7. CONCLUSIONS AND RECOMMENDATIONS

We presented a class of random processes, called cyclostationary. The key characteristic of this class of processes is that their statistics are not constant in time, as a stationary model would assume, but vary periodically. For example, the standard deviation changes in time as the process evolves instead of being constant. These type of processes are encountered in many engineering problems involving systems subjected to periodic excitations, such as turbines, propellers, helicopter rotors, diesel engines, and cars on roads made of concrete slabs of fixed length. We analyzed stationary and cyclostationary models to show that a stationary process model tends to underestimate the maximum response of a system subjected to a periodic, random excitation. Therefore, it underestimates the risk of failure. Cyclostationary models have the following advantages:

1) In problems involving systems subjected to multiple excitations with a certain phase relationship, cyclostationary models account for this phase relationship. This relationship is lost when stationary models are used.
2) Cyclostationary models calculate the time variation in the statistics of the excitation and the response, whereas stationary models ignore this..

These advantages are important in the calculation of the probability of failure of a system.

We presented a methodology to calculate the statistics of the response of a linear system subjected to cyclostationary excitations and confirmed that the representation of the excitation with traditional stationary model is only an approximation.

We demonstrated that a cyclostationary model estimates the statistics of the response of a system more accurately than a stationary model using the following two examples:

1) **A vehicle traveling on a road made of slabs:** We showed that the excitation on a vehicle due to a road made of slabs is cyclostationary. Then, we calculated the standard deviation of the response of the vehicle. We found that a stationary model smears the peaks of the standard deviation of the response of the vehicle. These peaks are important in calculating the probability of failure.

We also studied the effect of the fundamental frequency, ω_e, of the autocorrelation of the excitation on the standard deviation of the response. It is observed that when ω_e equals to the natural frequency, the standard deviation of response is maximum. It is seen that the standard deviation is sensitive to ω_e for values less than the natural frequency whereas it is insensitive to ω_e for values larger than the natural frequency. This information can help in vehicle design.

2) **A propeller rotating in the wake of a ship hull:** Literature survey on this topic shows that the statistics of the excitation vary periodically in time. We analyzed the response of a propeller rotating in the wake of a ship in the presence of turbulence. Due to presence of the ship hull, the wake velocity in not uniform in the plane of the propeller. Due to the presence of turbulence, the propeller encounters a field that fluctuates randomly. The intensity of the fluctuation changes periodically in time. Using the vortex panel method and the vortex theory of propeller, we calculated the mean values of the excitation forces on each propeller blade. Then, we calculated the first and second order statistics, i.e., the mean and the covariance, of the forces. We developed a geometric model of the propeller and performed finite element analysis of the propeller using I-DEAS^{TM} to find the vibrational characteristics of the propeller. After that, using the covariance of the hydrodynamic forces, we calculated the standard deviation of the response (the displacement of the tip of a blade). We found that while a cyclostationary model shows the peaks in standard deviation of the blade deflection, a stationary model gives constant standard deviation. Due to these peaks, maximum displacement of the blade over a period will be larger than the maximum deflection considering the average RMS of the displacement. Therefore, a structure subjected to a CS excitation has higher first excursion and fatigue failure probabilities than an identical structure subjected to a stationary excitation with the same time-average RMS.

Failure analysis of a structure is very sensitive to the first and the second order statistics of its response (deflection, stress). Hence, a cyclostationary model of the excitation yields more realistic estimation of the failure probability.

Finally, we did the parametric analysis to demonstrate the effects of a few selected variables such as correlation of the axial and the tangential components of the wake velocity, scale of turbulence, and decorrelation time on the standard deviation of the blade response. We found that as the autocorrelation of the turbulent velocity increases, the standard deviation of the blade deflection also increases. Similarly, if the scale of turbulence and the decorrelation time of autocorrelation of the axial and the tangential components of the turbulent velocity increase, the standard deviation of the response increases from its mean level. We also find that standard deviation of the response attains its maximum value at the mean level of the standard deviation of the crosscorrelation of that axial and the tangential velocity components.

While these studies showed the importance of cyclostationary models of random processes for some engineering problems and demonstrated their advantages, some important issues should be studied in the future:

1) The input-output problem presented here is limited to the case of linear, time-invariant systems. It is important to develop a method to calculate the response of a nonlinear or/and time-variant system subjected to CS excitations.

2) In the study of the road vehicle response we only calculated the displacement of the center of gravity using a one-degree-of freedom model of the vehicle. More realistic models of a vehicle should be considered. It is also important to study the stresses in critical components of the vehicle suspension and predict the probabilities of failure of these components due to first excursion and fatigue. Finally, we assumed constant slab length. In a more realistic case, the lengths of the slabs may vary randomly. One may want to explore the effects of randomness in slab length on the statistics of response and the probability of failure of the critical components of the vehicle.

3) As mentioned earlier, the calculation of the hydrodynamic forces acting on a propeller blade and their statistics are very difficult. The theory for predicting the characteristics of the turbulent flow field encountered by a propeller due to the interaction of the ship hull and the propeller should be further developed. Also, experiments are needed to obtain data for estimating the statistics of the forces acting on the propeller and velocity field encountered by the propeller.

4) One should also explore the effects of the randomness in the radial component of the velocity field, and the effects of bio fouling on the flow field encountered by the propeller and on the statistics of the blade responses.

5) One of the primary goals of random vibration analysis of a structure is to calculate its probability of failure. One should try to develop a general methodology for the calculation of the probability of failure of structures subjected to CS excitations due to fatigue and first excursion.