

A non-clinical method to simultaneously estimate  
thermal conductivity, volumetric specific heat, and perfusion  
of *in-vivo* tissue

By  
Marie Catherine Madden

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APPROVED:

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Elaine P. Scott, Advisor, Committee Chair

---

Thomas Diller, Committee Member

---

Brian Vick, Committee Member

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# A non-clinical method to simultaneously estimate thermal conductivity, volumetric specific heat, and perfusion of *in-vivo* tissue

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Abstract

Many medical therapies, such as thermal tumor detection and hypothermia cancer treatments, utilize heat transfer mechanisms of the body. The focus of this work is the development and experimental validation of a method to simultaneously estimate thermal conductivity, volumetric specific heat, and perfusion of *in-vivo* tissue. The heat transfer through the tissue was modeled using a modified Pennes' equation. Using a least-squares parameter estimation method with regularization, the thermal properties could be estimated from the temperature response to the known applied heat flux.

The method was tested experimentally using a new agar-water tissue phantom designed for this purpose. A total of 40 tests were performed. The results of the experiments show that conductivity can be successfully estimated for perfused tissue phantoms. The values returned for volumetric specific heat are lower than expected, while the estimated values of perfusion are far greater than expected. It is believed that the mathematical model is incorrectly accounting between these two terms. Both terms were treated as heat sinks, so it is conceivable that it is not discriminating between them correctly.

Although the method can estimate all three parameters simultaneously, but it seems that the mathematical model is not accurately describing the system. In the future, improvements to the model could be made to allow the method to function accurately.

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## Nomenclature

$\alpha$	regularization term
$\beta$	inverse estimation parameter
$c$	volumetric specific heat, $J/m^3K$
$C_p$	specific Heat, $J/kg/K$
$g$	heat generation, $W/m^2$
$h_b$	convective heat, $W/m^2$
$h_m$	metabolic heat, $W/m^2$
$I$	identity matrix
$k$	thermal conductivity, $W/mK$
$\rho$	density, $kg/m^3$
$q_0$	heat flux at boundary, $W/m^2$
$T$	temperature, $K$
$t$	time, $s$
$V$	perfusion flow, $mL/mL/s$
$V$	volume, $m^3$
$\omega$	perfusion term, $W/m^3K$
$X$	sensitivity matrix

## Super and subscripts

$b$	blood
$E$	east
$i$	current node
$i+1$	eastern node
$i-1$	western node
$N$	north
$p$	current time step
$p-1$	previous time step
$S$	south
$W$	west

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