

Dedication

To my parents, Lata Devi and Maya Kant Jha, and to my wife, Anjali Jha

Acknowledgements

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