.4) is replaced by $\tilde{F}(t, \gamma) = F(t, \gamma) - \frac{1}{2}$. This implies $\tilde{F}(t, 0) = 0$, and $H_0 : \gamma = 0$ becomes a natural hypothesis of parameter constancy (see Lin and Teräsvirta 1994). The alternative hypothesis is $H_1 : \gamma > 0$. There is one caveat, though: the STR model is not identified under the null hypothesis because of the nuisance parameters $\theta$ and $\boldsymbol{c}$. Thus the classical asymptotic distribution theory for testing the null hypothesis $\gamma = 0$ does not work. To circumvent this problem a Taylor series approximation to the transition function is used to obtain an appropriate test. Equation $(2.6)$ shows the auxiliary regression for the tests when $k = 2$. In Equations (2.7) and (2.8), the null hypotheses of the parameter constancy tests are presented:

$$\boldsymbol{y}_t = \delta_0^{'}\boldsymbol{x}_t + \delta_1^{'}\boldsymbol{x}_t t + \delta_2^{'}\boldsymbol{x}_t t^2 + v_t \tag{2.6}$$

$$LM_2 : \delta_1 = \delta_2 = 0 \tag{2.7}$$

$$LM_1 : \delta_1 = 0 \mid \delta_2 = 0 \tag{2.8}$$

If $k = 1$, the test to be applied is $\text{LM}_1$; if $k = 2$ it is $\text{LM}_2$. Thus, the size and power of the $\text{LM}_1$ statistic are reported when an LSTR(1) model is used for simulating the series, and $\text{LM}_2$ is used in simulations with the LSTR(2) model.

Testing the hypotheses of parameter constancy, a Lagrange multiplier (LM)–type test statistic can be obtained by using the linear auxiliary regression $(2.6)$. The test statistic will have an asymptotic chi-square distribution under the null hypotheses. In practice, an $F$ test is preferred to the chi-square test, since the former has better small sample properties (see Lin and Teräsvirta 1994).

### *2.3 Simulation setup*

The following two-dimensional system is used in the experiment:

$$\Delta\boldsymbol{y}_t = \theta\Delta\boldsymbol{y}_{t-1} + \delta\left(\boldsymbol{y}_{t-1} - \boldsymbol{z}_{t-1}\right)F(t) + u_t \tag{2.9}$$

$$\Delta\boldsymbol{z}_t = \omega_t \tag{2.10}$$

where $F(t)$ is the transition function, $u_t \sim \text{n.i.d.}(0, 0.25)$, $\omega_t \sim \text{n.i.d.}(0, 1)$, and $u_t$ and $\omega_t$ are mutually independent. Equation (2.9) represents a time-varying equilibrium correction model, and equation (2.10) generates a random walk. The magnitude of the short-run dynamics is allowed to vary, $\theta = \{0.2, 0.4, 0.9\}$, and so is the coefficient of the cointegration relationship, $\delta = \{-0.1, -0.4, -0.8\}$. To reduce the importance of the starting values, the first third of the observations are excluded from the reported results. That is, $T + \frac{1}{2}T$ observations are generated, and the sample sizes used are $T = 100,\ 200$. Each test is performed $N = 10{,}000$ times.

The simulated series are all generated by the time-varying equilibrium correction model (2.9) and (2.10). The transition function $F(t)$ is given by Equation ( 2.5, with either $k = 1$ or $k = 2$. In the former case one has an LSTR(1) model, and in the latter one has an LSTR(2) model.

The LSTR(1) model will be generated for different speeds of transition. When $\gamma = 1$ the transition is very smooth, when $\gamma = 10$ the transition function has the typical $S$-shape, and for $\gamma = 100$ the transition is rather quick (see Figures 1–3). Setting $\gamma = 1$, the strength of attraction increases almost linearly over time, and cointegration will be present in the whole observation period (see Figure 1). In this article the strength of the cointegrating relationship is increasing over time. It does not matter for the test, however, if the strength of the cointegrating relationship is increasing or decreasing over time. The location parameter $c$ is also allowed to vary and the series are simulated for $\boldsymbol{c} = \{0.25, 0.50, 0.75\}$. When $\gamma = 10$ or $\gamma = 100$ the length of the observation period including cointegration will vary with $c$. For larger values of $c$ the observation period including cointegration ($F > 0$) will be shorter (see Figures 2 and 3).

Generating an LSTR(2) model, the speed of transition is always high, $\gamma = 100$ (see Figure 4). There are two location parameters in the LSTR(2) model (see Equation (2.5)), $k = 2$, and they are allowed to vary too. The

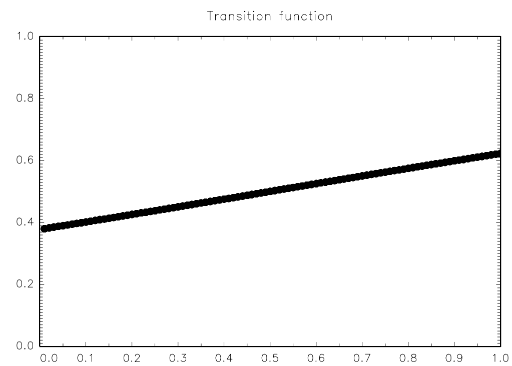


**Figure 1**
Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 1$.

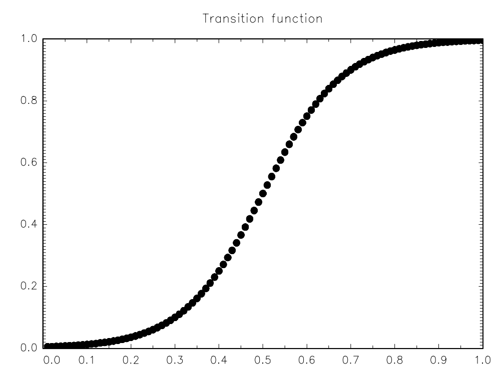


**Figure 2**
Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 10$.

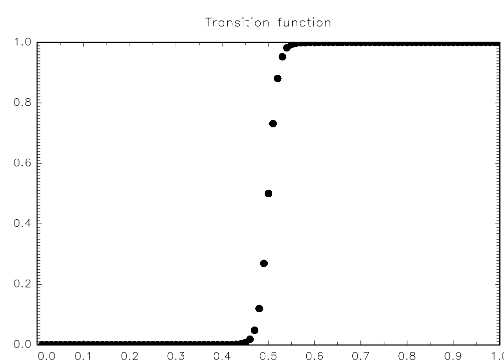


**Figure 3**
Transition function of an LSTR(1) model where $c = 0.5$ and $\gamma = 100$.

series are generated with $\boldsymbol{c}_1 = \{0.25, 1/3\}$ and $c_2 = 1 - c_1$. For the LSTR(2) model cointegration will be present in the beginning and the end of the sample. Hence, there will not be any cointegration ($F = 0$) in the middle of the sample (see Figure 2.4). The situation resembles that in threshold cointegration; the difference is that in this experiment, time is the transition variable. One may order the observations in a threshold cointegration model according to the threshold variable and draw a graph of the strength of attraction. If this is done it turns out that the figure is analogous to that corresponding to an LSTR(2) model where $\gamma \to \infty$.

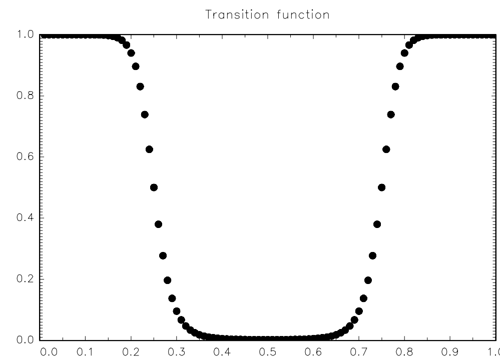


**Figure 4**
Transition function of an LSTR(2) model where $c_1 = 0.25$, $c_2 = 0.75$ and $\gamma = 100$.

## 3 Monte Carlo Simulations

In this section the testing procedure and the results of the simulations are presented. The purpose of the simulations is to find out how often a hypothetical researcher would arrive at the conclusion that there exists a cointegrating relationship whose strength varies over time when such a conclusion is correct. As mentioned in the introduction, there are no tests for simultaneously testing the hypothesis of no cointegration and parameter constancy; hence two tests will be applied: the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test. The results of the simulations will be presented for the two tests separately, but also jointly, since the results of the joint tests are the major concern. The parameter constancy test will be performed only when the Johansen test rejects the null hypothesis of no cointegration. If the null cannot be rejected, the estimated cointegrating vector would not create a stationary combination of the two $I(1)$ variables, making the Lin and Teräsvirta test misspecified. For the sake of simplicity, the rejection frequencies of the tests will be referred to as "power" throughout the article.

### *3.1 The testing procedure*

The testing procedure starts by estimating the null hypothesis of no cointegration. If the null hypothesis is rejected, the procedure continues by exploring the constancy of the cointegrating parameter using the Lin and Teräsvirta parameter constancy test. Whether the cointegration term enters the linear equilibrium correction equation significantly is also tested. This will be referred to as the "significance test."

Three different scenarios are considered when testing for parameter constancy: in Case 1 the cointegrating vector is assumed to be unknown, in Case 2 it is known, and in Case 3 no cointegration relationship is defined. In addition, the size of the Johansen cointegration test and the parameter constancy test are explored.

A more detailed description of the testing procedure is as follows:

- **Size of Johansen cointegration test and the Lin and Teräsvirta parameter constancy test.** When the size of the first test is investigated, the series are generated according to Equations (2.9) and (2.10), keeping the coefficient of the cointegrating relationship $\delta$ equal to zero in (2.9). The results are reported as "J size" in Table 1. This yields the empirical size of the Johansen test for the sample sizes $T = 100$ and $T = 200$. When the size of the parameter constancy test is examined, the series are generated according to Equations (2.9) and (2.10), assuming that the transition function $F(t)$ is equal to unity. Hence the model is linear. The hypotheses of linearity are tested, and the size results for Case 1 are reported as "LT-size" in Tables 2–5. This yields the empirical size of the parameter constancy test for Case 1. The selected nominal size is 5% for both tests.

**Table 1a**
Size of the Johansen cointegration test

| | | | |
|---|---|---|---|
| T = 100 | $\theta = 0.2$ | $\theta = 0.4$ | $\theta = 0.9$ |
| J size | 3.68 | 3.46 | 5.70 |
| T = 200 | $\theta = 0.2$ | $\theta = 0.4$ | $\theta = 0.9$ |
| J size | 3.54 | 3.33 | 4.67 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Number of repetitions: 10,000.

**Table 1b**
Power of the Johansen cointegration test: Linear model, $F(t) = 0$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $\theta$ : | **0.20** | **0.40** | **0.90** | **0.20** | **0.40** | **0.90** | **0.20** | **0.40** | **0.90** |
| Johansen | 99.98 | 90.17 | 99.97 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $\theta$ : | **0.20** | **0.40** | **0.90** | **0.20** | **0.40** | **0.90** | **0.20** | **0.40** | **0.90** |
| Johansen | 90.15 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Number of repetitions: 10,000.

**Table 2a**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.2$, $\gamma = 1$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| *c:* | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 54.62 | 48.28 | 40.16 | 100.0 | 99.94 | 99.76 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 6.52 | 6.48 | 6.53 | 11.98 | 13.70 | 14.38 | 16.89 | 19.81 | 22.42 |
| $H_0: \delta = 0$ | 88.50 | 89.23 | 89.74 | 74.22 | 75.77 | 78.08 | 51.76 | 57.39 | 62.36 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 6.74 | 7.33 | 6.80 | 14.50 | 16.11 | 17.00 | 30.34 | 32.12 | 33.66 |
| $H_0: \delta = 0$ | 99.56 | 99.38 | 99.07 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 9.67 | 10.34 | 10.28 | 12.55 | 13.71 | 14.02 | 23.03 | 24.80 | 26.33 |
| $H_0: \delta_1 = \delta_2 = 0$ | 71.84 | 67.77 | 64.42 | 99.98 | 99.95 | 99.79 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.59 | 4.79 | 4.56 | 4.73 | 5.04 | 4.84 | 4.78 | 4.58 | 4.71 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| *c:* | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 96.96 | 93.41 | 88.30 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 8.19 | 8.71 | 9.17 | 21.24 | 22.94 | 24.49 | 28.98 | 34.68 | 38.33 |
| $H_0: \delta = 0$ | 86.17 | 87.22 | 88.57 | 74.22 | 76.51 | 77.83 | 50.05 | 56.46 | 62.12 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 8.67 | 9.06 | 10.08 | 26.91 | 29.08 | 30.07 | 54.23 | 59.08 | 60.29 |
| $H_0: \delta = 0$ | 99.99 | 99.98 | 99.90 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 9.86 | 10.28 | 10.55 | 21.66 | 22.40 | 23.64 | 44.61 | 48.12 | 50.08 |
| $H_0: \delta_1 = \delta_2 = 0$ | 95.12 | 92.41 | 89.24 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.78 | 4.90 | 4.77 | 4.75 | 4.88 | 4.95 | 4.85 | 4.89 | 5.02 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

- **The Johansen cointegration test.** The numbers of rejections of the null hypothesis will be referred to as the power of the test and are reported as "Johansen" in Tables 2–5. If, and only if, the null hypothesis of no cointegration is rejected, the testing procedure continues with the parameter constancy test for three different scenarios named Cases 1, 2, and 3 as follows.

**Table 2b**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.2$, $\gamma = 10$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 56.68 | 22.46 | 14.61 | 95.89 | 31.62 | 13.30 | 99.22 | 31.64 | 11.28 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 34.33 | 24.00 | 10.81 | 79.32 | 75.96 | 50.98 | 62.62 | 69.12 | 64.63 |
| $H_0$: $\delta = 0$ | 91.95 | 89.76 | 88.36 | 81.08 | 81.63 | 85.41 | 61.34 | 69.43 | 77.30 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 48.71 | 61.98 | 70.77 | 96.73 | 99.34 | 93.53 | 99.90 | 100.0 | 99.82 |
| $H_0$: $\delta = 0$ | 99.70 | 97.33 | 87.13 | 100.0 | 99.78 | 94.89 | 100.0 | 99.87 | 96.28 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 38.64 | 49.29 | 60.37 | 91.76 | 97.15 | 90.38 | 99.79 | 100.0 | 99.02 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 75.72 | 52.05 | 49.08 | 99.76 | 87.98 | 59.62 | 100.0 | 92.95 | 77.04 |
| LT size: | 4.87 | 4.48 | 5.08 | 5.04 | 4.77 | 5.23 | 4.60 | 4.87 | 5.19 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 91.41 | 40.59 | 21.06 | 99.99 | 51.05 | 21.94 | 100.0 | 52.94 | 17.69 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 70.44 | 58.17 | 21.70 | 84.09 | 83.80 | 64.04 | 59.98 | 76.79 | 71.40 |
| $H_0$: $\delta = 0$ | 95.26 | 93.27 | 90.50 | 83.31 | 86.33 | 87.33 | 58.65 | 76.20 | 83.15 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 80.10 | 89.53 | 79.77 | 99.89 | 100.0 | 99.32 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta = 0$ | 100.0 | 99.24 | 91.98 | 100.0 | 99.94 | 98.27 | 100.0 | 99.98 | 99.04 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 75.52 | 84.60 | 68.47 | 99.77 | 100.0 | 98.72 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 94.44 | 74.92 | 60.97 | 100.0 | 95.75 | 77.44 | 100.0 | 97.15 | 89.49 |
| LT size: | 4.90 | 4.77 | 4.88 | 5.03 | 5.01 | 4.81 | 5.15 | 4.57 | 4.72 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

- **Case 1: The cointegrating vector is unknown.** After the hypothesis of no cointegration is tested and rejected, the cointegrating vector is estimated. The maximum-likelihood estimator is given by the eigenvector corresponding to the $h$ largest eigenvalues $\hat{\lambda}_1$ (see Johansen 1995, chapter 6) and is included in the linear model, subject to testing, which becomes

$$\Delta \boldsymbol{y}_t = \theta \Delta \boldsymbol{y}_{t-1} + \delta \left( \boldsymbol{y}_{t-1} - \hat{\xi} \boldsymbol{z}_{t-1} \right) + u_{1t}$$

  The existence and parameter constancy of the estimated cointegration coefficient $\hat{\delta}$ are tested; note that this is done only if the null of no cointegration is rejected. The number of times a false null hypothesis is rejected (power) is reported under "Case 1" in Tables 2–5. The only test results that will be reported for the parameter constancy tests are $LM_1$ for the LSTR(1) model and $LM_2$ for the LSTR(2) model.

- **Case 2: The cointegrating vector is known.** The real cointegrating relationship is assumed to be known. This assumption is not realistic, but the purpose is to create a case that can be used for comparisons with the more realistic scenarios. By doing the comparison, it is easier to detect where the major weaknesses in the testing procedure occur. The linear model subject to tests becomes

$$\Delta \boldsymbol{y}_t = \theta \Delta \boldsymbol{y}_{t-1} + \delta \left( \boldsymbol{y}_{t-1} - \boldsymbol{z}_{t-1} \right) + u_{2t}$$

  The power of the tests is reported under "Case 2" in Tables 2–5.

- **Case 3: Unrestricted cointegration.** The integrated variables are included without any restriction in the linear model. That is, the variables are still cointegrated, but the cointegrating vector is not estimated. The estimated linear model becomes

$$\Delta \boldsymbol{y}_t = \theta \Delta \boldsymbol{y}_{t-1} + \delta_1 \boldsymbol{y}_{t-1} + \delta_2 \boldsymbol{z}_{t-1} + u_{3t}$$

**Table 2c**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.2$, $\gamma = 100$

| T = 100 | | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 32.44 | 11.55 | 6.33 | 29.91 | 8.82 | 3.26 | 23.41 | 6.40 | 3.06 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 28.61 | 26.75 | 18.00 | 63.66 | 64.06 | 38.04 | 39.85 | 46.09 | 42.16 |
| $H_0$: $\delta = 0$ | 88.22 | 83.20 | 73.93 | 65.86 | 68.03 | 54.60 | 40.62 | 44.53 | 44.44 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 44.88 | 59.05 | 54.82 | 97.06 | 99.77 | 73.31 | 99.96 | 100.0 | 87.58 |
| $H_0$: $\delta = 0$ | 99.32 | 92.29 | 71.41 | 99.67 | 95.80 | 70.55 | 99.32 | 95.16 | 67.65 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 41.25 | 47.79 | 33.18 | 96.05 | 99.09 | 61.04 | 99.87 | 99.84 | 79.08 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 69.27 | 45.54 | 30.96 | 95.29 | 75.85 | 34.05 | 93.21 | 79.38 | 47.39 |
| LT size: | 5.03 | 4.57 | 4.84 | 4.85 | 5.10 | 4.81 | 4.76 | 5.03 | 4.93 |
| **T = 200** | | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 45.41 | 16.39 | 7.58 | 32.15 | 9.26 | 3.87 | 24.63 | 6.59 | 2.54 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 64.52 | 51.68 | 22.82 | 71.32 | 69.33 | 51.42 | 41.74 | 51.44 | 51.18 |
| $H_0$: $\delta = 0$ | 91.96 | 87.49 | 81.00 | 71.51 | 74.08 | 63.82 | 41.58 | 50.22 | 53.15 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 83.97 | 88.90 | 65.17 | 100.0 | 100.0 | 89.95 | 100.0 | 100.0 | 92.52 |
| $H_0$: $\delta = 0$ | 99.85 | 95.42 | 78.63 | 99.88 | 96.76 | 74.94 | 99.27 | 95.76 | 69.69 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 80.33 | 80.54 | 42.35 | 99.97 | 100.0 | 74.16 | 100.0 | 100.0 | 86.61 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 85.40 | 63.09 | 40.63 | 96.98 | 84.88 | 46.77 | 94.48 | 82.25 | 46.46 |
| LT size: | 4.85 | 4.67 | 4.68 | 4.78 | 4.66 | 5.16 | 4.71 | 5.27 | 5.21 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

The former cointegrating coefficient is divided into two separate estimates, one for each integrated variable. The estimated coefficients $\hat{\delta}_1$ and $\hat{\delta}_2$ ought to satisfy $\hat{\delta}_1 \approx -\hat{\delta}_2$, since data are generated according to Equation (2.9). The hypothesis $\delta_1 = \delta_2 = 0$ and that of parameter constancy are tested. Note that the assumptions of the parameter constancy tests are violated, as the test requires stationarity of the regressors. But since the $I(1)$ variables are cointegrated, the combination of the two is still stationary. The power of the tests is reported under "Case 3" in Tables 2–5.

### *3.2 Simulation results*

The simulation results of the LSTR(1) model can be found in Tables 2–4, and the results of the LSTR(2) model are shown in Table 5. Tables 2a–2c present the results when the coefficient of the lagged first-difference $\Delta\boldsymbol{y}_{t-1}$ is small ($\theta = 0.2$); in Tables 3a–3c, the coefficient is higher ($\theta = 0.4$); and in Tables 4a–4c, the coefficient of $\Delta\boldsymbol{y}_{t-1}$ is $\theta = 0.9$. The letters in the tables' titles refer to the speed of transition. Thus, *a* denotes the smoothest transition ($\gamma = 1$), *b* corresponds to $\gamma = 10$, and *c* stands for a transition function close to a step function ($\gamma = 100$) (see Figures 1–4).

The size and power results of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test are discussed below. The results of the latter test will be considered separately for each of the three different cases. The two tests are initially discussed separately for simplicity of exposure, since some parameter changes affect the power of the two tests in opposite ways. The joint performance of the tests will be discussed in Section 3.2.5.

**3.2.1 The size of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test** Before the power of the Johansen cointegration test and the Lin and Teräsvirta parameter constancy tests ($LM_1$ and $LM_2$)

**Table 3a**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.4$, $\gamma = 1$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 52.43 | 43.60 | 36.04 | 100.0 | 99.99 | 99.88 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 6.29 | 6.08 | 6.60 | 7.88 | 9.56 | 9.92 | 6.82 | 8.84 | 10.31 |
| $H_0$: $\delta = 0$ | 75.22 | 75.34 | 76.44 | 47.08 | 50.90 | 54.77 | 16.35 | 22.56 | 28.47 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 6.28 | 6.01 | 5.85 | 14.59 | 15.89 | 16.84 | 31.00 | 33.64 | 34.48 |
| $H_0$: $\delta = 0$ | 99.31 | 99.54 | 98.97 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 8.98 | 8.62 | 9.46 | 12.52 | 13.74 | 14.20 | 24.13 | 26.61 | 26.56 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 67.92 | 61.81 | 56.83 | 100.0 | 99.94 | 99.83 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.59 | 4.87 | 4.70 | 4.80 | 4.97 | 5.06 | 4.48 | 5.03 | 4.84 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 97.55 | 93.74 | 87.84 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 7.36 | 8.05 | 8.06 | 12.62 | 15.69 | 17.38 | 8.51 | 13.69 | 17.35 |
| $H_0$: $\delta = 0$ | 73.45 | 74.39 | 76.71 | 45.06 | 50.00 | 53.57 | 13.73 | 21.08 | 26.15 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 8.39 | 9.58 | 9.06 | 26.13 | 29.24 | 31.27 | 56.51 | 60.75 | 63.14 |
| $H_0$: $\delta = 0$ | 100.0 | 99.99 | 99.94 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 9.48 | 10.36 | 9.56 | 20.73 | 22.62 | 24.34 | 46.17 | 51.21 | 52.52 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 95.40 | 92.76 | 88.50 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.61 | 4.76 | 4.37 | 4.56 | 4.74 | 5.16 | 4.83 | 5.10 | 5.26 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

is considered, the size of the tests is examined. It turns out that the size of the Johansen test varies between 3.3% and 5.7% and is usually slightly below the chosen significance level of 5% (see Table 1a). The size of the Lin and Teräsvirta parameter constancy test, on the other hand, is in the close neighborhood of the nominal significance level of 5%: the smallest value is 4.33% and the largest 5.40% (see Tables 2–5).

**3.2.2 The power of the Johansen cointegration test** The results of the simulations show that the power of the Johansen cointegration test is usually quite high when there is time-varying equilibrium correction. The test is sensitive to some of the parameter changes, however, and in this section the effect of parameter changes on the power of the Johansen cointegration test will be discussed in more detail. To enable us to assess the loss of power due to time-varying parameters, the empirical power of the Johansen test for a linear model (transition function equal to unity) is shown in Table 1b.

When the magnitude of the cointegrating coefficient $\delta$ increases (in absolute value), the power increases for both the LSTR(1) and LSTR(2) models (see, e.g., Tables 2a and 5). An increase in $\delta$ makes the cointegrating relationship more distinct and therefore easier for the test to detect, and hence the power increases. For the LSTR(1) models the power of the Johansen test is also high when the speed of transition is low ($\gamma = 1$), which is not surprising considering that the test is developed for a linear model. Besides, the parameter change is almost a linear function, making the equilibrium correction present during the whole observation period. For a more rapid transition speed ($\gamma = 10$ or 100), the cointegration enters only when $F > 0$, and the observation period during which the system equilibrium corrects is only a subperiod of the original one. Because of this, the power also depends on the location parameter $c$. A time series generated with a higher $c$ will have lower power when $\gamma = 10$ or 100, since there will be fewer observations available with information about the cointegration relationship (see, e.g., Table 2b). The same result also appears when the observations

**Table 3b**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.4$, $\gamma = 10$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 50.80 | 16.62 | 8.13 | 97.20 | 30.71 | 10.85 | 99.93 | 37.53 | 11.02 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 30.06 | 22.32 | 8.86 | 48.60 | 50.21 | 39.91 | 26.28 | 34.85 | 44.19 |
| $H_0$: $\delta = 0$ | 75.93 | 76.47 | 73.92 | 48.57 | 52.33 | 60.18 | 26.13 | 34.51 | 48.00 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 45.33 | 53.67 | 49.08 | 96.69 | 99.67 | 90.14 | 99.92 | 100.0 | 99.46 |
| $H_0$: $\delta = 0$ | 99.74 | 96.15 | 78.60 | 100.0 | 99.80 | 92.17 | 100.0 | 99.87 | 95.64 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 42.15 | 45.67 | 34.81 | 94.87 | 99.19 | 84.06 | 99.87 | 100.0 | 99.46 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 70.63 | 43.32 | 28.91 | 99.80 | 88.70 | 57.97 | 100.0 | 92.97 | 81.03 |
| LT size: | 4.60 | 5.10 | 4.51 | 4.74 | 4.65 | 4.81 | 4.93 | 5.17 | 5.16 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 88.21 | 31.09 | 13.00 | 99.98 | 47.95 | 16.96 | 100.0 | 58.93 | 16.08 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 61.46 | 50.08 | 21.62 | 49.64 | 57.88 | 56.72 | 17.11 | 41.80 | 47.57 |
| $H_0$: $\delta = 0$ | 79.75 | 78.39 | 81.77 | 48.74 | 56.87 | 67.39 | 16.77 | 41.08 | 49.56 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 80.18 | 85.46 | 70.62 | 99.90 | 100.0 | 99.46 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta = 0$ | 100.0 | 99.16 | 90.46 | 100.0 | 99.98 | 97.46 | 100.0 | 100.0 | 98.32 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 75.31 | 79.22 | 57.92 | 99.78 | 100.0 | 98.76 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 93.55 | 67.39 | 43.69 | 100.0 | 95.43 | 77.95 | 100.0 | 96.95 | 86.32 |
| LT size: | 4.77 | 4.95 | 5.10 | 5.48 | 5.01 | 4.97 | 4.95 | 5.02 | 4.62 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

are simulated according to an LSTR(2) model (see Table 5). When the location parameter $c_1 = \frac{1}{3}$, the period including equilibrium correction will be longer than when $c_1 = 0.25$, since it is assumed that $c_2 = 1 - c_1$, and the power of the test is higher. The power is also positively affected by the sample size for both the LSTR(1) and LSTR(2) models; hence the tests appear consistent.

More surprisingly, the power of the cointegration test increases when the strength of the short-run dynamics ($\theta$) is high. This is especially noticeable when $\gamma = 100$ (see Tables 2c and 4c). The power is at least three times as high in Table 4c as in Table 2c. A reason for this might be that the change in the strength of the equilibrium correction disturbs the test less when the relative weight of the cointegrating relationship is small. Besides, a previous experiment comparing the power of the Johansen and the Engel-Granger cointegration tests showed that the power of the Johansen test was higher when data were generated according to Equations (2.4) and (2.5) instead of $\boldsymbol{y}_t = \theta \Delta \boldsymbol{z}_{t-1} + \delta \left(\boldsymbol{y}_{t-1} - \boldsymbol{z}_{t-1}\right) F(t) + u_t$ and (2.5). Hence, the Johansen test performs better when the short-run dynamics are given by lagged values of the dependent variable than when they are given by an exogenous variable. Keeping this in mind, the improved power of the cointegration test when $\theta = 0.9$ is less surprising.

**3.2.3 The power of the Lin and Teräsvirta parameter constancy test for Case 1: The cointegrating vector is estimated** The performance of the Lin and Teräsvirta parameter constancy test is strongly dependent on the choice of parameter values. For most setups the power is fair, but when the short-run dynamics are very strong ($\theta = 0.9$), it is close to zero. A more detailed discussion of how the parameter changes affect the power is given below.

When the model is generated by an LSTR(1) model, an increase (in absolute value) in the coefficient of the cointegrating relationship ($\delta$) improves the power (see, e.g., Table 2b). When the speed of transition increases

**Table 3c**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.4$, $\gamma = 100$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 26.73 | 8.98 | 4.41 | 32.73 | 10.82 | 4.57 | 32.24 | 9.80 | 4.50 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 25.63 | 20.60 | 11.79 | 31.01 | 36.23 | 26.91 | 14.14 | 27.76 | 28.00 |
| $H_0$: $\delta = 0$ | 70.37 | 67.15 | 56.92 | 31.38 | 35.03 | 35.23 | 15.79 | 21.33 | 24.22 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 42.76 | 52.56 | 31.52 | 97.68 | 99.72 | 76.59 | 100.0 | 100.0 | 90.89 |
| $H_0$: $\delta = 0$ | 99.51 | 90.31 | 60.32 | 99.60 | 94.55 | 66.74 | 99.32 | 91.02 | 62.00 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 37.75 | 41.09 | 23.81 | 97.22 | 98.89 | 58.64 | 100.0 | 99.90 | 75.56 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 66.55 | 39.64 | 24.04 | 94.29 | 76.34 | 29.76 | 92.56 | 76.12 | 42.22 |
| LT size: | 4.87 | 4.96 | 4.55 | 4.89 | 4.71 | 4.81 | 4.64 | 5.13 | 4.79 |
| **T = 200** | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 38.37 | 12.46 | 4.64 | 33.34 | 10.48 | 3.81 | 30.86 | 9.07 | 3.08 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 54.96 | 44.94 | 18.97 | 32.63 | 41.41 | 30.71 | 13.90 | 26.24 | 32.79 |
| $H_0$: $\delta = 0$ | 72.58 | 70.55 | 62.07 | 31.97 | 41.22 | 41.21 | 16.14 | 23.59 | 30.52 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 83.89 | 87.40 | 48.06 | 100.0 | 100.0 | 77.43 | 100.0 | 100.0 | 87.34 |
| $H_0$: $\delta = 0$ | 99.79 | 94.54 | 72.41 | 99.67 | 96.56 | 70.08 | 99.58 | 93.16 | 58.44 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 80.43 | 78.41 | 32.33 | 100.0 | 99.81 | 69.03 | 100.0 | 99.89 | 84.09 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 84.15 | 59.95 | 26.50 | 95.71 | 83.78 | 39.37 | 94.49 | 80.50 | 42.53 |
| LT size: | 4.90 | 4.37 | 4.77 | 4.72 | 4.58 | 5.28 | 4.94 | 4.63 | 5.23 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

($\gamma > 1$), the nonlinearity becomes more distinct, and the power of the test increases (see Tables 2a–2c). There seems to be a peak, however, when $\gamma = 10$, since the power is often slightly weaker when $\gamma = 100$. This is because the test is conditional on the rejection of the hypothesis of no cointegration. Moreover, changing the location of the transition ($c$) does not generally seem to affect the power much. But in some cases the power is slightly smaller when $c = 0.75$ and $\gamma$ is high (see Table 2c). In this case, the period during which the equilibrium correction is operating is short. This might make it hard to estimate the parameters of the cointegrating vector, and it becomes difficult for the test to detect the time variation. An increase in the sample size improves the power, especially when $\delta$ is small (see, e.g., Table 2b).

Finally, increasing the coefficient of the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ reduces the power sharply (see Tables 2b–4b). The reason for the poor performance when $\theta = 0.9$ is that the dominating short-run dynamics make the cointegrating vector difficult to estimate. The power of the significance test is also very low for this scenario, and if the cointegrating relationship does not enter significantly, it is hardly surprising that the parameter constancy test has low power. When data are generated according to an LSTR(2) model, the behavior is similar. A stronger coefficient of the cointegrating relationship and a larger sample size improve the power of the parameter constancy test, whereas it seems to be unaffected by the value of the location parameter (see Table 5).

#### 3.2.4 The power of the Lin and Teräsvirta parameter constancy test for Case 2: The cointegrating vector is known

In Case 2, the cointegrating vector is known, and the overall performance of the Lin and Teräsvirta parameter constancy test is better than for Case 1. The power of the significance test also improves considerably. This demonstrates the importance of high-quality estimates of the cointegrating vector for the second test.

**Table 4a**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.9$, $\gamma = 1$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 98.58 | 96.93 | 94.25 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 3.04 | 3.19 | 2.98 | 1.43 | 1.51 | 1.89 | 1.02 | 0.97 | 0.94 |
| $H_0$: $\delta = 0$ | 2.51 | 2.40 | 2.53 | 0.28 | 0.38 | 0.51 | 0.03 | 0.07 | 0.06 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 9.80 | 10.60 | 10.90 | 39.64 | 42.05 | 43.27 | 79.06 | 81.49 | 82.18 |
| $H_0$: $\delta = 0$ | 99.98 | 99.98 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 9.65 | 10.74 | 11.05 | 31.49 | 33.69 | 34.58 | 71.42 | 74.15 | 74.35 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 97.18 | 95.90 | 92.89 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.33 | 4.78 | 4.60 | 5.04 | 5.01 | 4.82 | 4.85 | 4.52 | 4.87 |
| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 100.0 | 100.0 | 99.97 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 1.08 | 1.00 | 1.10 | 0.39 | 0.50 | 0.48 | 0.35 | 0.32 | 0.36 |
| $H_0$: $\delta = 0$ | 0.80 | 0.92 | 1.22 | 0.03 | 0.03 | 0.03 | 0.00 | 0.00 | 0.00 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 16.02 | 18.74 | 18.42 | 68.17 | 71.87 | 72.50 | 97.55 | 98.42 | 92.26 |
| $H_0$: $\delta = 0$ | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 13.32 | 15.86 | 14.71 | 58.59 | 62.93 | 64.03 | 95.58 | 96.85 | 96.73 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 100.0 | 100.0 | 99.97 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| LT size: | 4.88 | 4.51 | 4.68 | 4.95 | 5.09 | 5.23 | 5.00 | 5.40 | 5.09 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

Generating the series according to an LSTR(1) model, the power of the parameter constancy test increases for a higher coefficient of the cointegrating relationship, for a larger sample size, and when the speed of transition is high ($\gamma > 1$). The power is usually not affected by changes in the location parameter $c$. But as in Case 1, the power is slightly smaller when $c = 0.75$ and $\gamma$ is high (see, e.g., Table 3c). Moreover, an increase in the short-run dynamics from $\theta = 0.2$ to $\theta = 0.9$ really improves the power, which is quite puzzling. When the cointegrating relationship is known, however, and the short-run dynamics are strong, it might be easier for the test to separate the dynamics of the two components, and the time variation may be more easily found. This can be compared to the results of Case 1, in which the power was close to zero for strong short-run dynamics, pointing at the importance of a correct estimate of the cointegrating vector when trying to detect the time-varying coefficient.

When data are simulated according to an LSTR(2) model, the power properties are similar to those of the LSTR(1) model. A higher coefficient of the equilibrium correction term and a larger sample size have a positive effect on the power. The performance of the test when the location parameter ($c_1$) is changed is puzzling. Because it is assumed that $c_2 = 1 - c_1$, increasing $c_1$ implies a longer operating period for the equilibrium correction. But as is seen from Table 5, the power is not higher for higher values of $c_1$. This is due to the selection bias, since the test is carried out only if the null of no cointegration is rejected.

**3.2.5 The power of the Lin and Teräsvirta parameter constancy test for Case 3: Unrestricted cointegration** In Case 3 the cointegrating vector is not estimated, and the long-run variables enter unrestricted into the linear model. Hence, since the cointegrating vector will not be estimated, there are two coefficients for the long-run variables. The power is measured as the number of times parameter constancy can be rejected for either one of the two parameters.

**Table 4b**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.9$, $\gamma = 10$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 97.79 | 45.96 | 20.26 | 100.0 | 74.29 | 28.26 | 100.0 | 87.21 | 25.75 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 3.82 | 3.76 | 5.82 | 1.07 | 2.53 | 4.21 | 0.58 | 2.00 | 3.81 |
| $H_0$: $\delta = 0$ | 1.17 | 2.59 | 4.64 | 0.16 | 1.10 | 3.22 | 0.03 | 0.80 | 2.60 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 87.79 | 95.56 | 85.93 | 99.73 | 100.0 | 99.65 | 99.99 | 100.0 | 100.0 |
| $H_0$: $\delta = 0$ | 100.0 | 99.56 | 92.20 | 100.0 | 99.79 | 96.89 | 100.0 | 100.0 | 97.71 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 85.09 | 92.93 | 80.45 | 99.57 | 100.0 | 99.50 | 99.99 | 100.0 | 100.0 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 95.46 | 65.67 | 47.14 | 100.0 | 77.90 | 62.81 | 100.0 | 84.53 | 65.86 |
| LT size: | 5.11 | 4.61 | 4.70 | 5.07 | 4.72 | 4.61 | 4.88 | 4.93 | 4.76 |
| **T = 200** | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 99.96 | 53.91 | 18.90 | 100.0 | 92.46 | 30.29 | 100.0 | 99.23 | 31.25 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 1.41 | 2.15 | 2.96 | 0.35 | 1.46 | 2.64 | 0.11 | 1.16 | 3.55 |
| $H_0$: $\delta = 0$ | 0.49 | 1.67 | 3.49 | 0.02 | 0.61 | 1.85 | 0.00 | 0.35 | 2.24 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 97.91 | 99.91 | 94.23 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta = 0$ | 100.0 | 99.87 | 94.60 | 100.0 | 100.0 | 98.28 | 100.0 | 100.0 | 99.10 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 96.75 | 99.80 | 91.43 | 100.0 | 100.0 | 99.97 | 100.0 | 100.0 | 100.0 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 99.89 | 76.63 | 56.72 | 100.0 | 92.70 | 70.55 | 100.0 | 99.00 | 74.66 |
| LT size: | 4.86 | 4.93 | 4.68 | 5.04 | 4.81 | 44.69 | 4.64 | 4.97 | 4.93 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

Considering the LSTR(1) model, the power properties for Case 3 are very similar to those in Case 2. Hence, the power increases when the cointegrating coefficient is stronger and the speed of transition increases. The power is quite sensitive to large values of the location parameter, which, as discussed above, makes the observation period with an equilibrium correction short. An increase in the sample size improves the power, and very strong short-run dynamics also have a positive effect on the power. The LSTR(2) model has very similar features to those of Case 2, also in the sense that an increase in the location parameter, which makes the period including the equilibrium correction longer, has a negative effect on the test. The high power is surprising at first, given that the basic assumption of stationarity is violated: the power is also good when the basic assumption of stationarity is violated: Case 3, in which two of the variables are characterized by a unit root. Obviously, this is because when the two $I(1)$ variables are cointegrated, a combination of the two variables is still stationary. This may affect the outcome, even though the parameters of the cointegrating vector are not restricted. The basic models for the $LM_1$ and $LM_2$ tests are Equations (2.4) and (2.5), with $k = 1, 2$.

**3.2.6 How often will the conclusion be a time-varying equilibrium correction?** The overall power of the two tests, the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test, is rather low. This might be surprising at first, since both tests perform well separately. But since some parameter changes affect the power of the tests in opposite ways, the joint power will not be as high as for each test separately, and it will vary a lot depending on the parameter choices.

When the series are generated according to an LSTR(1) model, the highest joint power occurs when the transition speed is quite fast ($\gamma = 10$), the coefficient of $\Delta_1 \boldsymbol{y}_{t-1}$ is moderate ($\theta = 0.2$ or $0.4$), the magnitude of the cointegrating coefficient is strong ($\delta < -0.1$), the location parameter is small ($c = 0.25$), and the sample

**Table 4c**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(1), $\theta = 0.9$, $\gamma = 100$

| T = 100 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 93.58 | 69.58 | 54.53 | 94.59 | 59.80 | 49.33 | 84.90 | 43.15 | 30.98 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 1.70 | 1.16 | 2.59 | 0.34 | 0.72 | 1.48 | 0.31 | 1.02 | 2.84 |
| $H_0$: $\delta = 0$ | 0.56 | 0.82 | 1.82 | 0.25 | 0.48 | 0.75 | 0.26 | 0.76 | 1.71 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 95.91 | 99.17 | 97.21 | 99.95 | 100.0 | 99.74 | 100.0 | 100.0 | 99.71 |
| $H_0$: $\delta = 0$ | 100.0 | 99.57 | 97.41 | 99.99 | 99.38 | 94.65 | 100.0 | 98.22 | 86.67 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 95.03 | 98.58 | 95.47 | 99.94 | 99.98 | 99.70 | 100.0 | 100.0 | 99.55 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 91.12 | 79.72 | 72.82 | 88.56 | 73.51 | 70.18 | 77.13 | 65.17 | 54.84 |
| LT size: | 4.50 | 4.58 | 4.84 | 5.21 | 4.99 | 4.72 | 5.31 | 5.04 | 5.08 |

| T = 200 | $\delta = -0.1$ | | | $\delta = -0.4$ | | | $\delta = -0.8$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $c$: | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** | **0.25** | **0.50** | **0.75** |
| Johansen | 89.49 | 59.39 | 48.20 | 78.43 | 40.01 | 27.79 | 57.49 | 25.31 | 12.90 |
| Case 1: | | | | | | | | | |
| $LM_1$ | 0.77 | 0.96 | 1.45 | 0.32 | 1.32 | 2.37 | 0.37 | 2.92 | 4.96 |
| $H_0$: $\delta = 0$ | 0.27 | 0.61 | 1.22 | 0.26 | 0.67 | 1.84 | 0.57 | 2.02 | 3.33 |
| Case 2: | | | | | | | | | |
| $LM_1$ | 99.83 | 99.93 | 97.26 | 100.0 | 100.0 | 98.92 | 100.0 | 100.0 | 99.30 |
| $H_0$: $\delta = 0$ | 99.99 | 99.29 | 95.35 | 100.0 | 97.88 | 87.51 | 99.98 | 94.19 | 75.47 |
| Case 3: | | | | | | | | | |
| $LM_1$ | 99.68 | 99.76 | 95.75 | 100.0 | 99.93 | 98.38 | 100.0 | 99.92 | 97.21 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 88.97 | 77.18 | 73.42 | 79.05 | 66.73 | 55.31 | 77.42 | 61.36 | 40.85 |
| LT size: | 5.23 | 4.41 | 4.65 | 4.67 | 4.96 | 4.64 | 5.03 | 4.51 | 5.18 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

size is large. The joint power deteriorates, however, when any of the parameter values change. It is especially sensitive to changes in $\gamma$, $c$, and $\theta$. That is, when $\gamma = 1$, the power of the parameter constancy test is strongly negatively affected, and when $\gamma = 100$ the performance of the Johansen cointegration test deteriorates, and hence a change in either direction results in a decrease in the joint power. Moreover, a higher $c$ implies a shorter period, including the cointegrating relationship, which results in a sharp fall of the power of the Johansen test. Another variable that severely affects the joint power is a high $\theta$, when Case 1 is considered. It turns out, however, that by not estimating the cointegrating relationship, Case 3, and including only the cointegrated variables (in levels) in the equation, the joint power will almost be as high as for Case 2, where the cointegrating vector is assumed to be known.

When the series is generated according to an LSTR(2) model, the joint power is also generally low. In this case the power is most strongly affected by the value of the location parameter $c$. A high $c$ implies a longer period, including the cointegrating relationship, and the Johansen cointegration test is especially sensitive to this, which makes the joint power smaller.

## 4 Conclusions

In this article the probability of detecting a time-varying equilibrium correction by using the Johansen cointegration test and the Lin and Teräsvirta parameter constancy test is explored. It turns out that the joint power of the two tests is quite low and that the power varies a lot, depending on the choice of parameter values.

The most interesting result is perhaps the fact that the power of the parameter constancy tests strongly improves when the cointegrated variables enter without restrictions. This improves the power, which becomes

**Table 5**
Size and power of the Johansen cointegration test and the parameter constancy tests, Model LSTR(2), $\theta = 0.2$, $\gamma = 100$, $c_2 = 1 - c_1$

| T = 100 | $\delta = -0.1$ | | $\delta = -0.4$ | | $\delta = -0.8$ | |
|---|---|---|---|---|---|---|
| $c$: | **0.25** | **1/3** | **0.25** | **1/3** | **0.25** | **1/3** |
| Johansen | 19.93 | 43.27 | 22.73 | 74.19 | 18.40 | 78.10 |
| Case 1: | | | | | | |
| $LM_1$ | 18.41 | 14.93 | 60.36 | 60.97 | 54.57 | 48.35 |
| $H_0$: $\delta = 0$ | 80.63 | 82.20 | 66.74 | 67.52 | 54.57 | 47.63 |
| Case 2: | | | | | | |
| $LM_1$ | 48.86 | 28.15 | 99.16 | 94.33 | 100.0 | 99.92 |
| $H_0$: $\delta = 0$ | 96.09 | 99.19 | 99.43 | 100.0 | 99.84 | 100.0 |
| Case 3: | | | | | | |
| $LM_1$ | 37.43 | 24.55 | 97.22 | 90.25 | 100.0 | 99.77 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 51.73 | 74.79 | 89.75 | 99.77 | 97.34 | 100.0 |
| LT size: | 4.37 | 4.86 | 4.37 | 4.88 | 5.03 | 4.77 |
| T = 200 | $\delta = -0.1$ | | $\delta = -0.4$ | | $\delta = -0.8$ | |
| $c$: | **0.25** | **1/3** | **0.25** | **1/3** | **0.25** | **1/3** |
| Johansen | 34.91 | 76.79 | 31.66 | 96.11 | 24.79 | 98.41 |
| Case 1: | | | | | | |
| $LM_1$ | 45.92 | 40.19 | 72.33 | 71.11 | 59.06 | 50.56 |
| $H_0$: $\delta = 0$ | 82.15 | 83.28 | 73.12 | 69.59 | 57.68 | 49.47 |
| Case 2: | | | | | | |
| $LM_1$ | 81.18 | 65.24 | 100.0 | 99.94 | 100.0 | 100.0 |
| $H_0$: $\delta = 0$ | 98.85 | 99.95 | 99.84 | 100.0 | 100.0 | 100.0 |
| Case 3: | | | | | | |
| $LM_1$ | 71.50 | 56.75 | 100.0 | 99.77 | 100.0 | 100.0 |
| $H_0$: $\delta_1 = \delta_2 = 0$ | 74.25 | 92.84 | 94.32 | 100.0 | 98.95 | 100.0 |
| LT size: | 4.76 | 4.97 | 4.72 | 4.71 | 4.76 | 4.96 |

*Note:* Rejection frequencies of the null hypothesis are presented in the table. Numbers of repetitions: 10,000. Note that $\delta_1$ and $\delta_2$ are the coefficients for $\boldsymbol{y}_{t-1}$ and $\boldsymbol{z}_{t-1}$ when no cointegrating relationship is estimated.

almost as high as it is when the cointegrating vector is assumed to be known. This result may have important implications for empirical work.

Finally, based on the results above, it seems fair to conclude that the probability that a potential researcher will detect a time-varying equilibrium correction when using the general modeling strategy of testing for cointegration, estimating a cointegrating vector and detecting time variation of the equilibrium correction term, is low. This calls for a new test that simultaneously tests the joint hypothesis of no cointegration and parameter constancy, but formulating such a test is beyond the scope of this article.

## References


Balke, N. S., and T. B. Fomby (1997). "Threshold cointegration." *International Economic Review*, 38: 627–645.

Engle, R. F., and C. W. J. Granger (1987). "Cointegration and error-correction: Representation, estimation and testing." *Econometrica*, 55: 251–276.

Ericsson, N. R., D. F. Hendry, and K. M. Prestwich (1998). "The demand for broad money in the United Kingdom, 1878–1993." *Scandinavian Journal of Economics,* 100: 289–324.

Granger, C. W. J., and T. Teräsvirta (1993). *Modelling Nonlinear Economic Relationships*. Oxford: Oxford University Press.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.

Lin, C. J., and T. Teräsvirta (1994). "Testing the constancy of regression parameters against continuous structural change." *Journal of Econometrics*, 62: 211–228.

Michael, P., D. Nobay, and M. Taylor (1997). “Ajustement non linéaire vers le taux de change d’équilibre à long terme. Le modèle monétaire revisité” (Nonlinear adjustment towards the long-run equilibrium exchange rate: The monetary model revisited). *Revue Économique,* 48: 653–659.

Ripatti, A., and P. Saikkonen (forthcoming). “Vector autoregressive processes with nonlinear time trends in cointegrating relations.” *Macroeconomic Dynamics.*

Teräsvirta, T. (1998). “Modeling economic relationships with smooth transition regressions.” In A. Ullah and D. E. A. Giles (eds.), *Handbook of Applied Economic Statistics.* New York: Marcel Dekker, pp. 507–552.

Teräsvirta, T., and A. C. Eliasson (2001). “Nonlinear equilibrium correction and the UK demand for broad money, 1878–1993,” *Journal of Applied Econometrics,* 16: 277–288.

van Dijk, D., and P. H. Franses (2000). “Nonlinear equilibrium correction models for interest rates in the Netherlands.” In W. A. Barnett, D. F. Hendry, S. Hylleberg, T. Teräsvirta, D. Tjøstheim, and A. Würst (eds.), *Nonlinear Econometric Modelling.* Cambridge: Cambridge University Press, pp. 203–227.

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