6. RESPONSE OF THE PROPELLER

In this chapter we demonstrate the methods developed in chapters 2, 4, and 5 by calculating the standard deviation of the propeller blade deflection.

6.1 Hydrodynamic Forces

Figure 6.1 shows the distribution of axial component of speed of advance for a ship, $C_B = 0.65$, as a function of radial distance r, and the angular position θ. The speed of the ship is 19.4 knots (10 m/s). $\theta = 0$ radian corresponds to the vertical position of the blade and the wake is symmetrical about the vertical axis. It can be seen from the figure that the speed of advance is minimum at $\theta = 0$ and maximum at $\theta \approx \pm \pi/2$. This is because the effect of ship hull on the wake velocity is maximum at $\theta = 0$ and minimum at $\theta \approx \pm \pi/2$.

As mentioned earlier, we use Vortex Panel Method to calculate the lift at different sections of the blade. Actual calculations were done using the software developed by Devenport and Vadapalli that calculates the flow field around an airfoil and lift using this method. The software has sufficient accuracy and Vortex Panel Method is applicable for thin hydrofoils like that of a propeller blade. Since the hydrofoil shape changes across the blade, the lift coefficient will also change. Figure 6.2 shows the value of lift curve slope, a_o, at different radial position on the blade. The theoretical value for a_o for 2-D airfoil is 2π per radian. In general, the calculation of the drag coefficient is very complicated and cannot be handled completely by the ideal flow solutions such as the vortex panel method. However, its value is small compared to the lift coefficient. The ratio of drag to lift coefficients is taken to be 0.05, which gives a fairly good estimate of the drag force.

Table 6.1: Data of the wake and the propeller

Parameter	Value
Speed of ship V, knots	19.6
Density of water ρ, kg/m^3	1025
Lift curve slope a_o	6 - 7
Ratio of drag and lift coefficient ϵ	0.05
Advance Ratio J	1
Rotation velocity of the propeller Ω, rad/sec	6.24
Modulus of Elasticity of the propeller E, pa	2.068×10^{11}
Poisson ratio ν	0.29
Density of the propeller ρ_p, kg/m^3	7820

After calculating the lift and drag coefficients, we follow the formulae developed in vortex theory of propeller to calculate the lift and drag forces. Figure 6.3 shows the variation of Prandtl's tip loss factor F along the radius (Eq. 4-7). This factor takes into account the end effect of the blade and makes the lift gradually decreasing to zero towards the tip section of the blade. Following Eqs. 4-16, 4-17, and 4-18, we calculate the induced angle of attack α_i. Next, we calculate tangential components of the induced velocity W_T by Eq. 4-15. Using Eq. 4-12 and 4-14, we calculate the axial component of the velocity W_a and total velocity V_e, respectively. Figure 6.4 shows the resultant velocity distribution with radial and angular positions. By Eqs. 4-21, and 4-23, we calculate the reduction in the angle of attack due to reduction in camber of the airfoil as the water traces a curved path inside the propeller. Now we apply Eq. 4-24 and 4-25 to calculate the lift and drag. Figure 6.5 shows the variation on the lift at section 0.7 R of the blade as it rotates. Lift is highest at $\theta = 0$ and lowest at $\theta \approx \pm \pi/2$. This change in the lift force causes vibration in the blade we calculate later. To verify the above procedure and to see the performance of the propeller we calculate the thrust coefficient, K_T, the torque coefficient, K_Q, and the open water efficiency, η_o as a function of the advance ratio J from Eqs. 4-30 to 4-33. Figure 6.6 shows the plots of these non-dimensional quantities. They are similar to the standard performance curves of a propeller having an average pitch by diameter ratio equal to *1.16* (Lewis, 1988). At $J = 1$, we see the highest open water efficiency equal to *0.7*. We have set this as the operating point of the propeller for the further calculations.

6.2 Mean of the Blade Response

As mentioned earlier, we use TET-10 elements for the finite element analysis of the propeller. Each node of the element has three-degrees-of-freedom as three displacements. We break the lift and drag forces into *fx, fy,* and *fz* components along the displacements of the nodes. Calculations of forces are done at five locations along the blade. As mentioned earlier, the points of applications of the forces have been chosen to be the quarter chord points of the hydrofoils at the five locations. These locations are 0.886 m, 1.587 m, 2.162 m, 2.874 m, and 3.505 m. Force on one blade is related to the force in the same direction and at the same relative location on another blade by a phase angle of $2\pi/3$. The next step is to obtain the mass, the stiffness matrices from the FEA of the propeller. Figure 6.7 shows the propeller mesh with 1532 elements. Figures 6.8 and 6.9 show the first eight modes shapes of the propeller and the first eight natural frequencies. The first natural frequency of the propeller is 28.12 Hz. As mentioned in chapter 5, it's computationally beneficial to ignore the higher order modes if the excitation frequency is significantly lower than the frequency of the highest considered mode. We consider the first fifteen modes. The fifteenth natural frequency is 167 Hz. Using Eq. 5-9 and Eq. 5-10 we obtain the modal mass, the modal stiffness, the modal damping matrix, and the modal force vector. The damping coefficient, ξ, is chosen to be 0.01. We represent the nodal forces in Fourier series taking 15 terms. Using Eq. 5-14, we obtain the steady state response of the propeller in the principal coordinate system and then transfer it back to the physical coordinate system using Eq. 5-7. Figure 6.10 shows the deflection time history of a node located at the tip section of a blade in z-direction. This response of the propeller can be also termed as the mean value of the response because so far we have not assumed any randomness in the velocity field and hence in the excitation. Our next task is to find the standard deviation of the response when there is randomness in the velocity field.

6.3 Standard Deviation of the Blade Response

To calculate the standard deviation of the response we first set the numerical values of the parameters in the model for Rv_av_a, and Rv_tv_t (Eq. 4-50). Crosscorrelation has been assumed zero. The effect of the crosscorrelation is considered in the next section. Table 6.2 shows the parameters chosen for the Rv_av_a, and Rv_tv_t. Figure 6.11 shows the plot of Rv_av_a and Fig. 6.12 shows the plot of the standard deviation of the velocity in the axial direction. We calculate the covariance matrix of the forces using Eq. 4-55. The size of the covariance matrix was then reduced to 15 × 15 in the principal coordinate system (Eq. 5-15). We took 11 terms in the Fourier series representation of the covariance of forces. Coefficients in these terms are the cyclic correlation matrices of the forces. Next we calculate the cyclic power spectral density by taking Fourier transform of the cyclic correlation function. By the modal mass, modal stiffness, and the modal damping matrix we calculate the frequency response function matrix and its first element is shown in Fig. 6.13. Then we calculate the cyclic power spectral density of the response using Eq. 2-21. Covariance matrix of the response is calculated using Eq. 2-19 and Eq. 2-20. Standard deviation of the deflection of a tip section of the blade is shown in the Fig. 6.14. The red curve shows the standard deviation calculated using CS model and the blue curve shows the standard deviation calculated using the stationary model. It can be seen that while the CS model shows periodic fluctuation, stationary model shows the constant standard deviation. The maximum value of the standard deviation of the response in this case is around 50% higher than the average standard deviation. This shows that stationary model underestimate the fluctuation in the standard deviation of the response. To get an idea of severity of the effect of the randomness in the wake velocity, we plot the fluctuation band of the axial wake velocity and the resulting fluctuation band of the blade deflection. For any time on the horizontal axis, the band shows the range of fluctuation of the quantity from {mean value - one standard deviation} to {mean value + one standard deviation}. Figures 6.15 and 6.16 show fluctuation band of the axial wake velocity and the fluctuation band of the tip deflection in the axial direction for time 0 to T, respectively. The band height is maximum at $t = 0$ because at this point of time the blade is in the wake of the hull and encounters the maximum fluctuation in the wake velocity. The band height is minimum at $t \approx T/4$ because at this time the blade is out of the wake of the hull and encounters the least fluctuation in the wake velocity.

Table 6.2: Values of the parameters of the correlation of velocity

Parameter	Values for $Rv_a v_a$	Value for $Rv_t v_t$
Decorrelation time D_t, s	*5*	*5*
Scale of turbulence D_s, m	*5*	*5*
σ_{max1}, m/s	*1.5*	*1.5*
σ_{max2}, m/s	*1.1*	*1.1*
σ_{av}, m/s	*1.0*	*1.0*

6.4 Parametric Analysis

In preliminary design stages, it is important to have an efficient tool for predicting the response of a propeller. This allows designers to evaluate a large number of alternative design configurations in a short period. One way to develop such a tool is to make a database with the values of the design variables of many designs and the responses of these designs. The information in this database can be used in constructing a response surface polynomial that relates the response of the propeller to the values of the design variables. A designer can then use this polynomial to find the response of a new design configuration instantaneously, and assess the effect of various design parameters on the response. This gives an indication to the designer as to which parameter is more important and how it affects the response. We study the effect of different parameters involved in the correlation of velocity on the standard deviation of the blade deflection. We calculate the standard deviation of the blade response for different values of the scale of turbulence (D_s), the decorrelation time (D_t), the average standard deviation (σ_{av}), the maximum standard deviation at $\theta = 0$ (σ_{max1}), and the maximum standard deviation at $\theta = \pi$ (σ_{max2}). These parameters are involved in the model of the autocorrelation of the axial component, autocorrelation of the tangential component, and the crosscorrelation of the axial and the tangential components of the velocity field (Eqs. 4-46 to 4-50).

Table 6.3 shows the values of the parameters along with the maximum and the average standard deviation of the tip deflection of a blade. All quantities are in SI units. The quantities written in percentage are expressed as a percentage of the ship velocity (= *10 m/s*). First, we set a mean level, which is Case 0 in table 6-3. The parameters are then changed from this mean level to observe its effect on the standard deviation of the response. For the mean level, we assume that the cross correlation is zero, and the autocorrelation of axial and the tangential component of the wake velocity is the same. For the crosscorrelation, the concept of the standard deviation is not applicable. S_{max1}, S_{max2} are defined as the crosscorrelation at $\{\tau = 0, \Delta S = 0, \theta = 0\}$ and at $\{\tau = 0, \Delta S = 0, \theta = \pi\}$, respectively. S_{av} is the average of the crosscorrelation at $\{\tau = 0, \Delta S = 0\}$. We use the correlation coefficient to determine the numerical values of the parameters in the crosscorrelation. Pearson's correlation coefficient is defined as:

$$\rho v_a v_t = \frac{\sigma v_a v_t}{\sigma v_a \sigma v_t} \tag{6-1}$$

We choose five levels (-0.7, –0.5, 0, 0.5, and 0.7) of the correlation coefficient. Given the value of the standard deviation of axial and the tangential component at $\theta = 0$, we determine S_{max1} for all the levels. $S_{max2,}$ and S_{av} are then chosen so that the shape of the crosscorrelation is reasonable and according to the guidelines (section 4.3.4) regarding the general structure of the correlation function.

For the mean level, maximum standard deviation at $\theta = 0$, maximum standard deviation at $\theta = \pi$, and the average standard deviation, are *15%*, *11%*, and *10%* of the ship velocity, respectively. The scale of turbulence and the decorrelation time are assumed to be *5 m* and 5 *s*, respectively. These values are the same for the autocorrelation of axial and tangential components of the turbulent velocity. The crosscorrelation is assumed to be zero. Maximum and the average standard deviation of the response are found to be 2.668 mm and 1.760 mm, respectively. After calculating the standard deviation for the mean level, we change the parameter for the correlations of the velocity.

Case 1 and Case 2 illustrate that if the scale of turbulence and the decorrelation time of autocorrelation of the axial velocity increase, the standard deviation of the response also increases from its mean level. This is because, as these two parameters increase, the cyclic correlation decays more slowly with time difference and spatial separation of the two points. This implies that the forces at any two points, which are even far away in time and space, are correlated to each other and their effects add up to produce a higher standard deviation of the blade deflection.

Case 3 and Case 4 show that if the intensity of the fluctuation of the axial velocity increases, the standard deviation of the response also increases. This can be also explained by Eq. 4-55. From this equation, the expression for the standard deviation of the forces can be given by

$$\sigma_f^{\ 2} = k_{ai}^{\ 2} \sigma v_a^{\ 2} + k_{ai} k_{ti} S v_a v_t^{\ 2} + k_{ti} k_{ai} S v_t v_a^{\ 2} + k_{ti}^{\ 2} \sigma v_t^{\ 2} \tag{6-2}$$

It can be seen from the above equation that since the coefficient k_{ai}^2 is always positive, an increase in the standard deviation of the axial component of the wake velocity will increase the standard deviation of the force and hence the standard deviation of the response.

The above conclusions, derived for the autocorrelation of the axial component, hold for the autocorrelation of the tangential component also (Case 5 to Case 8). However, it can be seen that the effect of change in the autocorrelation of tangential component is less than that due to the change in the autocorrelation of the axial component. This is because of the fact that the effect of the tangential velocity on the deflection of the blade is less than the effect of the axial velocity.

From the study of parametric analysis of the crosscorrelation of the axial and the tangential velocity components, we see that the standard deviation of the response attains the maximum value at the mean level. If we increase or decrease the scale of turbulence and the decorrelation time of the crosscorrelation from the mean level, the standard deviation of the response decreases (Case 9 and Case 10). Similarly, if we increase or decrease the value of the crosscorrelation, the standard deviation of the response decreases (Case 11 to Case 14).

The results from the parametric study suggest that a stronger autocorrelation of the velocity than the mean level increases the standard deviation of the response. Also, a stronger magnitude of the cross correlation of the velocities than the mean level decreases the standard deviation of the response.

Table 6.3: Data for the parametric analysis of the propeller

	Autocorrelation of the axial velocity					Crosscorrelation of the axial and the tangential velocities						Autocorrelation of the tangentiall velocity					Standard Dev. of the Response	
Case #	σ_{max1}	σ_{max2}	σ_{av}	D_t	D_s	S_{max1}	S_{max2}	$\rho_{vavt}(t_m)$	S_{av}	D_t	D_s	σ_{max1}	σ_{max2}	σ_{av}	D_t	D_s	$(\sigma_{res})_{max}$	$(\sigma_{res})_{av}$
	(%)	(%)	(%)			(%)	(%)		(%)			(%)	(%)	(%)			(10^-3)	(10^-3)
Mean Level																		
0	15	11	10	5	5	0	0	0	0	5	5	15	11	10	5	5	2.668	1.760
Change in Rv_av_a																		
1	15	11	10	1	1	0	0	0	0	5	5	15	11	10	5	5	1.468	0.975
2	15	11	10	10	10	0	0	0	0	5	5	15	11	10	5	5	3.020	2.076
3	10	8	7.5	5	5	0	0	0	0	5	5	15	11	10	5	5	1.832	1.338
4	20	16	15	5	5	0	0	0	0	5	5	15	11	10	5	5	3.490	2.510
Change in Rv_tv_t																		
5	15	11	10	5	5	0	0	0	0	0	0	15	11	1	1	1	2.190	1.366
6	15	11	10	5	5	0	0	0	0	0	0	15	11	1	10	10	2.753	1.808
7	15	11	10	5	5	0	0	0	0	0	0	10	8	7.5	5	5	2.594	1.720
8	15	11	10	5	5	0	0	0	0	0	0	20	16	15	5	5	2.749	1.864
Change in Rv_av_t																		
9	15	11	10	5	5	11.25	8	0.5	7.5	1	1	15	11	10	5	5	2.188	1.440
10	15	11	10	5	5	11.25	8	0.5	7.5	10	10	15	11	10	5	5	1.596	0.960
11	15	11	10	5	5	15.75	11	0.7	10	5	5	15	11	10	5	5	1.380	0.854
12	15	11	10	5	5	11.25	8	0.5	7.5	5	5	15	11	10	5	5	2.140	1.387
13	15	11	10	5	5	-11.25	-8	-0.5	-7.5	5	5	15	11	10	5	5	1.828	1.157
14	15	11	10	5	5	-15.75	-11	-0.7	-10	5	5	15	11	10	5	5	1.245	0.857