Studies in Nonlinear Dynamics and Econometrics

Quarterly Journal
January 2002, Volume 5, Number 4
The MIT Press

Studies in Nonlinear Dynamics and Econometrics (E-ISSN 1081-1826) is a quarterly journal published electronically on the Internet by The MIT Press, Cambridge, Massachusetts, 02142-1407. Subscriptions and address changes should be addressed to MIT Press Journals, Five Cambridge Center, Cambridge, MA 02142-1407; (617)253-2889; e-mail: journals-orders@mit.edu. Subscription rates are: Individuals $40.00, Institutions $135.00. Canadians add additional 7% GST. Submit all claims to: journals-claims@mit.edu. Prices subject to change without notice.

Nonlinearities and Inactivity in Aggregate Investment: Some Theoretical Analysis and Time-Series Evidence

Luisa Corrado
Faculty of Economics
University of Rome II "Tor Vergata"
Luisa.Corrado@uniroma2.it

Sean Holly
Department of Applied Economics
Cambridge University
Sean.Holly@econ.cam.ac.uk

Paul Turner
Department of Economics
University of Sheffield
pturner@sheffield.ac.uk.

Abstract. *The theoretical analysis of investment under uncertainty has been revolutionized over the last decade by the importation of ideas from finance. If investment is irreversible, there is a return to waiting. So although circumstances may suggest that it is profitable to invest, there may also be an incentive to postpone the decision until better opportunities arise. Identifying and valuing the option to invest has become the standard way to solve the firm's irreversible-investment problem. Empirical studies of investment that incorporate the insights of the real-options approach are now beginning to appear. These show that investment can have a nonlinear relationship to q and may show insensitivity for some threshold level to the shadow value of investment (Barnett and Sakellaris 1998). Abel and Eberly (1997) and Böhm and Funke (1999) have also shown how the real-options approach to investment can be combined with the traditional q approach. In this case the relationship between q and the rate of investment is discontinuous. Over a range of inaction there will be no investment, although q is in excess of one.*

This paper builds a theoretical model that explains the determinants of this investment discontinuity. In contrast to much of the literature, we use a mean-reverting stochastic process, of which the geometric Brownian motion process is a special case. Under the assumption of a production function with constant returns to scale and a specific functional form for the investment adjustment function, it is possible to derive a tractable analytical form for the shadow value of the investment project. We then analyze the comparative properties of the value of q under different assumptions about the stochastic process governing output. The advantage of using a mean-reverting process is that it better captures the undoubted persistence in the shocks that face firms, especially at the macroeconomic level.

We then consider what the implications would be for the aggregate relationship between investment, q, and the business cycle. We first carry out Monte Carlo simulations of a discrete version of the theoretical model. We *find that for many parameter values, aggregating suppresses any nonlinearities in the micro adjustment processes. Moreover, where we do detect nonlinearity at the aggregate level, it varies with the type of stochastic process. It is greatest when this is a random walk—corresponding to the Brownian motion in continuous time—and least when the stochastic process follows an i.i.d. process. Mean reversion lies in between. We turn finally to an empirical examination using aggregate data and explore how sensitive investment is to q in different regimes. To do this, we apply a generalization of the Granger-Lee method (Arden et al. 2000) that uses a linear spline function to approximate different regions for investment.*

Keywords. investment nonlinearities, Tobin's q, business cycle

Acknowledgments. This research was carried out under ESRC grant no. 116251017. We thank Yu-Fu Chen and all the participants at the workshop "Uncertainty and Factor Demands," Hamburg, 26–27 August 1999, for their comments on the work. The usual disclaimer applies.

1 Introduction

The theoretical analysis of investment under uncertainty has been revolutionized over the last decade by the importation of ideas from finance. If investment is irreversible, there is a return on waiting. Although circumstances may suggest that it is profitable to invest, there may also be an incentive to delay until better opportunities arise. Identifying and valuing the option to invest has become the standard way to solve the firm's irreversible-investment problem.

Another recent body of literature (e.g., Abel and Eberly 1997) also focuses on irreversible investment in the presence of convex costs. Abel and Eberly (1997) show the presence of regions where investment in a homogeneous capital good is insensitive to Tobin's q as well as regions where it is responsive to q. Böhm and Funke (1999) also illustrate that investment decisions may not be linearly related to the fundamentals. Increased uncertainty when there are asymmetric adjustment costs leads to lower investment if firms operate in markets characterized by imperfect competition.

The importance of deliberate delays in decision making has also been highlighted by Gale (1996), who stresses that when the profitability of investment depends on the level of economic activity, agents have an incentive to delay investment in a recession, which makes the recovery longer and magnifies the amplitude and depth of the cycle. Other relevant literature emphasizes the option value of delay in the presence of exogenous revelation of information over time (Bernanke 1983; Cukierman 1980; Pindyck 1988).

Investment delays may also be seen as a strategic response by the firm. Caballero and Engel (1998) focus on adjustment hazards across firms to explain the presence of investment delays. Depending upon the size of the shock, the estimated hazards have the potential to magnify or dampen the response of investment to an aggregate shock, and "the passivity of normal times is, occasionally, more than offset by the brisk response to large—current or cumulated—shocks" (Caballero and Engel 1998, 27).

The model outlined in this paper differs from existing models of investment with asymmetric adjustment costs (Abel and Eberly 1997; Böhm and Funke 1999), since it also incorporates a mean-reverting stochastic process designed to capture aggregate and industry fluctuations in demand. In fact, if it is assumed that the underlying fundamental follows a geometric Brownian motion, then this implies the "fundamental fluctuating randomly up and down" (Dixit and Pindyck 1994, 74). In other words, it rules out business cycles. It is possible, however, that the investment decision may be contingent on whether the economy is in a recession or in a boom and on the duration of the cycle. In general, a deeper recession and a slow recovery (i.e., a low speed of adjustment of the fundamental to its long-run value) may cause a delay in implementing the investment decision. On the other hand, a quick recovery (i.e., a relatively fast reversion of the fundamental to its long-run value) may provide an incentive to invest sooner than in the previous scenario.

Therefore, it is possible to consider the decision to invest as the result of two main factors. First, there is the opportunity cost of postponing profits rather than having the profits today (the risk of entry by other firms or simply foregone cashflows). Second, once the investment has been made, there is a risk of failure, which undermines future profits, since by exercising the option to invest, the firm actually gives up the possibility that new information may arrive that might affect the timing of implementation of the project.

The decision to delay an investment project depends also on the cost of becoming active and on the degree of asymmetry in adjustment costs. For example, if the purchase cost of capital is higher than the resale price, the investment becomes partially irreversible, and firms may optimally decide to postpone the investment.

Thus the investment decision is the result of a general assessment of the incremental value of transforming an idle project into an active investment. In such a case the possibility of delaying an irreversible investment can strongly affect the decision to invest. The reason is that a firm with an opportunity to invest (Dixit and Pindyck 1994) is actually holding an option analogous to a financial call option, that is, the right, but not the obligation, to buy an asset at some future time, paying for this option an exercise price (a firm with an investment opportunity has the option to spend money—the exercise price—in return for a project—the asset—of some value).

The first section of this paper outlines the basic theoretical model, in which the discontinuity in the investment decision is analyzed in a real-option framework. In order to reproduce some stylized business cycle patterns, we assume that the underlying fundamental that affects profits follows a mean-reverting process. The presence of an AK production function with constant returns to scale and a specific functional form for the adjustment cost function allows the optimization problem to have a relatively simple analytical form, so we can derive a tractable solution for the shadow value of the investment project.

The second section analyzes the implications for the aggregate relationship between investment, q, and the business cycle. We first carry out Monte Carlo simulations of a discrete version of the theoretical model. We find that for many parameter values, aggregating suppresses any nonlinearities in the micro adjustment processes. We turn finally to an empirical examination using aggregate data and explore how sensitive investment is to q in different regimes. To do this, we apply a generalization of the Granger-Lee method (Arden et al. 2000) that uses a linear spline function to approximate different regions for investment.

2 The Model

For each firm, let K denote the stock of capital. Following Hayashi (1982) we assume that current profits $\Pi(K)$ are proportional to K:

$$\Pi(K) = A\theta K \tag{2.1}$$

where A is a parameter that represents technological progress and θ is an aggregate shock that follows a geometric mean-reverting process without drift:[1]

$$d\theta = \mu(\overline{\theta} - \theta)\theta\, dt + \sigma\theta\, dz \tag{2.2}$$

[1]For the mean-reverting process

$$d\theta = \mu(\overline{\theta} - \theta)\theta\, dt + \sigma\ \theta\, dz \tag{N.1}$$

where $\overline{\theta}$ is the equilibrium level of the fundamental, z is a Wiener process and σ is its variance, and the expected rate of change of θ is $\mu(\overline{\theta} - \theta)$. If the value of θ is currently θ_0 and θ follows (N.1), then its expected value at any future time t is

$$E[\theta_t] = \frac{\overline{\theta}}{1 + \left(\frac{\overline{\theta}-\theta_0}{\theta_0}\right) e^{-\overline{\theta}\mu t}} \tag{N.2}$$

In this model the return on investment is supposed to be dependent on a composite fundamental that embodies an indicator of economic activity θ that affects demand (which is mean-reverting with a speed of adjustment μ) and on a stochastic shock $\sigma\theta\, dz$, where σ is the standard deviation of the process uncorrelated across time and at any time satisfying[2]

$$E(dz) = 0, \qquad E(dz^2) = dt \tag{2.3}$$

Note that Equation (2.2) can be derived from the more general specification

$$d\theta = \left\{\alpha + \left[\mu(\overline{\theta} e^{\alpha t} - \theta)\theta\right]\right\}\, dt + \sigma\theta\, dz \tag{2.4}$$

by assuming that the drift term, α, is equal to zero (see also Metcalf and Hassett 1995).

The dynamics for the fundamental θ specified in (2.2) implies that when the level of economic activity is away from its long-run value $\overline{\theta}$, the fundamental value θ will adjust to its long-run value with speed of adjustment μ. If $\theta < \overline{\theta}$, for example, firms may optimally decide to postpone investment until the recession is over in order to take advantage of increases in demand in the future.

The firm undertakes gross investment I and incurs depreciation at a constant rate $\delta \geq 0$. Thus the change in capital stock follows the process

$$dK = (I - \delta K)\, dt \tag{2.5}$$

Let $C(I)$ be a convex adjustment cost function[3]

$$C(I) = \left\{\left[p^+ I + \frac{1}{2} a I^2\right] \Upsilon + \left[p^- I + \frac{1}{2} a I^2\right](1 - \Upsilon)\right\} \tag{2.6}$$

where the cost function has a quadratic component, aI^2, with $a > 0$, and a linear component that depends asymmetrically on investment I. This asymmetry is meant to capture the irreversibilities in investment that magnify the amplitude of delays in implementing any project. If installed capital may be purchased at a price p^+ and be sold at a price p^-, with $p^+ > p^-$, only part of the cost of purchase may be recovered upon reselling, which implies that the investment is partially irreversible; Υ is an indicator function that takes values one if the investment is positive and zero if the investment is negative.

2.1 The maximization problem

The assumption is that firms operate in complete markets and try to maximize the expected present value of the cash flow at a constant rate $r > 0$. The fundamental value at time t is

$$V(K, \theta) = \max_{\{I_t\}} E_t \int_0^{\infty} \left[\Pi(K_t, \theta_t) - C(I_t)\right] e^{-rt}\, dt \tag{2.7}$$

Equation (N.2) is also known as the stochastic Verlhurst equation (Kloeden and Platen 1992, 125). The value of θ at time t is therefore

$$\theta = \frac{\theta_0 \exp\left[\left(\mu\overline{\theta} - \sigma^2/2\right) t + \sigma z\right]}{1 + \mu\theta_0 \int_0^t \exp\left[\left(\mu\overline{\theta} - \sigma^2/2\right) s + \sigma z\right] ds} \tag{N.3}$$

When $\sigma \to 0$, the relationship (N.3) becomes

$$\theta = \frac{\overline{\theta}}{1 + \left[\left(\overline{\theta} - \theta_0\right)/\theta_0\right] e^{-\overline{\theta}\mu t}} \tag{N.4}$$

In (N.4) as $\mu \to \infty$, $\theta = \overline{\theta}$, which means that θ can never deviate from $\overline{\theta}$, even momentarily. Finally as $\mu \to 0$, θ becomes a simple Brownian motion.

[2]Note that the process for $E(d\theta)^2 = \frac{1}{2}\sigma^2\, dt$.

[3]For a detailed analysis of the role of convex adjustment costs, uncertainty and investment, see Abel and Eberly 1997.

From this point we can suppress the time subscript. The fundamental value of the firm also satisfies the following Bellman equation:

$$rV(K,\theta) = \max_{\{I\}} \left[\Pi(K,\theta) - C(I) + \frac{EdV}{dt} \right] \tag{2.8}$$

where the right-hand side contains the instantaneous cash flow $\Pi(K,\theta) - C(I)$ and the expected capital gain $\frac{EdV}{dt}$. The no-arbitrage condition requires the sum of these components to be equal to the return $rV(K,\theta)$. The relationship indicates that there is an opportunity cost to undertaking the investment ($r > 0$, where r is the return that the firm could get on its capital if it invested in the financial market).

The expected capital gain is calculated by applying Ito's lemma; using (2.2) and (2.5) describing the evolution of θ and K, it follows that

$$\frac{EdV}{dt} = (I - \delta K)V_k + \mu(\overline{\theta} - \theta)\theta V_\theta + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta} \tag{2.9}$$

In (2.9) the expected capital gain depends on the marginal valuation of a unit of installed capital V_k. We define $q \equiv V_k$, which is the shadow value of installed capital.[4] Substituting q for V_k in (2.9) and then substituting (2.9) in (2.8) gives

$$rV(K,\theta) = \max_{\{I\}} \left\{ AK\theta - C(I) + (I - \delta K)q + \mu(\overline{\theta} - \theta)\theta V_\theta + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta} \right\} \tag{2.10}$$

The revenue level, $\Pi(K,\theta)$, has been replaced by $AK\theta$, $V_\theta = \frac{\partial V(\theta)}{\partial \theta}$, and $V_{\theta\theta} = \frac{\partial^2 V(\theta)}{\partial^2 \theta}$.

To solve the problem we apply the solution method of Abel and Eberly (1997). Therefore, we can rewrite (2.10) by first "maximizing out" the level of investment to obtain

$$rV(K,\theta) = AK\theta + \phi - \delta Kq + \mu(\overline{\theta} - \theta)\theta V_\theta + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta} \tag{2.11}$$

where

$$\phi = \max_I \left[Iq - C(I) \right] \tag{2.12}$$

Note that ϕ is the excess of additional investment value over costs. In fact, when the firm invests at rate I over an interval dt of time, it acquires $I\,dt$ unit of capital. Because q is the shadow price of this capital, the firm acquires capital worth $qI\,dt$ but pays $C(I)$ to increase its capital stock by $I\,dt$.

Given the cost function specified in (2.6), the maximizing level of investment is

$$I^* = \left\{ \begin{matrix} I^* = \frac{q-p^+}{a} & \text{for} & I > 0 \\ I^* = \frac{q-p^-}{a} & \text{for} & I < 0 \end{matrix} \right\} \tag{2.13}$$

where the optimal level of investment I^* is determined by differentiating the term in brackets on the right-hand side of (2.12) with respect to I and setting the derivative equal to zero. The relationship (2.13) indicates that the optimal level of investment is a function of q.

To determine the value of ϕ, substitute equation (2.13) in (2.12) to obtain

$$\phi = \left\{ \begin{matrix} \frac{(q-p^+)^2}{2a} & \text{for} & I > 0 \\ \frac{(q-p^-)^2}{2a} & \text{for} & I < 0 \end{matrix} \right\} \tag{2.14}$$

Because the investment cost function is convex, the firm earns rents on an inframarginal unit of investment only when investment is nonzero.

[4]We are also assuming the replacement cost of new capital is normalized to one.

2.2 The solution for $q(\theta)$

The previous section derived the optimal rate of investment as a function of q, the marginal value of installed capital. The next step is to determine q as a function of the fundamental, θ, and then to determine the value of the project $V(K,\theta)$. We make the assumption that the solution is a linear function of the capital stock:

$$V(K,\theta) = q(\theta)K + G(\theta) \tag{2.15}$$

where $q(\theta)$ is the shadow value of the investment; $G(\theta)$ is the intercept term and, as will be shown later, it equals the present value of the expected rents, ϕ, accruing to the adjustment technology. The reason q depends only on θ is that $V(K,\theta)$ is linear in K, so $q = V_K(K,\theta)$ is a function of θ.

The relationship (2.15) is substituted in (2.14), and considering the expression for $C(I)$ in (2.6), it follows that

$$\begin{aligned} r(qK+G) \;=\; & AK\theta + \left(\Upsilon\frac{(q-p^{+})^2}{2a} + (1-\Upsilon)\frac{(q-p^{-})^2}{2a}\right) - \delta K q \\ & + \mu(\overline{\theta}-\theta)\theta(q_{\theta}K + G_{\theta}) + \frac{1}{2}\sigma^2\theta^2(q_{\theta\theta}K + G_{\theta\theta}) \end{aligned} \tag{2.16}$$

Since the differential equation (2.16) must hold for all values of K, the terms multiplying K on the left-hand side must equal the terms multiplying K on the right-hand side:

$$rq = A\theta - \delta q + \mu(\overline{\theta}-\theta)\theta q_{\theta} + \frac{1}{2}\sigma^2\theta^2 q_{\theta\theta} \tag{2.17}$$

In addition the terms *not* involving K on the left-hand side must equal the sum of terms not involving K on the right-hand side:

$$rG = \left(\Upsilon\frac{(q-p^{+})^2}{2a} + (1-\Upsilon)\frac{(q-p^{-})^2}{2a}\right) + \mu(\overline{\theta}-\theta)\theta G_{\theta} + \frac{1}{2}\sigma^2\theta^2 G_{\theta\theta} \tag{2.18}$$

These equations have a recursive structure. The differential equation for $q(\theta)$ in (2.17) does not depend on $G(\theta)$, but the differential equation for $G(\theta)$ in (2.18) depends on $q(\theta)$. Therefore we will solve (2.17) for $q(\theta)$ and then proceed to solve (2.18) for $G(\theta)$.

2.3 Investment nonlinearities with a mean-reverting fundamental

Inaction in investment decisions can play a crucial role in explaining asymmetries and investment nonlinearities. With uncertainty about the level of real and financial activity, firm inaction is endogenous, since it increases the marginal value of the idle investment.

The assumption of a mean-reverting fundamental implies that the inactivity area may widen. Firms may wish to postpone their investment, for example, until a recession has ended, to take advantage of the future increase in economic activity; they will try not to be the first to implement the project in order to avoid unwanted risks that undermine the profit that they might earn in the future.

Depending on the value of the stochastic shock that affects the cash flow of the investment project, $\Pi(K,\theta)$, the investment decision may be either to cancel, to wait, or to activate the investment. If the level of θ that affects profitability (e.g., positive demand shocks) is greater than an upper threshold θ_I, then investment is implemented ($\Upsilon = 1$), whereas for a level of θ that is less than a lower threshold θ_E, disinvestment occurs. For intermediate levels of $\theta \in [\theta_E, \theta_I]$, firms may decide to delay the investment decision.

It follows that it is possible to identify the following three regimes: one in which the size of the level of the fundamental is such that $\theta \in [0, \theta_I]$ and $I < 0$, a second in which $\theta \in (\theta_E, \theta_I)$ for the inactivity area and $I = 0$, and a third in which $\theta \in [\theta_E, \infty)$ and $I > 0$. The investment value can be expressed as

$$\begin{Bmatrix} rV_I(\theta,K) = \Pi(K,\theta) + \phi + \frac{E[dV_I(\theta,K)]}{dt} & \text{if } \theta \in [\theta_E, \infty) \\ rV_E(\theta,K) = \phi + \frac{E[dV_E(\theta,K)]}{dt} & \text{if } \theta \in [0, \theta_I] \end{Bmatrix} \tag{2.19}$$

The subscript indicates whether the firm decides to invest (I) or to abandon the investment (E). Note that for low fundamental levels, we assume that revenue is zero, whereas for quantity ranges above the critical upper bound, θ_I, the profit flow equals revenue $\Pi(K,\theta)$ net of the investment cost $C(I)$—as embedded in $\phi-$.

By linking the fundamental solution (2.17) and (2.18) to (2.15), it follows that

$$\left\{\begin{array}{ll} rq^I = A\theta - \delta q^I + \mu(\overline{\theta}-\theta)\theta q^I_\theta + \frac{1}{2}\sigma^2\theta^2 q^I_{\theta\theta} & \text{if } \theta \in [\theta_E, \infty) \\ rq^E = -\delta q^E + \mu(\overline{\theta}-\theta)\theta q^E_\theta + \frac{1}{2}\sigma^2\theta^2 q^E_{\theta\theta} & \text{if } \theta \in [0, \theta_I] \end{array}\right\} \tag{2.20}$$

where $q^i_\theta = \frac{\partial q^i(\theta)}{\partial\theta}$, $q^i_{\theta\theta} = \frac{\partial^2 q^i(\theta)}{\partial^2\theta}$ for $i = I, E$, and

$$\left\{\begin{array}{ll} rG^I = \frac{(q^I-p^+)^2}{2a} + \mu(\overline{\theta}-\theta)\theta G^I_\theta + \frac{1}{2}\sigma^2\theta^2 G^I_{\theta\theta} & \text{if } \theta \in [\theta_E, \infty) \\ rG^E = \frac{(q^E-p^-)^2}{2a} + \mu(\overline{\theta}-\theta)\theta G^E_\theta + \frac{1}{2}\sigma^2\theta^2 G^E_{\theta\theta} & \text{if } \theta \in [0, \theta_I] \end{array}\right\} \tag{2.21}$$

where $G^i_\theta = \frac{\partial G^i(\theta)}{\partial\theta}$ and $G^i_{\theta\theta} = \frac{\partial^2 G^i(\theta)}{\partial^2\theta}$ for $i = I, E$. Note that the intercept term $G(\theta)$ in the fundamental value of the firm (2.15) equals the present value of the expected rents, ϕ, accruing to the adjustment technology represented by the convex cost function.

2.4 The solution for $q(\theta)$ and $G(\theta)$

The solution for $q^i(\theta)$ is obtained in Appendix A and is merely stated here:

$$q^I(\theta) = \frac{1}{r+\delta-\mu\overline{\theta}}A\theta + B^I_-\theta^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b\right) \qquad \text{if } \theta \in [\theta_E, \infty) \tag{2.22}$$

$$q^E(\theta) = B^E_+\theta^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b\right) \qquad \text{if } \theta \in [0, \theta_I] \tag{2.23}$$

where to rule out explosive bubbles we impose the condition $B^I_+ = 0$, and since zero is an absorbing barrier, we have $B^E_- = 0$.

The values for λ_+ and λ_- are the solutions of the quadratic equation

$$\frac{1}{2}\sigma^2\lambda(\lambda-1) + \mu\overline{\theta}\lambda - (r+\delta) = 0 \tag{2.24}$$

and the value for the parameter b is given by

$$b = 2\lambda + \frac{2\mu}{\sigma^2}\overline{\theta} \tag{2.25}$$

For $G(\theta)$ we have:

$$G^I(\theta) = \frac{1}{r}\frac{(q^I-p^+)^2}{2a} + C^I_+\theta^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b\right) + C^I_-\theta^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b\right) \tag{2.26}$$

$$G^E(\theta) = \frac{1}{r}\frac{(q^E-p^-)^2}{2a} + C^E_+\theta^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b\right) + C^E_-\theta^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b\right) \tag{2.27}$$

Note that the expected growth rates of $C^i_-\theta^{\lambda_-}H(\frac{2\mu}{\sigma^2}\theta; \lambda_-; b)$ and $C^i_+\theta^{\lambda_+}H(\frac{2\mu}{\sigma^2}\theta; \lambda_+; b)$ for $i = I, E$ are both equal to r. As in Abel and Eberly 1997, we refer to these terms as bubbles, since, unlike the shadow value q, they are unrelated to the underlying fundamental (cash flow) and are only a function of the particular adjustment technology assumed in (2.6). Restricting attention to the fundamental value of G (i.e., ruling out bubbles in the adjustment technology) implies that $C^i_- = C^i_+ = 0$ (for $i = I, E$). Therefore

$$G^I(\theta) = \frac{1}{r}\frac{(q^I-p^+)^2}{2a} \tag{2.28}$$

$$G^E(\theta) = \frac{1}{r}\frac{(q^E-p^-)^2}{2a} \tag{2.29}$$

which implies that the intercept of the value function (2.15) equals the present value of expected rents, ϕ.

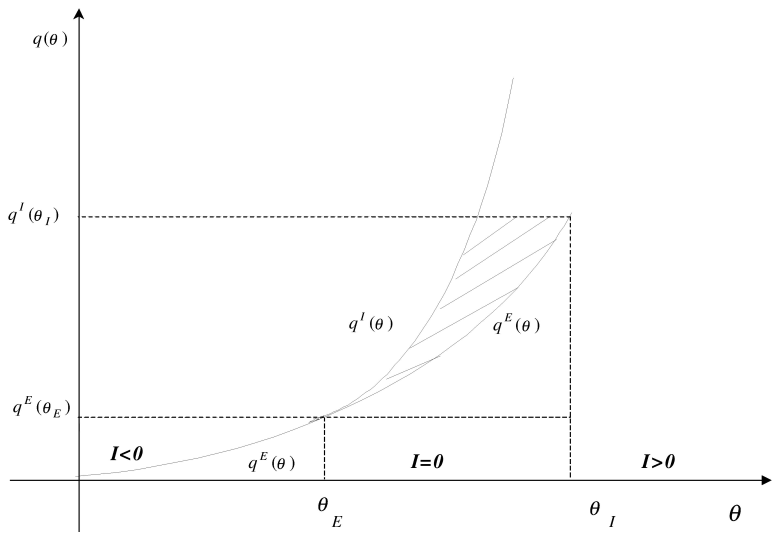

Figure 1
The q value in the three regimes.

Given the recursive relation between q and G, the resulting value for the investment project is

$$V^I(\theta) = G^I(\theta) + q^I(\theta) \qquad \text{if } \theta \in [\theta_E, \infty) \tag{2.30}$$

$$V^E(\theta) = G^E(\theta) + q^E(\theta) \qquad \text{if } \theta \in [0, \theta_I] \tag{2.31}$$

In fact, because we have obtained the value of q in Equations (2.22) and (2.23), the value of the adjustment technology, $G^i(\theta)$ (for $i = I, E$), which depends on q, is uniquely determined as well.

2.5 The shadow value of the investment

Given the previous results we can now focus on a qualitative analysis of the shadow value of investment $q^i(\theta)$ (for $i = I, E$) when the firm either is active or has decided to abandon the investment.

Inactivity in implementing an investment project occurs for intermediate levels of the fundamental $\theta \in [\theta_E, \theta_I]$, where delay is a response to uncertainty about the level of the fundamental (Gale 1996). Since investment depends upon the level of economic activity, firms may be willing to "wait and see" until the recession has ended and, in order to take advantage of the future increase in economic activity, they may switch to the investment decision close to the end of the recession.

Figure 1 shows the dynamics of project value as a function of the level of economic activity θ. The path labeled $q^E(\theta)$ defines the shadow value of the investment project when $\theta \in [0, \theta_I]$, and $q^I(\theta)$ is the project value when $\theta \in [\theta_E, \infty)$.

Note that it is possible to impose a barrier control on the shadow value of investment defining an upper and a lower ceiling denoted, respectively, $q^I(\theta_I)$ and $q^E(\theta_E)$. The critical value of q is calculated approximately by equating the marginal shadow value of the investment to the cost of acquiring one additional unit of capital.

The marginal cost of capital can be derived by considering that, because of partial irreversibility,

$$\lim_{I \uparrow 0} C_I(I) = \lim_{I \uparrow 0} \left(p^+ + aI\right) = p^+ \tag{2.32}$$

$$\lim_{I \downarrow 0} C_I(I) = \lim_{I \downarrow 0} \left(p^- + aI\right) = p^-$$

The implied boundary conditions are

$$q^I(\theta_I) = p^+ \qquad \text{if } I > 0 \tag{2.33}$$

$$q^E(\theta_E) = p^- \qquad \text{if } I < 0 \tag{2.34}$$

Conditions (2.33) and (2.34) imply that at θ_I the marginal benefit of becoming active must be equal to the purchase cost of capital p^+ and that at θ_E the marginal benefit of becoming inactive must be equal to p^-, where p^- is the resale price of capital.

The two branches of the solution must smooth-paste and value-match at θ_E:

$$q^E(\theta_E) = q^I(\theta_E) \tag{2.35}$$

$$q^E_\theta(\theta_E) = q^I_\theta(\theta_E) \tag{2.36}$$

Conditions (2.33), (2.34), (2.35), and (2.36) define the values of the coefficients B_+ and B_- and of the thresholds θ_I, θ_E for the idle investment area $[\theta_E, \theta_I]$ (see section A.1 of Appendix A for details).

Recent research in this area (Barnett and Sakellaris 1998) stresses how investment may be insensitive to movements in q for long periods of time followed by discrete adjustments to firms' desired level of capital. Faced with an irreversible decision to make under uncertainty, firms value positively the option of waiting for more information. This option value explains why, over a certain range, firms decide not to react in response to variations in the costs of investment unless such costs reach certain upper thresholds.

2.6 Comparative properties of the solution

Consider the mean-reverting process without drift, already described in (2.2):

$$d\theta = \mu(\overline{\theta} - \theta)\theta\, dt + \sigma\theta\, dz \tag{2.37}$$

As $\mu \to 0$, θ becomes a simple Brownian motion without drift. Hence, when $\mu \to 0$, this gives a benchmark solution for the shadow value of investment:

$$d\theta = \sigma\theta\, dz \tag{2.38}$$

The solutions to (2.39) and (2.41), derived in Appendix B, represent the benchmark for $q(\theta)$ and $G(\theta)$ with asymmetric adjustment costs:

$$q^I(\theta) = \frac{1}{r+\delta}A\theta + B_-\theta^{\lambda_-} \tag{2.39}$$

$$G^I(\theta) = \frac{1}{r}\frac{(q^I - p^+)^2}{2a} \tag{2.40}$$

$$q^E(\theta) = B_+\theta^{\lambda_+} \tag{2.41}$$

$$G^E(\theta) = \frac{1}{r}\frac{(q^E - p^-)^2}{2a} \tag{2.42}$$

where λ_- and λ_+ are the roots of the quadratic equation $\frac{1}{2}\sigma^2\lambda(\lambda - 1) - (r + \delta) = 0$.

The values of the triggers θ_I and θ_E and of the two coefficients B_+ and B_- are derived by imposing again the conditions from (2.33) to (2.36). Once $q^i(\theta)$ is determined (for $i = I, E$), the value of the adjustment technology, $G^i(\theta)$, is uniquely determined as well, as illustrated above.

2.7 Investment nonlinearities and Tobin's q

This section focuses on a qualitative analysis of the properties of the shadow value of investment q, which, together with the rents accruing to the investment technology, G, determine the investment value function V.

Figure 2 plots the value of q under different assumptions on the degree of asymmetry in adjustment costs and on the speed of mean reversion of the fundamental. The path labeled AA' defines the benchmark case, in which there are no asymmetries in adjustment costs ($p^+ = p^-$) and there is no region of inactivity. In this benchmark case a rise in the shadow value of investment leads to an increase in investment for all values of q.

If there is mean reversion, but there is a wedge between the purchase cost of capital p^+ and the resale price p^-, then this generates two paths, one labeled BB and associated with the solution (2.22) for $q^I(\theta)$, and

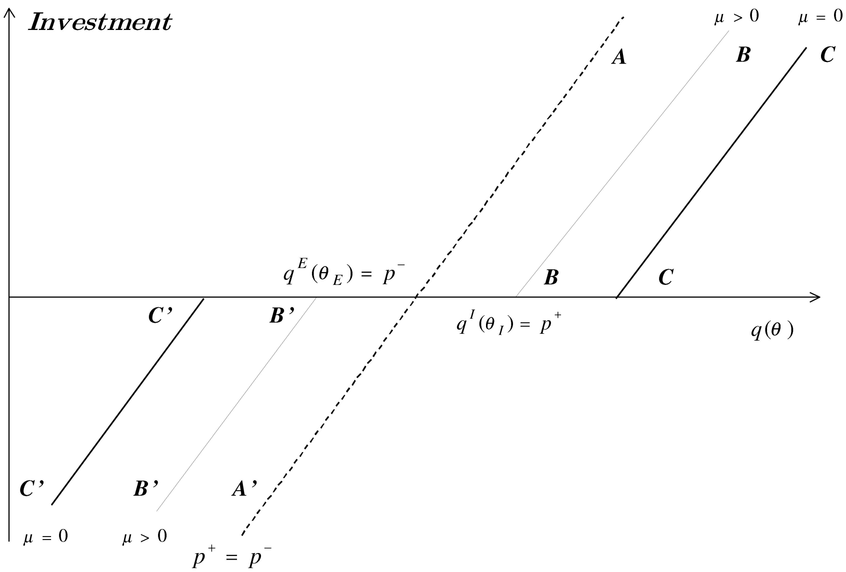

Figure 2
Relationship between investment and q with asymmetric adjustment costs and mean reversion.

a second labeled $B'B'$, associated with the solution (2.23) for $q^E(\theta)$. Because of partial irreversibility ($p^+ > p^-$), there is a region where investment may be insensitive to q, since we consider a nonlinearity in the relationship between investment and q coming from kinked adjustment costs.

If $\mu = 0$, then the fundamental θ becomes a simple Brownian motion. A simple geometric Brownian motion (see Dixit and Pindyck 1994, 75) tends to fluctuate randomly around its initial value, whereas in a mean-reverting process, the larger μ is, the less θ deviates from $\overline{\theta}$. Given this property we expect that the inactivity area increases as $\mu \to 0$, since this scenario is associated with the maximum level of uncertainty for a firm, with the "fundamental fluctuating randomly up and down" (Dixit and Pindyck 1994, 74). Combining a pure Brownian motion process with the presence of asymmetric adjustment costs, the area of inactivity widens as the paths CC, associated with the solution (2.39) for $q^I(\theta)$, and $C'C'$, associated with the solution (2.41) for $q^E(\theta)$, show.

2.8 Numerical results

This section analyzes the effect of changes in the long-run value, $\overline{\theta}$, and in the speed of mean reversion, μ, on the value of the idle investment, $V^I(\theta) - V^E(\theta)$. This value can also be interpreted as the firm's incremental value of becoming active in the range $\theta \in [\theta_E, \theta_I]$, that is, how much the investment decision, V^i (with $i = I, E$), is worth in the active rather than in the inactive state. Using numerical simulations,[5] we can show that if the speed of mean reversion of the fundamental to its long-run value, μ, increases and/or the long-run value, $\overline{\theta}$, shifts upward, the value of the idle investment, $V^I(\theta) - V^E(\theta)$, increases. In fact, if the economy is in a recession, an increase in the degree of mean reversion implies that the fundamental will revert more quickly to the long-run trend, and the investment value increases. Also a shift upward of the long-run trend implies higher returns in the future, such that the value of the investment (and of implementing the project later) is worth more.

We have simulated the case where $\overline{\theta} > 0$ and the adjustment costs are asymmetric, which implies ($p^+ - p^-) > 0$. Note that, under these assumptions, the values of the Tobin's q, embedded in V, follow the paths BB and $B'B'$ in Figure 2.

[5]The simulations in this section have been performed using Mathematica version 3.0.

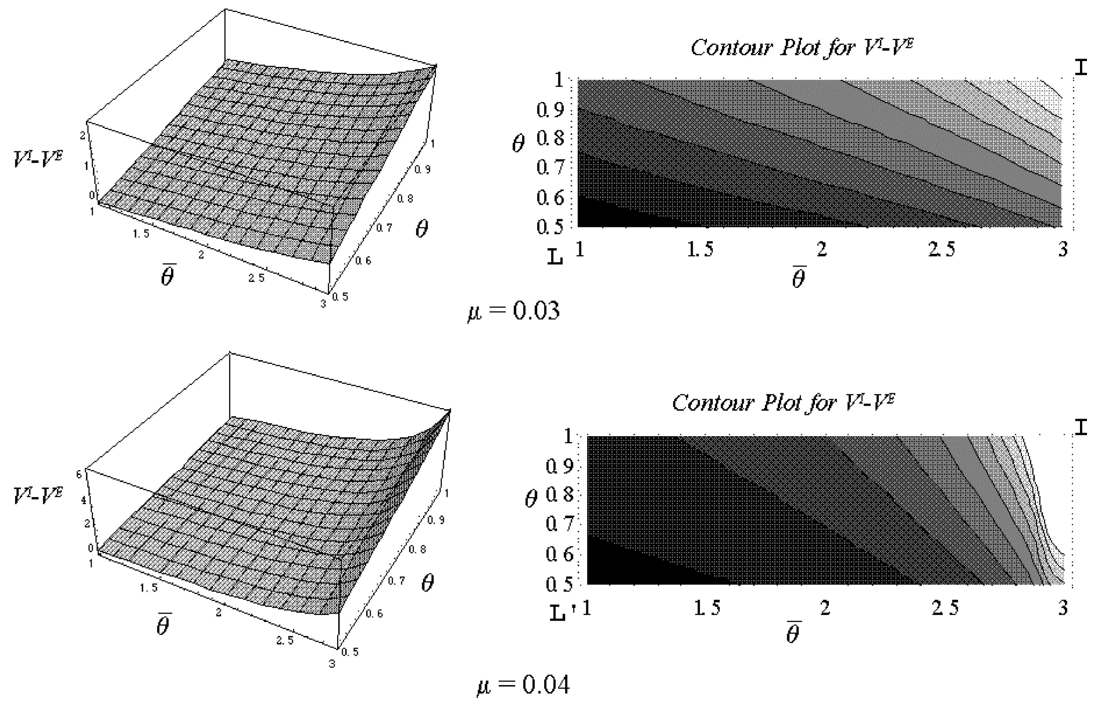

Figure 3
Idle investment value $V^I(\theta) - V^E(\theta)$ as a function of the long-run equilibrium, $\overline{\theta}$, and of the fundamental, θ. *Note*: The lighter regions of the contour plot indicate higher values of the idle investment. Parameter values: $A = 0.1$, $\delta = 0.04$, $p^+ = 1$, $p^- = 0.1$, $\sigma = 0.2$; $r = 0.09$.

Let us consider the solutions reported in (2.30) and (2.31). We assume two scenarios in which the fundamental θ has a drift rate, μ, of 3% and 4%. The volatility parameter is 20% ($\sigma = 0.2$), the standard value used by Dixit and Pindyck (1994), and the firm discounts the future profit stream at a constant risk-free interest rate of 9% ($r = 0.09$). The depreciation rate, δ, is 4%. In the simulation the net value of the idle investment, $V^I(\theta) - V^E(\theta)$, depends on the long-run value, $\overline{\theta}$, and on the fundamental level, θ.

We have considered the case in which the composite fundamental θ is below its long-run value $\overline{\theta}$. As Figure 3 shows, given μ, a higher level of $\overline{\theta}$ implies a higher expected rate of growth of θ (i.e., higher future level of the fundamental) such that an option to invest is worth more.[6]

The darkest regions, such as point L on the contour plot, where $\overline{\theta}$ is at its lowest level, are points where the value of the investment is at its minimum. The lightest regions, around point I, correspond to higher values for the long-run value, $\overline{\theta}$, and are associated with the highest value of the investment project.

Increasing the speed of reversion, μ, from 3% to 4% implies that the fundamental will revert much more quickly to its long-run value. In business cycle terms, the phase of the cycle will be shorter, which in turn implies that for a given value of the fundamental, θ, an increase in the long-run value of the economic activity, $\overline{\theta}$, will result in an increase of the value of the investment project.

3 Aggregation Issues

The model of investment described in previous sections generates two main insights. The range of inaction in the response of investment to changes in the environment facing the firm depends on both the amount of

[6] Metcalf and Hassett (1995) compare investment returns under alternative price dynamics and find that when prices follow a geometric mean reversion process without drift, this has the effect of increasing cumulative investment over time, compared to the simple geometric Brownian motion process.

uncertainty and asymmetries in the adjustment cost function. It also depends on the degree of mean reversion in the stochastic process. We have interpreted this to represent the delay that Gale (1996) identifies in his model. In order to explore the consequences of this approach for aggregate investment, however, we need to consider aggregation. To do this we employ a discrete approximation to the highly nonlinear form of the microeconomic model. Assume that the relationship between investment and q for the ith firm can be approximated by the function

$$i_{it} = \omega(D_1 q_{it} + D_2 q_{it} + D_3 q_{it}) + \varepsilon_{it} \tag{3.1}$$

where

$$D_1 = \{1 \text{ for } q_{it} < \phi_{i1}, 0 \text{ otherwise}\} \tag{3.2}$$

$$D_2 = \{1 \text{ for } \phi_{i1} < q_{it} < \phi_{i2}, 0 \text{ otherwise}\} \tag{3.3}$$

$$D_3 = \{1 \text{ for } q_{it} > \phi_{i2}, 0 \text{ otherwise}\} \tag{3.4}$$

and where ε_{it} is an idiosyncratic shock to the investment plans of the ith firm; the ϕ_{ij}, for $j = 1, 2$, are the thresholds specific to each firm; and the D_j are indicator functions. Assume that the mean-reverting process (deviations about $\bar{\theta}$) is of the first-order form

$$(1 - \mu L) y_t = \epsilon_t \tag{3.5}$$

where now μ is a metric for the degree of business cycle persistence, and L is the lag operator. For $\mu = 0$, y follows an i.i.d. process, where ϵ_t and $\varepsilon_{it} \sim N(0, 1)$. Now let the range of inaction for the ith firm, $\phi_{i1} - \phi_{i2}$, depend on y_{t-1}, so firms observe the state of the business cycle only after one period, in the form

$$\phi_{i1} - \phi_{i2} = -\lambda_i y_{t-1} \tag{3.6}$$

Since aggregate investment is a component of aggregate income, we can calibrate a recursive partial equilibrium (consumption is not modeled here). We simulate an economy with 1,000 firms for 150 periods. We assume that ϕ_{i1} and ϕ_{i2} are $\sim N(\phi_j, 0.01)$, for $j = 1, 2$, λ_i is $\sim N(0.5, 0.01)$, and $\mu = 0.6$. We then measure the degree of nonlinearity of the aggregate relationship between investment and q using a portmanteau test for the independence of the residuals from a regression of aggregate investment on q employing the aggregates constructed from the microeconomic variables. The BDSL test (Brock, Dechert, Sheinkman, and LeBaron 1996) is used to test for whether the residuals are i.i.d. In Table 1 we report the p-values for the bootstrapped probability with the dimension of the test set to three. We report different values for the width of the band of inaction, $\phi_1 - \phi_2$, and the degree of asymmetry, ϕ_1/ϕ_2. We did a grid search over mean values of ϕ_1 and ϕ_2 in the range $(-1.7, 2.2)$ and for three values of μ. For particular values of ϕ_1 and ϕ_2, we found only two regions where the bootstrapped p-values indicated rejection of the null of independence at less than a 10% critical value. In Table 1 we report the results for only one region, since the second region gave values of ϕ_1 and ϕ_2 greater than $(-1.6, 1.3)$. This implies, for a standardized normal, that the range of inaction would span some 85% of the area under the curve. Since this appears unreasonable, we concentrate on the other region we detected. It is clear that aggregation swamps nonlinearities at the micro level except for values of ϕ_1 and ϕ_2 around $(-0.1, 0.6)$ without mean reversion and $(-0.1, 0.8)$ when we allow for mean reversion. This implies that the range of inaction for a standardized normal is around 26–33%, which is more reasonable. The results also suggest that as with the simulations of the previous section, the inclusion of a mean-reverting process makes the results for nonlinearity much more significant.

4 Some Empirical Results

In this section we turn to time-series evidence on the types of nonlinearity suggested by the model of the previous section. We use aggregate quarterly measures of investment in the United Kingdom from 1967 to

Table 1
Bootstrapped p-values from Monte Carlo simulations

i.i.d. stochastic process, $\mu = 0.0$							
$\phi_1 \backslash \phi_2$	1	0.9	0.8	0.7	0.6	0.5	0.4
0	0.334	0.218	0.106	0.078	0.072	0.118	0.104
−0.1	0.350	0.224	0.104	0.070	**0.064**	0.100	0.086
−0.2	0.384	0.230	0.126	0.114	0.106	0.146	0.138
−0.3	0.372	0.220	0.128	0.112	0.124	0.260	0.228
−0.4	0.420	0.244	0.174	0.180	0.146	0.282	0.284
−0.5	0.438	0.266	0.172	0.186	0.191	0.332	0.370
Geometric mean reversion, $\mu = 0.6$							
$\phi_1 \backslash \phi_2$	1	0.9	0.8	0.7	0.6	0.5	0.4
0.2	0.326	0.252	0.044	0.056	0.110	0.114	0.152
0.1	0.322	0.232	0.040	0.072	0.110	0.142	0.162
0	0.374	0.226	0.038	0.064	0.110	0.128	0.136
−0.1	0.368	0.202	**0.036**	0.064	0.096	0.104	0.124
−0.2	0.410	0.212	0.050	0.076	0.124	0.128	0.168
−0.3	0.456	0.262	0.064	0.126	0.174	0.194	0.208
−0.4	0.468	0.278	0.102	0.180	0.234	0.246	0.254
Geometric mean reversion, $\mu = 0.8$							
$\phi_1 \backslash \phi_2$	1	0.9	0.8	0.7	0.6	0.5	0.4
0.2	0.324	0.224	0.06	0.074	0.102	0.12	0.146
0.1	0.318	0.222	0.052	0.068	0.112	0.12	0.126
0	0.296	0.260	0.048	0.074	0.116	0.118	0.148
−0.1	0.332	0.232	**0.038**	0.066	0.106	0.128	0.126
−0.2	0.454	0.228	0.062	0.098	0.132	0.166	0.200
−0.3	0.474	0.240	0.062	0.142	0.182	0.228	0.244
−0.4	0.406	0.258	0.078	0.152	0.27	0.214	0.284

Note: Boldface numbers represent the bootstrapped p-values with a rejection of the null of independence at less than a 10% critical value.

1994. Carrying over the insights of the theoretical model to an empirical application presents the immediate difficulty that the nonlinearities at the level of the firm or establishment may, as we indicated in the previous section, wash out through aggregation (Barnett and Sakellaris 1998). There is the added difficulty that the theoretical analysis is in terms of marginal q, when empirically we only observe average q. The conditions that Hayashi (1982) has identified under which marginal and average q will be the same are unlikely to hold in our framework. Moreover, we need a framework within which we can test for the types of nonlinearity predicted by the model but using aggregate time-series data, which may be nonstationary. So to test for the possibly nonlinear relationship between investment and q, we adopt the approach used in the modeling of nonstationary time series, involving cointegrating processes and error-correcting dynamics.

In general, let us write a nonlinear loss minimization problem defined over some variable x as

$$\min_{x_t} E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\frac{1}{2} (f(x_{t+i} - x^*_{t+1}) + \gamma \Delta x^2_{t+i}) \right) \right] \tag{4.1}$$

where $f(.)$ can capture a variety of different forms of adjustment around the attractor (x^*_t). Under various assumptions (i.e., quadratic adjustment), when the first derivative of $f(.)$ is linear, the intertemporal adjustment function implies the symmetric error correction formulation

$$\Delta x_t = \phi_1(x_{t-1} - x^*_{t-1}) + \phi_2 \Delta x_{t-1} + \cdots + \phi_p \Delta x_{t-p} \tag{4.2}$$

One special case of (4.1) due to Granger and Lee (1989) would imply the error correction model

$$\Delta x_t = \phi_{11}(x_{t-1} - x^*_{t-1})^- + \phi_{12}(x_{t-1} - x^*_{t-1})^+ + \phi_2 \Delta x_{t-1} + \cdots + \phi_p \Delta x_{t-p} \tag{4.3}$$

where the terms $(x_{t-1} - x^*_{t-1})^-$ and $(x_{t-1} - x^*_{t-1})^+$ represent the negative and positive deviations, respectively, from the attractor. Granger and Lee make the implicit assumption that the error correction function is piecewise linear, with the break in slope occurring at the mean of the attractor. There is no particular reason, however, to believe that this will always be the case. It is entirely plausible to imagine cases in which the change in adjustment costs occurs only when the deviation from equilibrium has reached some unknown critical level, as the theoretical model of this paper shows; indeed, it is also possible that there may be more than one break in adjustment costs. For example, we may wish to consider cases in which adjustment costs change for sufficiently large deviations from equilibrium in both a positive and a negative direction. The fact that the break points are likely to be unknown in most circumstances complicates matters considerably. Therefore we use the following generalization of the Granger-Lee empirical framework to deal with these issues.

First, we assume that the decision variable, x, has a long-run relationship with some other variable (or vector of variables), z, defined by the vector of cointegrating parameters, γ; γz then corresponds to x^* in Equation (3.2). At any rate we assume that the value(s) of γ are known prior to the estimation of the error correction model. Second, we model the behavior of x using an error correction framework, allowing for a maximum of two breaks in the error correction parameter. The values of the error correction term at which these breaks occur are estimated using a grid search procedure to minimize the residual sum of squares. A similar approach has recently been applied by Escribano and Pfann (1998).

The intuitive idea behind this approach is that there are some critical deviations of the control variable from its equilibrium value at which behavior changes and error correction shifts into a "higher gear." The likeliest scenario is one in which adjustment is relatively slow close to equilibrium but speeds up when deviations get sufficiently large. Note that there is no reason to expect behavior to be symmetric in terms of the slope of the error correction function for extreme positive and negative deviations or in terms of the critical values at which error correction shifts into a higher gear.

To allow for this type of behavior we adopt a general specification in which the error correction term is a linear spline function with three sections. Although it is possible to consider more general functional forms, it is not easy to provide them with a convincing economic rationale. In our empirical work we therefore estimate the three-section function as our most general case and test to see if the alternatives of a two-section function, or a simple linear function, are acceptable alternatives. The general form of the model can be written

$$\begin{aligned}\Delta x_t \; = \; & \gamma \Delta z_t + (\alpha_1 + \beta_1 (x - \gamma z)_{t-1})\, D_1 + (\alpha_2 + \beta_2 (x - \gamma z)_{t-1})\, D_2 \\ & + (\alpha_3 + \beta_3 (x - \gamma z)_{t-1})\, D_3 + u_t\end{aligned} \tag{4.4}$$

where

$$D_1 \; = \; \{1 \text{ for } (x - \gamma z)_{t-1} < \phi_1, 0 \text{ otherwise}\} \tag{4.5}$$

$$D_2 \; = \; \{1 \text{ for } \phi_1 < (x - \gamma z)_{t-1} < \phi_2, 0 \text{ otherwise}\} \tag{4.6}$$

$$D_3 \; = \; \{1 \text{ for } (x - \gamma z)_{t-1} > \phi_2, 0 \text{ otherwise}\} \tag{4.7}$$

and where ϕ_1 and ϕ_2 are the critical values of the error correction term at which changes in behavior occur. Continuity of the error correction function requires us to impose the following constraints:

$$\begin{aligned}\alpha_1 + \beta_1 \phi_1 \; &= \; \alpha_2 + \beta_2 \phi_1 \\ \alpha_2 + \beta_2 \phi_2 \; &= \; \alpha_3 + \beta_3 \phi_2\end{aligned} \tag{4.8}$$

Expanding (4.4) and substituting in the continuity constraints yields:

$$\Delta x_t = \alpha_2 + \gamma \Delta z_t + \phi_1(\beta_2 - \beta_1)D_1 + \phi_2(\beta_2 - \beta_3)D_3 + \beta_1(x - \gamma z)_{t-1}D_1 + \beta_2(x - \gamma z)_{t-1}D_2 + \beta_3(x - \gamma z)_{t-1}D_3 + u_t \tag{4.9}$$

In our empirical work we estimate (4.9) by least squares for given ϕ_1 and ϕ_2 and conduct a grid search over the ϕ parameters for those values that minimize the residual sum of squares. If the estimates of the general model indicate either $\beta_1 = \beta_2$ or $\beta_2 = \beta_3$, we adopt a specification with two piecewise linear segments, and if both restrictions hold, we adopt a simple linear function.

This linear spline function can now be used to test for a nonlinear relationship between investment and q. As a first stage we regress the ratio of business investment to the business capital stock against a measure of Tobin's q. The data are quarterly from 1967:3 to 1994:4. Since both series contain trends but are not cointegrated, we first detrend using the Hodrick-Prescott filter. Although the data are detrended, the resulting series exhibit a substantial degree of serial correlation, and a simple regression of one on the other is significantly dynamically misspecified. Therefore we make use of an error correction equation to model their relationship. We obtain the following regression (analogous to path AA' in Figure 2):

$$\begin{aligned}\Delta \ln(i/k)_t &= \underset{(0.17)}{-0.0006} + \underset{(2.92)}{0.0758}\Delta q_t - \underset{(-4.99)}{0.3478}(\ln(i/k)_{t-1} - \underset{(3.54)}{0.2232}q_{t-1})\\ R^2 &= 0.23,\ DW = 2.13,\ \sigma = 0.0349,\ LM(2) = 1.21\ (0.35), ARCH = 2.64\ (0.11), NORM = 6.60\ (0.03)\end{aligned}$$

Absolute values of the t-ratio are given in parentheses below coefficients, R^2 is the coefficient of determination, DW is the Durbin-Watson statistic, σ is the standard error of the regression, $LM(2)$ is the F-form of the Lagrange multiplier test for a second-order serial correlation process in the residuals, $ARCH$ is the F-form of the Lagrange multiplier test for a first-order auto regressive conditional heteroskedasticity process, and $NORM$ is the Jarque-Bera test statistic for normally distributed residuals. p-values for test statistics are given in parentheses following the statistics. This equation indicates a significant long-run relationship between the investment capital ratio and q (albeit in terms of deviations from trend). The diagnostic test statistics indicate no misspecification apart from the normality test, which indicates some deviation from the assumption of normally distributed residuals.

Next we test for the possibility of a nonlinear equilibrium relationship between the investment-capital ratio and q. The equation estimated is based on Equation (4.9), with a grid search over the parameters ϕ_1 and ϕ_2 determining the division of the sample that minimizes the residual sum of squares. We obtain the following specification:

$$\begin{aligned}\Delta \ln(i/k)_t &= \underset{(0.95)}{0.0027} + \underset{(4.14)}{0.0876}\Delta q_t - \underset{(7.62)}{1.5739}D_1ECT_{t-1} - \underset{(6.10)}{0.3500}D_2ECT_{t-1} - \underset{(5.11)}{1.1463}D_3ECT_{t-1} + \text{dummy variables}\\ ECT &= \ln(i/k)_t - \underset{(7.60)}{0.1826}q_t\\ R^2 &= 0.49,\ DW = 1.78,\ \sigma = 0.0288, LM(2) = 2.53\ (0.08), ARCH = 0.63\ (0.48),\ NORM = 0.95\ (0.62)\end{aligned}$$

This equation indicates a significant improvement over the symmetric specification. All the adjustment coefficients are significant, with the values of those associated with both large positive and large negative deviations from equilibrium being significantly larger than those for deviations around equilibrium. Figure 4 shows the sensitivity of Tobin's q in the three regimes on the basis of the estimated piecewise specification.

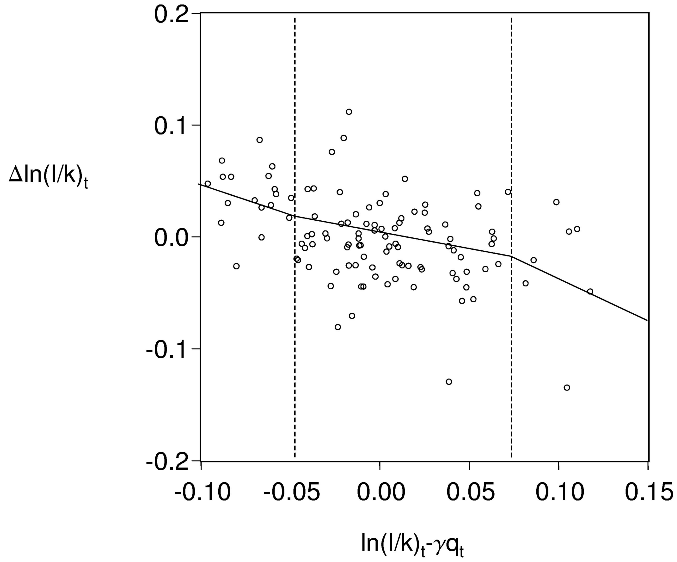

Figure 4
Sensitivity of investment to Tobin's q in the three regimes.

Wald tests confirm that we can reject the equality of both the extreme adjustment coefficients to the midsection coefficient.[7] The use of Wald tests (see Granger and Teräsvirta 1996) requires that the model be able to be estimated under the alternative, which is indeed feasible given our piecewise linear specification.[8] Interestingly however, we cannot reject the null that the adjustment coefficients for the two extreme sections are equal. In terms of the overall fit of the equation we note that the nonlinear specification results in a sharp rise in the coefficient of determination and a fall in the standard error of the regression. Note also that allowing for a nonlinear functional form removes the apparent nonlinearity in the residuals evident from the Jarque-Bera test in the symmetric adjustment model.

It is also interesting to examine the effects of the nonlinear specification in terms of the division of the sample into alternative adjustment regimes. The values of ϕ_1 and ϕ_2 that we obtain are -0.05 and 0.08, respectively. These compare with an overall range for the error correction term of -0.097 to 0.13. Figure 5 illustrates the division of the sample by superimposing the critical values of ϕ_1 and ϕ_2 on the graph of the error correction term. It is clear from this graph that most of the sample lies in the middle section of the kinked adjustment function. It is also evident, however, that allowing for the change in the adjustment parameter is important in terms of its effects on the overall explanatory power of the model.

It is also clear that we do not detect a region of inaction with respect to the relationship between investment and q, as suggested by the theory. Nevertheless, the aggregate effect seems to take the form of slower adjustment of the equilibrium capital stock to q, with much faster adjustment when the capital stock is well away from equilibrium.

5 Conclusions

This paper provides a theoretical model of investment under uncertainty and investment irreversibility that builds on previous work in this area by Abel and Eberly (1997). We augment the model by assuming a

[7]It is worth noting that Cook, Holly, and Turner (1999) provide some Monte Carlo evidence to suggest that the power of tests for asymmetry of the Granger-Lee kind have very low power against the null of a linear model for quite large samples.

[8]As Granger and Teräsvirta (1996) note, "Many of the nonlinear models are not identified under the assumption of nonlinearity. Attempts to estimate the nonlinear alternative when the null hypothesis is in fact true may, and should, therefore fail" (62–63).

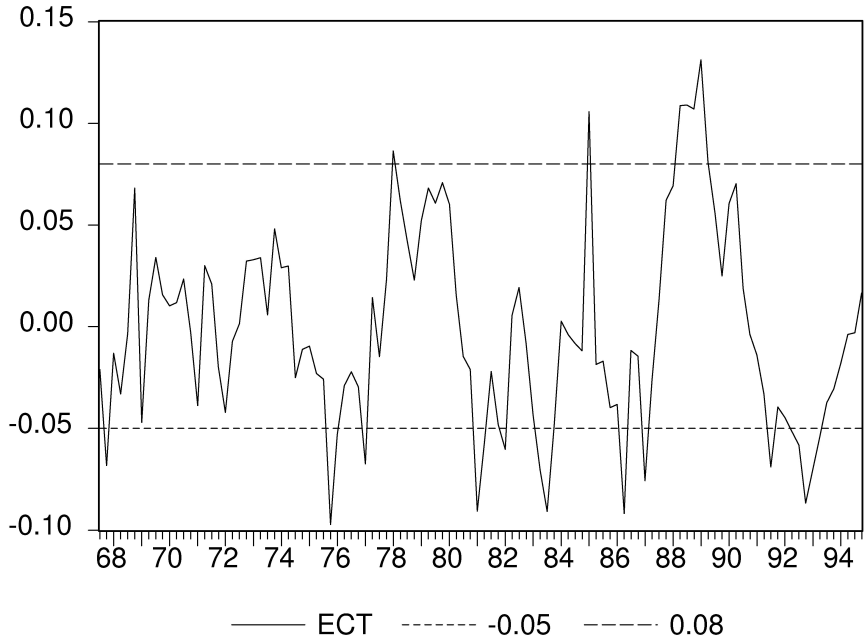

Figure 5
Critical values ϕ_1 and ϕ_2 for the change of regime in the error correction term specification.

mean-reverting process in the stochastic process governing fluctuations in demand and partial investment irreversibility. The model suggests that the position of the business cycle crucially matters for investment. We have shown that when output is below trend, a shift upward of the long-run trend implies higher returns in the future such that the option value of the investment and of implementing the project later is worth more.

We provide time-series evidence to suggest that there are different regimes describing the effect of q on investment. There are also indications, however, that much of the nonlinearity washes out because of aggregation. Empirically it would be better to carry out further examination of this model with mean reversion on firm-level data.

Appendix A The Solution with Mean Reversion

When the underlying fundamental follows a mean-reverting process, the solution of the homogenous part of the differential equations

$$(r+\delta)q^I = A\theta + \mu(\overline{\theta} - \theta)\theta q^I_\theta + \frac{1}{2}\sigma^2\theta^2 q^I_{\theta\theta} \tag{A.1}$$

in the region $\theta \in [\theta_E, \infty)$ and

$$(r+\delta)q^E = \mu(\overline{\theta} - \theta)\theta q^E_\theta + \frac{1}{2}\sigma^2\theta^2 q^E_{\theta\theta} \tag{A.2}$$

in the region $\theta \in [0, \theta_I]$ is of the form

$$q^i(\theta) = C\theta^\lambda h^i(\theta) \tag{A.3}$$

for $i = I, E$, where C and λ are constants that are chosen in order to make $h^i(\theta)$ satisfy a differential equation with a known solution. Substituting the expression for $q^i(\theta)$ in (A.3) into the homogeneous part of (A.1) and (A.2),

$$(r+\delta)q^i = \mu(\overline{\theta} - \theta)\theta q^i_\theta + \frac{1}{2}\sigma^2\theta^2 q^i_{\theta\theta} \tag{A.4}$$

gives

$$\theta^{\lambda} h^{i}(\theta)\left[\frac{1}{2}\sigma^{2}\lambda(\lambda-1)+\mu\overline{\theta}\lambda-(r+\delta)\right]+\theta^{\lambda+1}\left[\frac{1}{2}\sigma^{2}\theta h^{i}_{\theta\theta}(\theta)+h^{i}_{\theta}(\lambda\sigma^{2}+\mu\overline{\theta}-\mu\theta)-\mu\theta h^{i}(\theta)\right] \tag{A.5}$$

This equation can be solved as described in Dixit and Pindyck 1994 (162–63). Since (A.5) must hold for any value of θ, the two bracketed terms must equal zero. First, λ is chosen to set the first bracketed term equal to zero:

$$z(\lambda)=\frac{1}{2}\sigma^{2}\lambda(\lambda-1)+\mu\overline{\theta}\lambda-(r+\delta)=0 \tag{A.6}$$

From the second line of (A.5) we have

$$\frac{1}{2}\sigma^{2}\theta h^{i}_{\theta\theta}(\theta)+h^{i}_{\theta}(\lambda\sigma^{2}+\mu\overline{\theta}-\mu\theta)-\mu\theta h^{i}(\theta)=0 \tag{A.7}$$

We need to transform this equation into a standard form. We choose $w=\frac{2\mu\theta}{\sigma^2}$. Then we assume that $h^i(\theta)=m^i(w)$, which implies $h^i_{\theta}(\theta)=\frac{2\mu}{\sigma^2}m^i_{\theta}(w)$ and $h^i_{\theta\theta}(\theta)=(\frac{2\mu}{\sigma^2})^2 m^i_{\theta}(w)$. Then (A.7) becomes

$$wm^{i}_{\theta\theta}(w)+(b-w)m^{i}_{\theta}(w)-\lambda m^{i}(w)=0 \tag{A.8}$$

where $b=2\lambda+\frac{2\mu}{\sigma^2}\,\overline{\theta}$. Equation (A.8) is known as Kummer's equation, and its solution is the confluent hypergeometric function $H(w;\lambda;b)$.

If we define $w=\frac{2\mu}{\sigma^2}\theta$, then H is the confluent hypergeometric function

$$H(w;\lambda;b)=1+\frac{\lambda}{b}w+\frac{1}{2!}\frac{\lambda(\lambda+1)}{b(b+1)}w^{2}+\frac{1}{3!}\frac{\lambda(\lambda+1)(\lambda+2)}{b(b+1)(b+2)}w^{3}\ldots \tag{A.9}$$

which is a generalization of the exponential function if λ and b are equal to each other.
The solution $q^i(\theta)_H$ to the homogeneous differential equation (A.4) is of the form

$$q^{i}(\theta)_{H}=B_{+}\theta^{\lambda_{+}}H(w;\lambda_{+};b)+B_{-}\theta^{\lambda_{-}}H(w;\lambda_{-};b) \tag{A.10}$$

To rule out explosive dynamics in the homogenous differential solution for $q^I(\theta)$, the coefficient related to the positive root must be zero, and likewise in the solution related to $q^E(\theta)$, the coefficient related to the negative root must be zero.

For $q^I(\theta)$ the particular solution $q^i(\theta)_P=\frac{1}{r+\delta-\mu\overline{\theta}}A\theta$ is found by applying the method of undermined coefficients. The solution for $q^I(\theta)$ if $\theta\in[\theta_E,\infty)$ is therefore

$$q^{I}(\theta)=\frac{1}{r+\delta-\mu\overline{\theta}}A\theta+B_{-}\theta^{\lambda_{-}}H\left(\frac{2\mu}{\sigma^{2}}\theta;\lambda_{-};b\right) \tag{A.11}$$

and when $\theta\in[0,\theta_I]$, the solution for $q_E(\theta)$ is

$$q^{E}(\theta)=B_{+}\theta^{\lambda_{+}}H\left(\frac{2\mu}{\sigma^{2}}\theta;\lambda_{+};b\right) \tag{A.12}$$

A.1 Boundary conditions

The constants B_+, B_- and the trigger points θ_I and θ_E are derived by imposing smooth pasting and value matching at θ_I and θ_E:

$$q^{E}(\theta_E)\;=\;q^{I}(\theta_E) \tag{A.13}$$

$$q^{E}_{\theta}(\theta_E)\;=\;q^{I}_{\theta}(\theta_E) \tag{A.14}$$

$$q^{I}(\theta_I)\;=\;p^{+} \tag{A.15}$$

$$q^{E}(\theta_E)\;=\;p^{-} \tag{A.16}$$

which imply

$$\frac{1}{r+\delta-\mu\overline{\theta}}A\theta_E + B_-\theta_E^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_-;b\right) = B_+\theta_E^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_+;b\right) \tag{A.17}$$

$$\frac{A}{r+\delta-\mu\overline{\theta}} + B_-\left[\lambda_-\theta_E^{\lambda_--1}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_-;b\right) + \frac{2\mu}{\sigma^2}\theta_E^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_-;b\right)\right] \tag{A.18}$$

$$= B_+\left[\lambda_+\theta_E^{\lambda_+-1}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_+;b\right) + \frac{2\mu}{\sigma^2}\theta_E^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_+;b\right)\right]$$

$$\frac{1}{r+\delta-\mu\overline{\theta}}A\theta_I + B_-\theta_I^{\lambda_-}H\left(\frac{2\mu}{\sigma^2}\theta_I;\lambda_-;b\right) = p^+ \tag{A.19}$$

$$B_+\theta_E^{\lambda_+}H\left(\frac{2\mu}{\sigma^2}\theta_E;\lambda_+;b\right) = p^- \tag{A.20}$$

Given the complexity of the boundary conditions, the values for θ_E, θ_I, B_+, and B_- can be found only by solving the system numerically.

Appendix B The Solution without Mean Reversion

The solution without mean reversion implies that the stochastic process that affects profitability is a geometric Brownian motion process. The solutions are

$$q^I(\theta) = \frac{1}{r+\delta}A\theta + B_-\theta^{\lambda_-} \tag{B.1}$$

$$q^E(\theta) = B_+\theta^{\lambda_+} \tag{B.2}$$

where λ_- and λ_+ are the roots of the quadratic equation

$$\frac{1}{2}\sigma^2\lambda(\lambda-1) - (r+\delta) \tag{B.3}$$

The value of the triggers θ_I and θ_E and of the two coefficients B_+ and B_- are derived by imposing value-matching and smooth-pasting conditions at θ_I and θ_E, as reported in equations (2.33) to (2.36):

$$\frac{1}{r+\delta}A\theta_E + B_-\theta_E^{\lambda_-} = B_+\theta_E^{\lambda_+} \tag{B.4}$$

$$\frac{1}{r+\delta}A + \lambda_-B_-\theta_E^{\lambda_--1} = \lambda_+B_+\theta_E^{\lambda_+-1} \tag{B.5}$$

$$\frac{1}{r+\delta}A\theta_I + B_-\theta_I^{\lambda_-} = p^+ \tag{B.6}$$

$$B_+\theta_E^{\lambda_+} = p^- \tag{B.7}$$

Note that the conditions from (B.4) to (B.7) can be derived from the conditions (A.17) to (A.20), imposing $\mu \to 0$, which implies $H(\frac{2\mu}{\sigma^2}\theta_I;\lambda_-;b) \to 1$.

The previous equations give explicit solutions for the triggers and the coefficients. Multiplying equation (B.4) by $-\lambda_-\theta_E^{-1}$ and adding it to equation (B.5) we get the result for the exit trigger θ_E:

$$\theta_E = \frac{(r+\delta)}{A}\left[p^-\left(\frac{\lambda_+-\lambda_-}{1-\lambda_-}\right)\right] \tag{B.8}$$

Substituting in (B.6) the value for B_{-} derived by (B.4) gives

$$\theta_I = \frac{(r+\delta)}{A}\left[p^{+}\left(\frac{1-\lambda_{+}+\lambda_{-}}{1-\lambda_{-}}\right)\left(\frac{\theta_I}{\theta_E}\right)^{\lambda_{-}}\right] \tag{B.9}$$

The two coefficients B_{-} and B_{+} are, again, derived as a function of the known solution for θ_E and θ_I.

References

Abel, A. B., and J. C. Eberly (1997). "An exact solution for the investment and value of a firm facing uncertainty, adjustment costs, and irreversibility." *Journal of Economics Dynamics and Control*, 21: 831–852.

Arden, R., S. Cook, S. Holly, and P. Turner (2000). "The asymmetric effects of monetary policy: Some results from a macroeconometric model." *Manchester School*, 68(4): 419–441.

Barnett, S. A., and P. Sakellaris (1998). "Non-linear response of firm investment to q: Testing a model of convex and non-convex adjustment." *Journal of Monetary Economics*, 42: 261–288.

Bernanke, B. S. (1983). "Irreversibility, uncertainty, and cyclical investment." *Quarterly Journal of Economics*, 98(2): 85–106.

Böhm, H., and M. Funke (1999). "The uncertainty-investment relationship." Quantitative Macroeconomics Working Paper Series. Hamburg: Hamburg University.

Brock, William, Davis Dechert, Jose Sheinkman and Blake LeBaron (1996). "A test for independence based on the correlation dimension." *Econometric Reviews*, 15(3): 197–235.

Caballero, R. J., and E. M. R. A. Engel (1998). "Nonlinear aggregate investment dynamics: Theory and evidence." Working paper, Department of Economics. Cambridge: Massachusetts Institute of Technology.

Cook, S., S. Holly, and P. Turner (1999). "The power of tests for nonlinearity: The case of Granger-Lee asymmetry." *Economics Letters*, 62: 155–159.

Cukierman, A. (1980). "The effects of uncertainty on investment under risk neutrality with endogenous information." *Journal of Political Economy*, 88: 462–475.

Dixit, A., and R. S. Pindyck (1994). *Investment under Uncertainty.* Princeton: Princeton University Press.

Escribano, A., and G. Pfann (1998). "Non-linear error correction, asymmetric adjustment and cointegration." *Economic Modelling*, 15(2): 197–216.

Gale, D. (1996). "Delay and cycles." *Review of Economic Studies*, 63: 169–198.

Granger, C. W. J., and T. H. Lee (1989). "Investigation of production, sales and inventory relationships using multicointegration and non-symmetric error correction models." *Journal of Applied Econometrics*, 4 (suppl.): S145–S159.

Granger, C. W. J., and T. Teräsvirta (1996). *Modelling Nonlinear Economic Relationships.* Oxford: Oxford University Press.

Hayashi, F. (1982). "Tobin's marginal q and average q: A neoclassical interpretation." *Econometrica*, 50, 215–224.

Kloeden, P. E., and E. Platen (1992). *Numerical Solution of Stochastic Differential Equations.* Berlin: Springer-Verlag.

Metcalf, G. E., and K. A. Hassett (1995). "Investment under alternative return assumptions comparing random walks and mean reversion." *Journal of Economics Dynamics and Control*, 19: 1471–1488.

Pindyck, R. S. (1988). "Irreversible investment, capacity choice, and the value of the firm." *American Economic Review*, 78: 969–985.