

# CYCLOSTATIONARY RANDOM VIBRATION OF A SHIP PROPELLER AND A ROAD VEHICLE

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(ABSTRACT)

A special class of nonstationary processes with periodically varying statistics, called cyclostationary (CS), is investigated. These processes are encountered in many engineering problems involving rotating machinery such as turbines, propellers, helicopter rotors, and diesel engines. We analyze a cyclostationary process model in order to show its advantages compared to a traditional stationary process model and present a methodology for calculating the statistics of the response of a linear system subjected to CS excitations.

We demonstrate that a CS model estimates the statistics of the response of a linear dynamic system subjected to CS excitations more accurately by considering (1) a vehicle traveling on a rough road and (2) a propeller rotating in the wake of a ship in the presence of turbulence. In the case of the vehicle, the road consists of concrete plates of fixed length. We model the road excitation using a CS process and calculate the standard deviation (root mean square) of the vehicle response. In the case of the ship propeller, we calculate the hydrodynamic forces acting on the propeller using the vortex panel method and the vortex theory of propeller. Considering the randomness in the axial and the tangential components of velocity, we calculate the mean and the covariance of the forces. This analysis shows that the hydrodynamic forces acting on the propeller are CS processes. Then we perform finite element analysis of the propeller and calculate the mean and the standard deviation of the blade response. We do the parametric analysis to demonstrate the effects of some physical quantities such as the standard deviation, the correlation coefficient, the decorrelation time, and the scale of turbulence of the axial and the tangential components of the wake velocity on the standard deviation of the blade deflection. We found that the CS model yields the time-wise variation of the statistics of the excitation and the response (e.g., the root mean square) and their peaks correctly. This is important information for the calculation of probability of failure of the propeller. A traditional stationary model cannot provide this information.

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# NOTATIONS

## Latin Symbols

$a_o(r)$ : Lift curve slope at a blade section at radius  $r$ .

$A(t), B(t)$ : Zero-mean stationary random processes.

$c$ : Damping coefficient for the road-vehicle.

$c(r)$ : Chord length at a blade section at radius  $r$ .

$C$ : Damping matrix.

$C'$ : Modal damping matrix.

$C_B$ : Block coefficient.

$Cd$ : Drag coefficient.

$Cl$ : Lift coefficient.

$C_r$ : Element in  $r^{th}$  row and  $r^{th}$  column of the diagonal damping matrix.

CS: Cyclostationary.

$d$ : Length of a slab.

$D$ : Diameter of the propeller.

$D_r$ : Decorrelation distance.

$D_t$ : Decorrelation time.

$dL$ : Lift force on a hydrofoil section of infinitesimal length  $dr$ .

$dD$ : Drag force on a hydrofoil section of infinitesimal length  $dr$ .

$dr$ : Infinitesimal length of the blade section at radius  $r$ .

$E[ ]$ : Expected value of a random variable.

$f( )$ : Probability density function.

$h(t)$ : Matrix of impulse response functions.

- $H_i$ : Height of the  $i^{th}$  slab.
- $H(\omega)$ : Matrix of frequency response functions.
- $i$ : Index.
- $I$ : Set of integers.
- $j$ : Index, or,  
imaginary number ( $\sqrt{-1}$ ).
- $J$ : Advance ratio.
- $k$ : Spring constant.
- $\mathbf{K}$ : Stiffness matrix.
- $\mathbf{K}'$ : Modal stiffness matrix.
- $k_{ai}$ : Slope of  $f_i$  with respect to  $v_a$ .
- $k_{ti}$ : Slope of  $f_i$  with respect to  $v_t$ .
- $C_{f_j f_j}(r_i, t, r_j, \tau)$ : Element of covariance matrix of the blade force.
- $\mathbf{Cov}_{XY}(t_1, t_2)$ : Covariance matrix of the vector  $\mathbf{X}(t)$  and  $\mathbf{Y}(t)$ .
- $\mathbf{Cov}_{YY}(t_1, t_2)$ : Covariance matrix of the vector  $\mathbf{Y}(t)$ .
- $\mathbf{Cov}_{ZZ}(t_1, t_2)$ : Covariance matrix of the vector  $\mathbf{Z}(t)$ .
- $f_{mi}$ : Mean value of the force  $f_i$ .
- $f_{ro}$ : Element of amplitude vector of the sinusoidal forcing function.
- $f_{xi}$ : Component of total force in  $x$ -direction acting at  $i^{th}$  location on the blade.
- $f_{X(t)}(x)$ : Probability density function of the random vector process  $\mathbf{X}(t)$  at time  $t$ .
- $f_{X(t_1)Y(t_2)}(\mathbf{x}, \mathbf{y})$ : Joint probability density function of the two random vector processes  $\mathbf{X}(t)$ , and  $\mathbf{Y}(t)$  and times  $t_1$  and  $t_2$ .
- $f_{yi}$ : Component of total force in  $y$ -direction acting at  $i^{th}$  location on the blade.
- $f_{zi}$ : Component of total force in  $z$ -direction acting at  $i^{th}$  location on the blade.
- $F$ : Prandtl's tip loss factor.
- $\mathbf{F}'$ : Modal force vector.
- $\mathbf{F}_o$ : Amplitude vector of the sinusoidal forcing function.
- $h$ : Distance from the leading edge of a hydrofoil section.
- $k$ : Goldstein's kappa factor.
- $\mathbf{K}$ : Stiffness matrix of the propeller.
- $\mathbf{K}_r$ : Element in  $r^{th}$  row and  $r^{th}$  column of the diagonal stiffness matrix.

- $K_T$ : Thrust coefficient.
- $K_Q$ : Torque coefficient.
- $LTI$ : Linear time invariant.
- $m$ : Mass of the road-vehicle.
- $\mathbf{M}$ : Mass matrix.
- $\mathbf{M}'$ : Modal mass matrix.
- $MC$ : Monte Carlo.
- $MIMO$ : Multi-input multi-output.
- $M_r$ : Element in  $r^{th}$  row and  $r^{th}$  column of the diagonal mass matrix.
- $n$ : Index, or,  
The number of blades ( $n = 3$ ), or,  
The rotational speed of the propeller in revolutions per second.
- $N$ : Number of panel in vortex panel method.
- $\mathbf{N}(t)$ : Response vector in principal coordinate system.
- $P$ : A particular location on the blade.
- $pulse( )$ : Pulse function.
- $P(t)$ : A CS process.
- $Q$ : Torque produced by a propeller blade.
- $Q(t)$ : A CS process.
- $Q_T$ : Total torque produced by the propeller.
- $r$ : Radial distance of a blade section from the center of the propeller.
- $R$ : Radius of the propeller.
- $RMS$ : Root mean square value.
- $rf_{ij}(n, \tau, r_i, r_j)$ : Element of Cyclic covariance matrix of the blade force.
- $\mathbf{R}_{F'F'}(t_1, t_2)$ : Correlation matrix of the forces in principal coordinate system.
- $\mathbf{R}_{FF}(t_1, t_2)$ : Correlation matrix of the forces in physical coordinate system.
- $\mathbf{R}_{NN}(t_1, t_2)$ : Correlation matrix of the response in principal coordinate system.
- $r_r$ : Ratio of the forcing frequency to the  $r^{th}$  natural frequency.
- $r_{nij}(\theta_i - \theta_j)$ : Coefficient in the expression for correlation of turbulent velocity.
- $R_{ss}(t_1, t_2)$ : Autocorrelation of the random process  $S(t)$ .
- $Rv_a v_a$ : Autocorrelation of the axial component of the turbulent velocity.

- $R_{v_a v_t}$ : Cross correlation of the axial and tangential components of the turbulent velocity.
- $R_{v_t v_t}$ : Autocorrelation of the tangential component of the turbulent velocity.
- $r_{YY}(n, \tau)$ : Cyclic correlation matrix of the excitation vector.
- $R_{YY}(\tau)$ : Autocorrelation of a stationary random process  $Y(t)$ .
- $R_{YY}(t_1, t_2)$ : Autocorrelation of a nonstationary random process  $Y(t)$ .
- $R_{YY}(t_1, t_2)$ : Correlation matrix of the excitation vector.
- $R_{uu}(t_1, t_2)$ : Autocorrelation of the velocity in the wake of a ship.
- $R_{YY}(x + \Delta, x)$ : Autocorrelation of the road excitation measured at two spatial location  $x$  and  $x + \Delta$ .
- $r_{zz}(n, \tau)$ : Cyclic correlation matrix of the response vector.
- $R_{zz}(t_1, t_2)$ : Correlation matrix of the response vector.
- $R_{zz}^S(t_1, t_2)$ : Correlation matrix of the response vector of the equivalent stationary random processes.
- $S_{av}$ : Average of the crosscorrelation at  $(\tau = 0, \Delta S = 0)$
- $S_{max1}$ : Crosscorrelation at  $(\tau = 0, \Delta S = 0, \theta = 0)$
- $S_{max2}$ : Crosscorrelation at  $(\tau = 0, \Delta S = 0, \theta = \pi)$
- $S(t)$ : Resultant process after phase randomization.
- $S_{YY}(n, \omega)$ : Matrix of cyclic cross spectral density of the excitation vector.
- $S_{ZZ}(n, \omega)$ : Matrix of cyclic cross spectral density of the response vector.
- $t$ : Time.
- $t_1, t_2$ : Times at two instants.
- $T$ : Time period, also the thrust produced by a propeller blade.
- $T_T$ : Total thrust produced by the propeller.
- $u(\mathbf{r}, t)$ : Horizontal component of the velocity.
- $\mathbf{u}(\mathbf{r}, t)$ : Velocity field in the plane of a propeller.
- $u_o$ : Ship velocity.
- $v$ : Fluctuation in the velocity.
- $V$ : Velocity of the road-vehicle, also velocity of ship.
- $V_a$ : Axial component of the wake velocity.
- $V_e$ : Resultant velocity of the flow that a blade section encounters.

- $V_{sa}$ : Velocity of advance.
- $w$ : Wake fraction.
- $W$ : Induced velocity.
- $W_a$ : Axial component of the induced velocity.
- $W_{SCS}$ : Wide sense cyclostationary.
- $W_T$ : Tangential component of the induced velocity.
- $x$ : Distance that the the road vehicle has traveled in time  $t$ .
- $Y(x)$ : Excitation due to road roughness as function of distance traveled by the vehicle.
- $Y(t)$ : Excitation due to road roughness as function of time.
- $Y(t)$ : Excitation vector (size =  $m \times 1$ ).
- $Z(t)$ : Response Vector (size =  $p \times 1$ ).

## Greek Symbols

- $\alpha$ : Section angle of attack.
- $\alpha_i$ : Induced angle of attack.
- $\alpha_r$ : Phase shift in the sinusoidal response of a single degree-of-freedom system.
- $\beta$ : Geometric pitch angle of zero lift line of a hydrofoil section of blade.
- $\beta_{tip}$ : Geometric pitch angle of zero lift line of a hydrofoil section at the tip of the blade.
- $\Delta$ : Distance between two points where autocorrelation is being calculated.
- $\Delta S$ : Distance between two points where correlation is being calculated
- $\Delta\theta$ : The change in  $\theta$  from the leading edge to the trailing edge.
- $\Delta\alpha$ : Reduction in the angle of attack of the zero lift line since the flow traces a curve path between two blades.
- $\varepsilon$ : Ratio of lift to drag coefficients.
- $\phi$ :  $\tan^{-1} (V_a/\Omega r)$ .
- $\phi_1$ : Random phase angle of the CS signal  $P(t)$ .
- $\phi_2$ : Random phase angle of the CS signal  $Q(t)$ .

- $\Phi$ : Modal matrix.
- $\Phi_i$ :  $i^{th}$  modal vector.
- $\phi_T$ : Helix angle at the tip of the blade.
- $\gamma$ : Slope of the flow at any point at a distance  $h$  from the leading edge of the hydrofoil.
- $\Gamma$ : Circulation around a blade element.
- $\Gamma_\infty$ : Circulation around a blade element when number of blades approaches infinity.
- $\eta(t)$ : Mean of the excitation vector to a dynamic system.
- $\eta_o$ : Open water efficiency of the propeller.
- $\eta_r(t)$ :  $r^{th}$  element in the response vector of system in principal coordinate system.
- $\sigma$ : Standard deviation.
- $\sigma_{av}$ : Average standard deviation
- $\sigma_{max1}$ : Maximum standard deviation at  $\theta = 0$
- $\sigma_{max2}$ : Maximum standard deviation at  $\theta = \pi$
- $\sigma_{v_a v_t}$ : Crosscorrelation at  $(\tau = 0, \Delta S = 0)$
- $\sigma_{Z Average}$ : Average value of the standard deviation of the vehicle response.
- $\sigma_{Z Maximum}$ : Maximum value of the standard deviation of the vehicle response.
- $\sigma_h$ : Standard deviation of slab height.
- $\theta$ : Angular position of the blade ( $\theta = 0$  is vertical up), and also the slope of the flow at any point at a distance  $h$  from the leading edge of the hydrofoil.
- $\Sigma(x, y, z, t)$  A scalar random field
- $\rho$ : Density of water.
- $\rho_{v_a v_t}$ : Pearson's correlation coefficient of the axial and the tangential components of the wake velocity.
- $\tau$ : Time difference  $(t_1 - t_2)$ .
- $\omega$ : Frequency.
- $\omega_1$ : Frequency of the CS process  $P(t)$ .
- $\omega_2$ : Frequency of the CS process  $Q(t)$ .
- $\omega_e$ : Fundamental frequency.

- $\Omega$ : Rotational velocity of the propeller, or,  
Frequency of the forcing function.
- ' : Denotes transpose of a vector or matrix.