Wavelet-Based Multiresolution Surface Approximation from Height Fields

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Abstract

A height field is a set of height distance values sampled at a finite set of sample points in a two-dimensional parameter domain. A height field usually contains a lot of redundant information, much of which can be removed without a substantial degradation of its quality. A common approach to reducing the size of a height field representation is to use a piecewise polygonal surface approximation. This consists of a mesh of polygons that approximates the surfaces of the original data at a desired level of accuracy. Polygonal surface approximation of height fields has numerous applications in the fields of computer graphics and computer vision.

Triangular mesh approximations are a popular means of representing three-dimensional surfaces, and multiresolution analysis (MRA) is often used to obtain compact representations of dense input data, as well as to allow surface approximations at varying spatial resolution. Multiresolution approaches, particularly those moving from coarse to fine resolutions, can often improve the computational efficiency of mesh generation as well as can provide easy control of level of details for approximations.

This dissertation concerns the use of wavelet-based MRA methods to produce a triangular-mesh surface approximation from a single height field dataset. The goal of this study is to obtain a fast surface approximation for a set of height data, using a small number of approximating elements to satisfy a given error criterion. Typically, surface approximation techniques attempt to balance error of fit, number of approximating elements, and speed of computation. A novel aspect of this approach is the direct evaluation of wavelet coefficients to assess surface shape characteristics within each triangular element at a given scale. Our approach hierarchically subdivides and refines triangles as the resolution level increases.

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Introduction

1.1 Motivation

A height field is a set of height distance values sampled at a finite set of sample points in a two-dimensional parameter domain. A height field usually contains a lot of information, much of which is redundant and can be removed without a substantial degradation of its quality. A common approach to reducing the size of a height field representation is to use a piecewise polygonal surface approximation. This consists of a mesh of polygons that approximates the surfaces of the original data at a desired level of accuracy. Polygonal surface approximation of height fields has numerous applications in the fields of computer graphics and computer vision.

Computer graphics applications have used polygonal surface models for both simulation and display, while computer vision applications often utilize surface models focused on such tasks as surface reconstruction, pose estimation, and 3D object recognition. Accordingly, development of a fast and accurate approximation method for height fields has long been pursued in both arenas. In addition, many applications require the polygonal surface models to have a capability of level-of-detail control by which the surface models can be represented in various degrees of fidelity to the original data. The main focus of this dissertation is to design a fast algorithm that automatically constructs surface approximations from height fields acquired with an arbitrary input device. In order to achieve high speed as well as the level-of-detail control, this study exploits a wavelet-based approach.

Polygonal surface approximation from input data is a useful preprocessing step for further high level processing. In computer vision, the input data are mostly *range images* acquired with a variety of 3D (three-dimensional) sensing techniques such as *n*-camera stereo [Pulli et al. 98], structured light [Hu and Stockman 89], and time-of-flight laser range-finders [Journet and Bazin 00]. Other sources are *terrain data* acquired from satellite photographs in remote sensing [Pottier et al. 99], and *isosurfaces*, which are extracted from volume data with the "marching cubes" algorithm [Nielson and Hamann 91] in scientific visualization.

With effective approximation techniques, those measured datasets could be reduced in size considerably, and utilized in computer graphics and geometric computer-aided design [Chen and Medioni 92, Curless and Levoy 96, Ramamootrhi and Arvo 99]. Also, high level processing such as analysis of surface characteristics [Sacchi et al. 99], pose estimation [Linnainmaa et al. 88, Chen and Stockman 96, Ji et al. 98], and 3D object recognition [Stark and Bowyer 96, Johnson and Hebert 98] in computer vision, can benefit from such models. Accordingly, a fast polygonal surface approximation method can be very useful for computer vision applications.

In addition, multiresolution analysis (MRA) is a popular trend in 3D data analysis, offering the possibility of compactly representing the input data as well as allowing representation at variable resolution. Wavelets are naturally suited to extracting information at a desired level of resolution and they have very attractive features from a computational point of view [Stollnitz et al. 95a, b, Starck et al. 98]. Therefore, wavelet-based multiresolution analysis has broad range of applications; these include data compression [Lewis and Knowles 92, Shapiro 93], object recognition in 2D [Tieng and Boles 97] and 3D [Paulik and Wang 98], tracking [Hongwei and Zhongkang 96], 3D surface recovery [Yaou and Chang 94, Maeda et al. 97], multiresolution description of meshes in computer graphics and computer-aided geometric modeling [Gortler and Cohen 95, Takahashi et al. 97], as well as surface approximation [Gross et el. 95, Yu and Ra 99, Paster and Rodriguez 99]. Most MRA techniques in computer graphics have used a wavelet-based framework for the decomposition and reconstruction of polygonal meshes [Eck et al. 95, Schroeder and Sweldens 95, Lounsbery et al. 97, Madhuram et al. 97, Mielson et al. 97, Staadt et al. 97, Valette et al. 99, Bonneau 98, Bertram et al. 00]. However, most of earlier studies have not fully exploited *direct decomposition* methods, which directly decompose input data based on wavelet transform. Only the applications for 3D surface recovery have utilized the direct decomposition methods.

1.2 Problem Statement and Goal

The goal of this study is to develop a fast and effective algorithm that can generate a fast triangular-mesh surface approximation for an arbitrary set of height data. The algorithm must balance error of fit, number of approximating elements, and speed of computation. Thus, the surface approximation problem can be formulated as a minimization problem of an energy function

$$\epsilon = \alpha E_{fit}(\Gamma) + \beta E_{size}(\Gamma) + \lambda E_{speed}\,, \tag{1.1}$$

where Γ represents a set of approximating elements, and α , β , and λ are constants. The first term corresponds to fitting error between the approximation and the input data. The second term penalizes meshes that contain large numbers of approximation elements. The speed of the algorithm is formulated in the third term. It is not a function of Γ, because the speed of the algorithm depends on the algorithm itself.

1.3 Proposed Approach

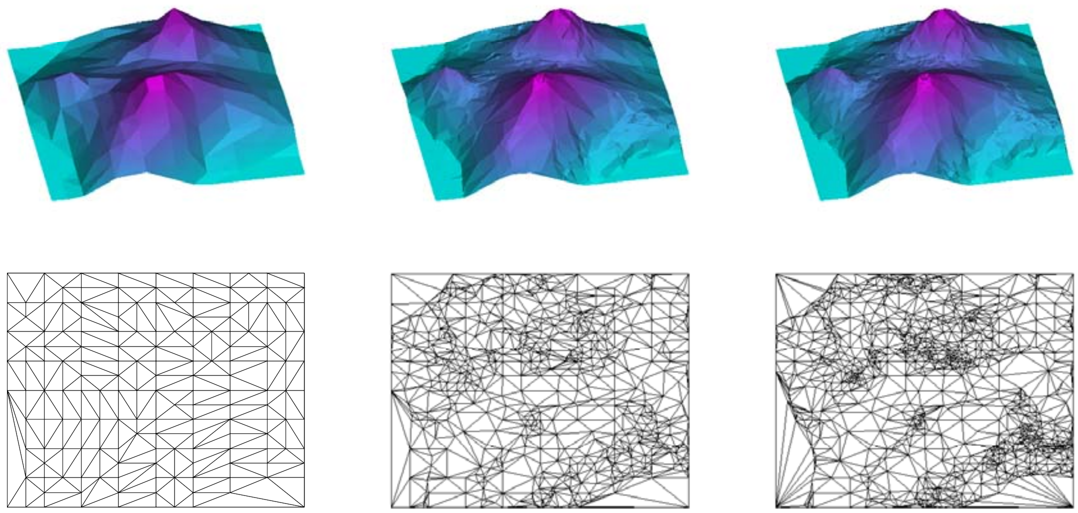
Many other researchers have considered the problem of surface approximation. Most previous work can be categorized as either refinement or decimation approaches. A refinement approach starts with a low-quality approximation and builds more and more accurate ones. The opposite case is decimation, a fine-to-coarse approach, starting from an exact fit to the data and discarding some details to create less and less accurate approximations.

This study focuses on the construction of a fast refinement (coarse-to-fine) method for surface approximation that directly utilizes wavelet information from input data. Instead of exploiting the entire dataset to approximate surfaces, which is quite computationally expensive and cannot easily deal with isolated regions of interest, the new algorithm will utilize wavelet decomposition in order to obtain a fast refinement scheme at various resolution levels. The high level outline of the proposed approach is as follows:

1. Decompose a given height field with a wavelet transform.
2. Construct an initial triangulation using predefined templates at the coarsest level.
3. Repeat until the finest level is reached or the approximation meets a user defined error criterion:
    (a) Select a candidate region.
    (b) Refine the triangles in the candidate region.
    (c) Reduce redundancy and regularize triangular mesh.

While most refinement algorithms have a structure similar to this, the proposed approach has some unique features such as multiscale decomposition of the height field and predefined templates. These features speed up the processing time while maintaining good fitness of the approximations. In addition, the proposed algorithm employs a redundancy removal to reduce the size penalty in the energy function (1.1).

As a result of the proposed approximation method, consider the surface approximations shown in Figure 1.1. A geographic terrain dataset is used in this example, and it contains values from a planar grid of size 256×256. This is an example of a digital elevation model (DEM). In order to obtain approximations, the two-dimensional domain is tessellated with a triangular mesh and then vertices of 3D triangular faces are placed at the height values. Accordingly, the approximated surfaces are piecewise planar patches connecting each triangular region through their common edges. Every 3D vertex of the resulting triangulation is taken directly from the initial dataset.

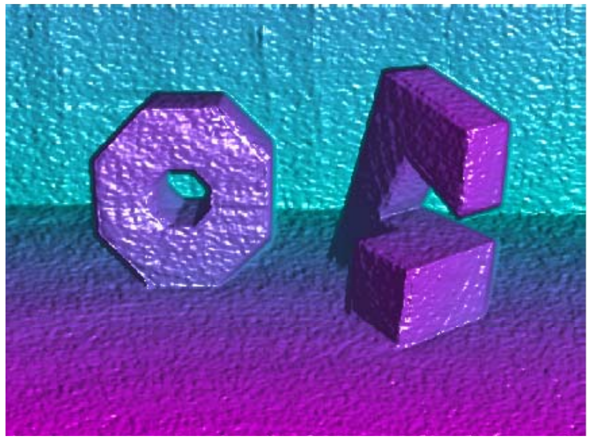


(a) (b) (c)

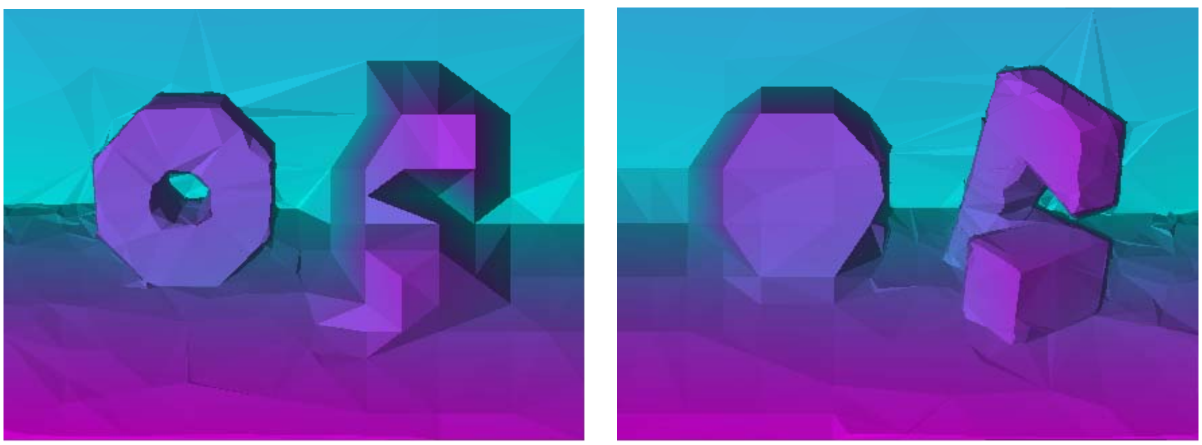
Figure 1.1 Surface approximations with different resolution levels. Wavelet decomposition of input data provides useful information for the surface approximations at various resolution levels. Rendered outputs are displayed in the first row and the corresponding triangular meshes are present in the second row. (a) The coarsest surface approximation. (b) An approximation at an intermediate resolution level. (c) The finest approximation. Only 1.8% of the input data points are used for this approximation.

The three example approximations are rendered with a Lambertian reflection model and displayed in the first row of Figure 1.1. Corresponding triangular meshes are present in the second row. The coarsest approximation (a), constructed from evaluating wavelet coefficients at the highest decomposition level, shows only the overall shape of the input data, and most details are not recovered at this resolution level. In some situations, the approximation at this resolution level is enough, while finer approximations are required for other situations. An intermediate approximation (b) represents many details of the terrain, and its triangular mesh contains tiny triangles. The finest approximation (c) has a much denser triangular mesh in areas of high variability, especially around the peaks of mountains and along the coastline. But even for the finest approximation, only 1.8% of the input data points have been used, and this represents a significant level of data compression.

The proposed approach is capable to have local control of level of detail by using a selective refinement technique that is readily accomplished with the wavelet transform. The most common approach for selective refinement in computer graphics applications is to use a view dependent refinement, by which the entire scene is represented with different levels of detail depending on viewpoint and other parameters such as lighting [Xia and Varshney 96, Hoppe 97, Hoppe 98]. Although there are numerous publications on the selective refinement for terrain datasets and other graphical objects, no such an effort has been performed for range images. In computer vision applications, the selective refinement of range datasets provides both fully reconstructed objects and roughly approximated objects in a single dataset. Figure 1.2 shows results for one case of selective refinement. One of two objects in a single dataset is fully refined while the other remains in a coarse approximation. Selective refinement may be a very attractive property for most computer vision applications.



(a)



(b)

Figure 1.2 Two examples of selective refinement. The proposed approach uses a selective refinement technique that is readily accomplished with the wavelet decomposition of input data. (a) Original dataset. (b) Selective refinement is capable of refining isolated regions of interest.

1.4 Contributions

Specific contributions of this study include the following:

- *A new fast algorithm for approximation of a dense height field using multiresolution analysis with wavelets*. The algorithm has potential application in the areas of computer graphics, computer-aided design, and computer vision. Detailed explanation of the algorithm appears in chapter 3 and chapter 4.

- *A new wavelet-based approach for shape analysis at different levels of detail*. If the 3D shape of a target object is known, it is possible to use the information to approximate the object. This dissertation has developed novel methods for initial mesh construction, and for a refinement scheme based on shape information inferred from the wavelet detail coefficients. This is described in section 3.5 and section 4.4.

- *A set of triangulation templates that can be used for fast triangular tessellation of the 2D domain, based on shape information of the input data*. These are used for fast triangulation at the coarsest resolution level. Section 3.5 will describe how to construct the templates.

- *A shape-preserving refinement algorithm for triangular meshes based on shape analysis of input height fields*. The proposed refinement algorithm for triangular meshes utilizes the shape information implicated in the wavelet detail coefficients so that approximated surfaces can represent the overall shape of the input height fields even at low-resolution levels. Section 4.4 includes details of the algorithm.

- *A method of selective triangulation*. The proposed surface approximation method can construct selective triangulations in which the entire dataset is represented with different levels of detail according to regions of interest. Figure 1.2 shows an example of the selective triangulation. This is described in section 5.6.

1.5 Prospective Applications

Multiresolution surface approximation with wavelets has a wide range of applications. These include:

- *Fast height field approximation*. A height field is a set of data sampled at points in a planar domain. Terrain data, a common type of height field, is used in many applications, including flight simulators, ground vehicle simulator, video games, and in computer graphics for entertainment. Computer vision uses height fields to represent range data acquired by stereo and laser range finders. In most of these applications, it is desirable to have a fast approximation method as well as an efficient data structure for representing and displaying the height field. Typically, the raw sample data is highly redundant. A fast approximation that preserves visual accuracy to a reasonable extent is desired. Using a multiresolution surface approximation with wavelets, it is possible to approximate the height field quickly, while eliminating redundancies.

- *Reverse engineering*. The approximated surfaces with triangular meshes can be used as geometric models that facilitate computer-aided design and subsequent numerical analysis and manufacturing.

- *Level-of-detail (LOD) control in various applications*. Like other MRA techniques, multiresolution surface approximation can control the level of detail of the surface approximation.

- *Analysis of surface characteristics at different resolution levels*. Since the approximation is done by successively refining the coarsest level approximation, it is possible to track surface characteristics at each resolution level. This scheme provides a fast and useful way to find important or critical characteristics on surfaces.

- *3D object recognition*. The proposed surface approximation method can be used in a 3D object recognition system to do coarse-to-fine matching

- *Fast 3D object display on remote terminal.* Transmission of a complex 3D object, to a remote terminal, especially linked with a wireless network, is slow. Thus, it is important to reduce the number of data elements being displayed. An attractive way for displaying such an object over a network is to begin with a low-resolution version that can be quickly rendered, and then progressively improve the display by transmitting detail information.

1.6 Overview of Material

The rest of this dissertation is organized as follows. Chapter 2 presents a brief literature review and the background of some concepts of surface approximation related to the research in this area. This chapter includes fundamental concepts and definition of height field and multiresolution surface approximation with triangular mesh, and introduces wavelet-based multiresolution analysis and some of local operators over triangular meshes. Error metrics and measurement methods are also covered in this chapter. Chapter 3 presents an overview of two-dimensional wavelet decomposition and brief concepts of graph theory. Based on these two ingredients, the chapter then presents a set of shape-preserving triangulation templates that are designed to reflect the underlying shapes of input height fields. The proposed surface approximation method uses these templates to create an initial triangular mesh. Chapter 4 explains the main algorithm for the wavelet-based iterative local refinement method that consists of three sub-procedures: candidate selection, refinement, and mesh enhancement. Section 4.3 through 4.5 will cover these sub-procedures. At the end of this chapter, the detailed version of the approximation method is summarized along with a flow diagram. In addition, all parameters used in the proposed approximation algorithm are summarized. In Chapter 5, time complexity and memory usage of the algorithm are analyzed before the extended experiments are performed. The experiments include the multiresolution surface approximation, performance on noisy datasets, selective triangulation, and empirical running time, showing that the proposed algorithm is feasible for fast surface approximation. Concluding remarks and future directions of this work are presented in Chapter 6.