

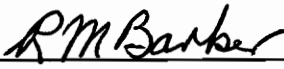
**YIELD LINE ANALYSIS OF AN AASHTO NEW JERSEY CONCRETE  
PARAPET WALL**

by

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Virginia Polytechnic Institute and State University  
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# YIELD LINE ANALYSIS OF AN AASHTO NEW JERSEY CONCRETE PARAPET WALL

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Civil Engineering

(ABSTRACT)

Concrete bridge rails are rated according to three performance levels. For classification at a given performance level, the rail must meet specific strength and geometric requirements. To meet the strength requirement, the rail must be able to satisfactorily withstand a transverse concentrated load applied at the top of the rail. This load is called  $F_t$  (kips) and is listed for each performance level in the Draft NCHRP Project 12-33 document entitled *Development of a Comprehensive Bridge Specification and Commentary*.

Researchers at the Texas Transportation Institute have developed equations to determine  $R_w$  (kips), the total transverse resistance of a rail (which must be greater than or equal to  $F_t$ ), and  $L_c$  (ft), the critical length of wall failure (Hirsch 1978). These equations (referred to as Hirsch equations in this study) were developed by yield line analysis for a constant thickness concrete parapet wall.

The purpose of this study is to develop similar equations for  $R_w$  and  $L_c$  based on yield line analysis of a variable thickness New Jersey concrete parapet wall instead of a constant thickness wall.

The results from this study indicate that the Hirsch equations significantly over estimate  $R_w$  for variable thickness concrete walls where  $M_c$ , the flexural resistance of the wall about the horizontal axis, varies substantially over the height of the wall. This study recommends that an average value for  $M_c$ , taken over the height of the wall, be used in the Hirsch equations when this situation arises.

## ACKNOWLEDGMENTS

### THE NEGRO MOTHER

Children, I come back today  
To tell you a story of the long dark way  
That I had to climb, that I had to know  
In order that the race might live and grow.  
Look at my face--dark as the night--  
Yet shining like the sun with love's true light.  
I am the child they stole from the sand  
Three hundred years ago in Africa's land.  
I am the dark girl who crossed the wide sea  
Carrying in my body the seed of the free.  
I am the woman who worked in the field  
Bringing the cotton and the corn to yield.  
I am the one who labored as a slave,  
Beaten and mistreated for the work that I gave--  
Children sold away from me, husband sold, too.  
No safety, no love, no respect was I due.  
Three hundred years in the deepest South:  
But God put a song and a prayer in my mouth.  
God put a dream like steel in my soul.  
Now, through my children, young and free,  
I realize the blessings denied to me.  
I couldn't read then. I couldn't write.  
I had nothing, back there in the night.  
Sometimes, the valley was filled with tears,  
But I kept trudging on through the lonely years.  
Sometimes, the road was hot with sun,  
But I had to keep on till my work was done:  
*I had* to keep on! No stopping for me--  
I was the seed of the coming Free.  
I nourished the dream that nothing could smother  
Deep in my breast--the Negro mother.  
I had only hope then, but now through you,

Dark ones of today, my dreams must come true:  
All you dark children in the world out there,  
Remember my sweat, my pain, my despair.  
Remember my years, heavy with sorrow--  
And make of those years a torch for tomorrow.  
Make of my past a road to the light  
Out of the darkness, the ignorance, the night.  
Lift high my banner out of the dust.  
Stand like free men supporting my trust.  
Believe in the right, let none push you back.  
Remember the whip and slaver's track.  
Remember how the strong in struggle and strife  
Still bar you the way, and deny you life--  
But march ever forward, breaking down bars.  
Look ever upward at the sun and the stars.  
Oh, my dark children, may my dreams and my prayers  
Impel you forever up the great stairs--  
For I will be with you till no white brother  
Dares keep down the children of the Negro mother.

--Langston Hughes

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# CHAPTER 1

## INTRODUCTION

### 1.1 CONCRETE PARAPET WALL DESIGN

There are two ways bridge rail designs can meet AASHTO specifications. For the first approach, bridge rails are acceptable if they have been designed according to a standard process. This design process is known as "prescriptive design" and consists of designing rails to meet specific geometric and strength requirements. The alternative is crash testing. If the bridge rail performs satisfactorily during the tests, then, it is also acceptable even though it may not meet the geometric or strength requirements of the previous approach. (Bronstad, et al., 1987)

#### 1.1.1 Performance Levels

Concrete bridge rails are rated according to three performance levels.

***PL1 - Performance Level One** - Used for short, low level structures on rural highway systems, secondary expressways, and areas where a small number of heavy vehicles are expected and speeds are either posted or reduced.*

***PL2 - Performance Level Two** - Used for high-speed main line structures on freeways, expressways, highways, and areas with a mixture of heavy*

*vehicles and maximum tolerable speeds.*

***PL3 - Performance Level Three - Used for freeways with variable cross slopes, reduced radius of curvature, higher volume of mixed heavy vehicles and maximum tolerable speeds.***

*(NCHRP Project 12 - 33, 1993)*

### **1.1.2 Strength Requirements**

In order for a concrete bridge rail to be classified at a given performance level, it has to meet geometric criteria and strength requirements. For the strength requirement, a concrete bridge rail must be able to satisfactorily resist a transverse load applied at the top of the rail. This transverse force is called  $F_t$  (kips) and is listed in Table 1.1 for each performance level.

## **1.2 PURPOSE AND ORGANIZATION**

Researchers at the Texas Transportation Institute have developed equations for calculating  $R_w$ , the total transverse resistance of a railing, and  $L_c$ , the critical length of wall failure, for a given parapet wall by using yield line analysis of a constant thickness wall (Hirsch, 1978).  $R_w$  must be greater than or equal to  $F_t$  to meet the strength requirement of a given performance level.

**TABLE 1.1**

**DESIGN FORCES FOR TRAFFIC RAILINGS (NCHRP Project 12 -33, 1993)**

Design Forces and Designations	Railing Performance Levels		
	PL-1	PL-2	PL-3
$F_T$ Transverse (KIP)	27.0	54.0	116.0
$F_L$ Longitudinal (KIP)	9.0	18.0	39.0
$F_V$ Vertical (KIP) Down	4.5	18.0	50.0
$L_T$ and $L_L$ (FT)	4.0	3.5	8.0
$L_V$ (FT)	18.0	18.0	40.0
$H_e$ (min) (IN)	20.0	32.0	40.0
Minimum H Height of Rail (IN)	20.0	32.0	40.0

This study develops similar equations for determining  $R_W$ , the total transverse resistance of a railing, and  $L_C$ , the critical length of wall failure, based on yield line analysis of a variable thickness AASHTO New Jersey concrete parapet wall instead of a constant thickness wall.

Chapter 2 presents a literature review of traffic barrier design and development. A review of the derivation of equations for  $R_W$  and  $L_C$  is given in Chapter 3. A variable thickness AASHTO New Jersey concrete parapet wall is analyzed in Chapter 4 and equations for  $R_W$  and  $L_C$  are developed. Examples are worked in Chapter 5 and conclusions and recommendations are in Chapter 6.

## **CHAPTER 2**

### **LITERATURE REVIEW**

#### **2.1 BARRIER RESEARCH**

Early barriers were modified versions of livestock fences. They were 3 to 4 ft high and made of wood. However, as the means of transportation progressed from horse and buggies to motorized vehicles and heavy trucks, improved barrier systems were needed to accommodate the changing vehicular traffic. In order to improve barrier design, the vehicle-barrier interaction during impact must be understood. To improve this understanding, full-scale crash tests, small-scale model tests, and analytical studies have been conducted.

##### **2.1.1 Crash Testing**

Full-scale crash tests were conducted as early as 1924 in Pennsylvania. "Report of Committee on Highway Guards" summarizes full-scale crash tests conducted on guard rails up until 1942 (Rahn, 1942). Early crash tests were fairly simple. Wooden tracks were built along inclines. The test barriers were constructed at a given test angle at the end of the tracks. The driverless, free-running test vehicle was accelerated by gravity and guided into the barrier by the tracks. (Rahn, 1942)

Later full-scale crash tests were more sophisticated and technically involved. Beaton and Field (1960) conducted crash tests of median barriers. Medium

weight four passenger sedans were driven into barriers at speeds of 60 mph and at 30 degree angles. The impacts were recorded by normal and high speed cameras.

Beaton and Field added the following instrumentation to the 1951 - 1955 automobiles.

- For brake control, attached a Bendix hydrovac booster to the master brake cylinder.
- Wired the ignition system to a remote control panel.
- Drained the gas tank and connected the gas line to a 1 gallon tank with a relief and cut-off valve.
- Mounted the steering motor to the front seat floor.
- Bolted storage batteries and steering pulsor to rear seat floor.
- Bolted remote control equipment to trunk bottom.
- Clamped adjustable pulley to steering wheel.

The following functions were required to control the car.

- Ignition on
- Ignition off
- Steer right
- Steer left
- Brakes on

The accelerator was wired to a full throttle position allowing a maximum speed of 58 - 62 mph. A controlled relay energized the ignition system. Radio control failure would result in opening of "ignition relay allowing car to stop under compression. . ." The steering motor pulsor actuated the steering motor in increments of 1/8" to 1" in each direction per pulse. The pulse rate ranged from 2 - 20 seconds per second.

Standard photographic coverage was used and included seven cameras in the following locations:

- Front data camera
- Rear data camera
- Two overhead data cameras
- Two documentary cameras panning vehicle through collision to terminal point
- Sequence camera operating at 20 frames per second

Michie (1981) gives more recent procedures for crash testing. He lists standard vehicles that should be used along with year limits. He states that dummies or sand can be used to "simulate occupant loading." As far as the actual tests are concerned, the vehicles "may be pushed, towed or self-powered" to the test speed. The use of brakes should be avoided for as long as possible to determine the "unbraked runout trajectory." He states that any type of guidance system may be used as long as the system does not effect the dynamics of the vehicle.

### **2.1.2 Small-scale Tests**

Small-scale model tests have also been used to further understand the phenomena of vehicle-barrier impacts. Small-scale modeling has several advantages over full-scale crash tests.

- More control over experiments
- Parameters can be easily varied
- Faster testing rate
- Lower cost

The disadvantage is some parameters are distorted when the size is scaled down. For this reason and others, small-scale modeling has only been done on a limited basis. (Michie, Calcote and Bronstad, 1971)

### **2.1.3 Mathematical Models**

Mathematical models have also been used to analyze vehicle-barrier collisions. Graham, Burnett and Gibson (1967) used the following steps to conduct a mathematical analysis:

1. Developed equations for vehicle motion in terms of applied moments and forces
2. Determined forces and moments as a function of barrier characteristics
3. Solved equations and compared to full-scale test results

Barrier characteristics used were post strength, post spacing, rail bending strength, and rail strength in tension. The vehicle motion had six degrees of freedom. These were lateral, longitudinal and vertical linear motions and roll, pitch, and yaw angular motions.

Several simplifying assumptions were made.

- Regarded vehicle as single body
- Used only horizontal motion; neglected vertical roll and pitch
- Forces on vehicle acted in horizontal plane

## **2.2 BARRIER TYPES**

Traffic barriers are classified according to two systems. These are longitudinal systems and crash cushion systems. Longitudinal systems consist of guardrails, median barriers and bridge rails. Crash cushion systems included steel barrels, entrapment nets and containers filled with sand or water. (Michie and Bronstad, 1971)

Guardrail and median systems are further categorized according to their lateral stiffness.

1. Rigid barriers - allow no lateral deflections  
example - New Jersey barrier
2. Semi-rigid barriers - allow small to moderate lateral deflections

- a. strong post/strong beam  
example - W-section beam on wood posts
  - b. weak post/strong beam  
example - box beam
3. Flexible barriers - allow large lateral deflections
- a. strong post/weak beam  
example - cables with wood posts
  - b. weak post/weak beam  
example - cables on weak steel posts (Michie, Calcote and Bronstad, 1971)

### **2.3 BARRIER FUNCTION AND SAFETY PERFORMANCE**

The purpose of traffic barriers is to protect vehicle occupants from dangerous roadside "features" and errant vehicles. Crash cushions provide this protection by reducing the severity of head-on collisions with fixed objects by decelerating the vehicle to a stop. Longitudinal barriers provide protection by meeting the following safety performance requirements (Bronstad, et al., 1987)

- Retain vehicle (no vaulting, penetration or rollover)
- Rail should remain intact during the collision
- Vehicle shall remain upright

- Rail should smoothly redirect vehicle (Smoothness is defined as no rotating or yawing of the vehicle's rear end while the vehicle is in contact with the barrier.)
- Rail should decelerate vehicle within acceptable limits
- Vehicle exit angle should be less than 10 degrees and within 100 ft of barrier after losing contact with the barrier

## **2.4 BARRIER USAGE**

As previously stated, barrier systems are used to protect vehicle occupants from roadside hazards and out-of-control vehicles. However, barrier systems are intrinsically hazardous. Barrier systems minimize accident severity, but they also increase accident frequency because they are usually a larger target than the roadside obstacle and are closer to the roadway. For these reasons, it is recommended that hazardous site features be eliminated before considering barrier installation. The decision to install barrier systems depends upon whether it is more dangerous to hit roadside obstacles or the barrier system (Michie and Calcote, 1971).

Barrier systems are required for the following hazardous highway features.

1. Lateral obstacles
  - a. bridge structures
  - b. abrupt embankments
  - c. sloped embankments

## 2. Roadside features

- a. rough rock cuts
- b. large boulders
- c. permanent bodies of water deeper than 2 ft
- d. large trees (6" diameter or greater) within 30 ft of roadway
- e. elevated gores
- f. gap between twin bridges

## 3. Fixed objects

- a. bridge parapet and rails depending upon bridge width  
traffic direction
- b. sign supports
- c. light poles
- d. bridge piers
- e. abutments
- f. retaining walls
- g. culvert headwalls
- h. wood poles and posts

For barriers in narrow medians one must consider the median width and traffic volume. (Michie and Bronstad, 1971)

## **2.5 DEVELOPMENT OF THE NEW JERSEY BARRIER**

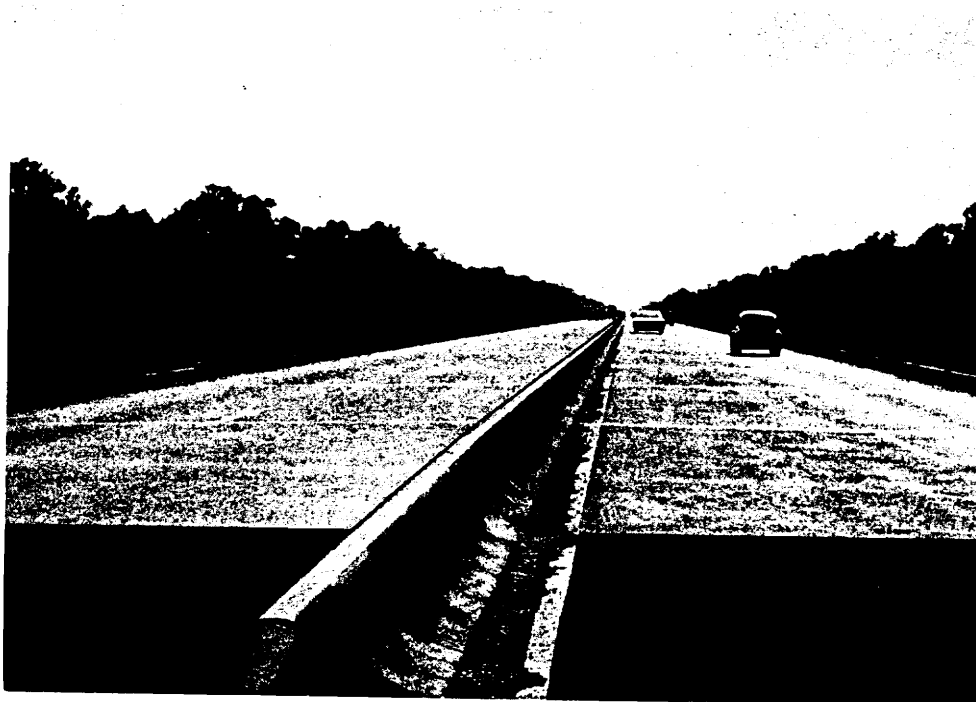
The New Jersey safety shape was based on concrete median barriers developed in Louisiana from 1942 - 1943 and California in 1946. The Louisiana barrier was 22.5 inches high with a curved face while the California barrier had a parabolic curved base. Both are shown in Figures 2.1 and 2.2. The New Jersey barrier has been used in New Jersey since 1955. By 1966 most states were using the modified design shown in Figure 2.3 (Lokken, 1974).

The height of the New Jersey barrier was chosen by trial and error. Initially, the height was 18 inches and was later raised to 20 inches. However, at this height vehicles were still able to vault the rail. The height was then raised to 32 inches. The width and thicknesses were determined to provide protection against vehicle damage and prevent overturning. (Lokken, 1974)

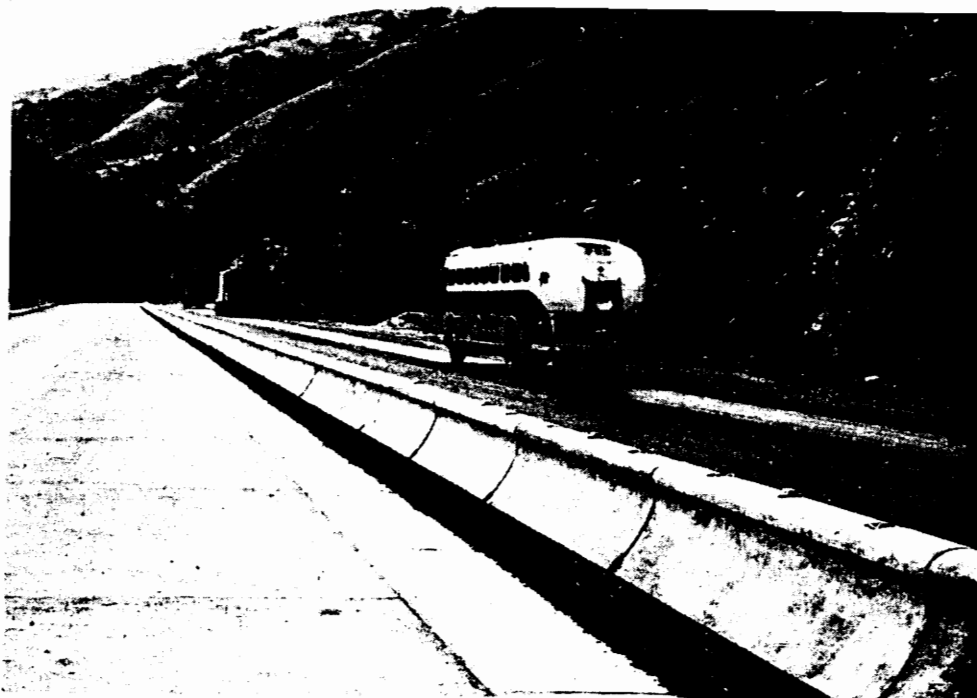
The resulting barrier is 32 inches high, has a 24 inch wide base and is 6 inches wide at the top. A 10 inch radius curve intersects the slopes of the two curves. The lower face has a 55 degree slope and a 2 to 3 inch vertical curb.

The New Jersey barrier works in the following manner:

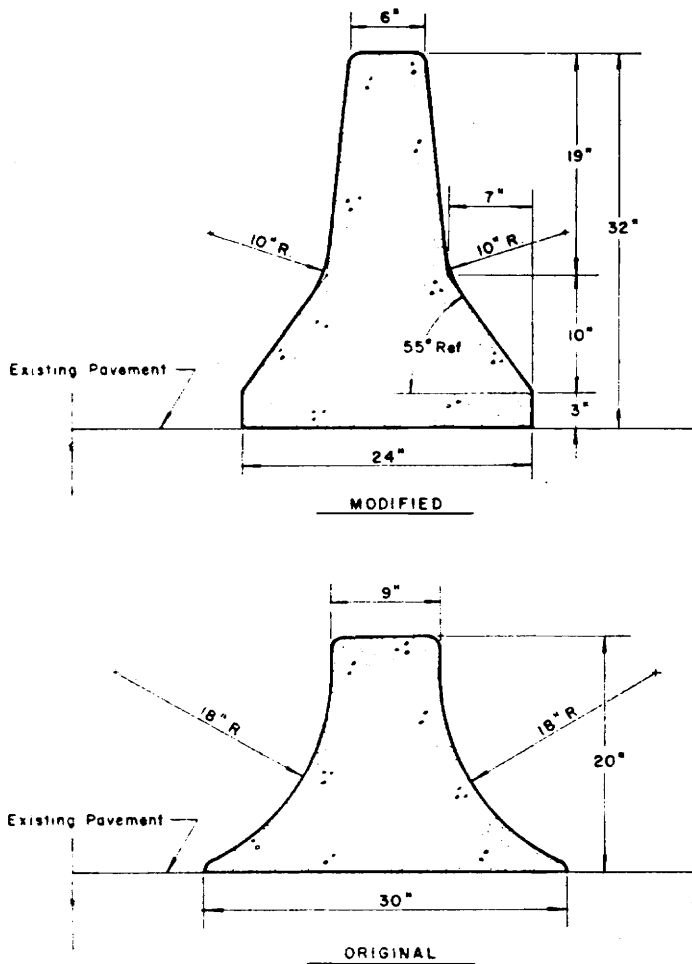
1. The first tire hits the 3 inch curb which slows the vehicle while redirecting it.



**FIGURE 2.1**  
**LOUISIANA CONCRETE BARRIER**  
*(Concrete Paving, 1969)*



**FIGURE 2.2**  
**CALIFORNIA CONCRETE BARRIER**  
*(California Highways and Public Works, 1947)*



**FIGURE 2.3**  
**NEW JERSEY CONCRETE BARRIER (Lokken, 1974)**

2. The front wheel climbs the 55 degree slope which results in one or both wheels and the side of the car being lifted 13 to 15 inches. The energy component perpendicular to the barrier is absorbed during lifting and the overturning moment is "overcome" by compression of the vehicle's suspension system.
3. At impact angles less than 10 degrees, redirection is by the vehicles' wheels. Compression of the suspension system absorbs energy and not the body of the vehicle.
4. For high speed and large angle impacts, the barrier functions differently. As the wheel reaches the almost vertical upper part of the wall, it is redirected, decelerated and returned to the roadway parallel to the barrier. The barrier imposes minimum damage to the vehicle and tolerable deceleration rates the vehicle's occupants (Lokken, 1974).

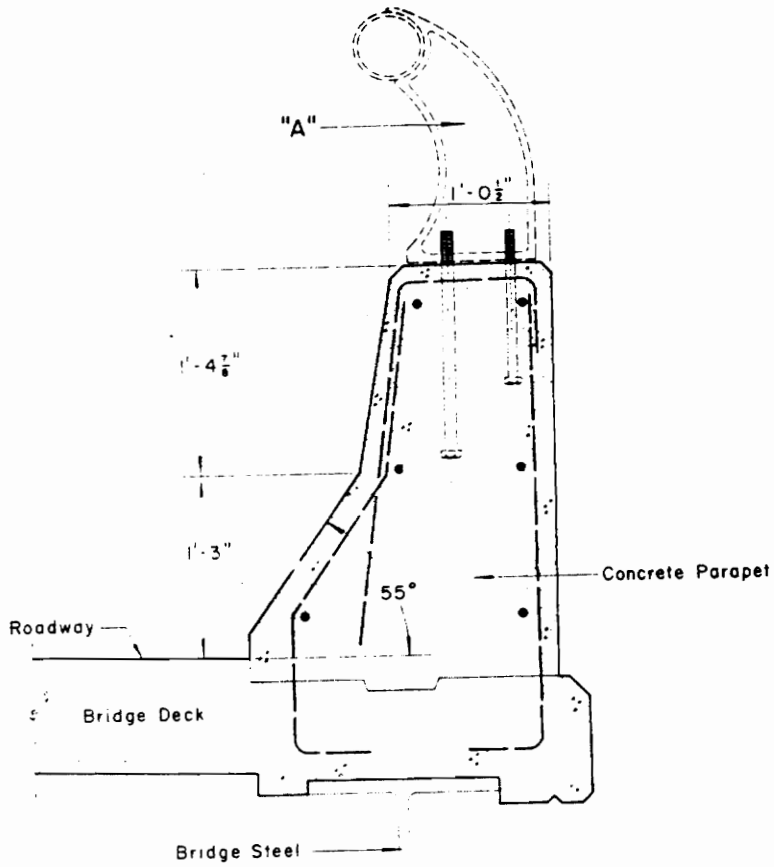
The New Jersey barrier has been crash tested by various groups. Lundstrom (1965) conducted twenty-one tests on New Jersey barriers at the General Motors Proving Ground. Lundstrom took measurements on vehicles to help determine at what elevation the two sloped faces should intersect to "eliminate sheet metal contact with the upper portion when the wheel contacts the base." The elevation of the slopes' intersection was raised to 15 inches. He also determined that the wheel friction force resulting from contact with the parapet did not affect the action of the vehicle. Lundstrom concluded that 32 inches was

not high enough for larger trucks so a rail was added to increase the height, bringing it closer to the center of gravity of the vehicle (see Figure 2.4).

The California Division of Highways (Nordlin and Field, 1968) also conducted tests on the New Jersey barrier in 1966 and 1967. They concluded that the New Jersey barrier effectively redirects medium weight sedans at angles less than 10 degrees with little or no damage to the vehicle or barrier. However, for medium weight sedans at speeds of 65 mph and 25 degree angles, the barrier redirects the vehicle but with damage to the vehicle and high deceleration rates.

Buth, et al., (1984) conducted full-scale crash tests on the New Jersey safety shape. They found that the safety shape did not meet the performance standards for a 4500 lb vehicle impacting at a speed of 60 mph and an angle of 25 degrees. The vehicle was not smoothly redirected and exhibited "excessive roll displacement." However, it did meet the given performance standards for impacts at lower speeds and angles.

Buth (1990) reported on additional crash tests on New Jersey barriers in 1990. The impact conditions were a 5390 lb vehicle at 57.7 mph and 20.6 degrees and a 18000 lb vehicle at 51.6 mph and 15.5 degrees. The rail performed satisfactorily meeting performance level 2 requirements.



**FIGURE 2.4**  
**GENERAL MOTORS PROVING GROUND NEW JERSEY PARAPET**  
 (Lundstrom, 1965)

## CHAPTER THREE

### REVIEW OF PREVIOUS WORK

#### 3.1 PRESENTATION OF HIRSCH EQUATIONS

T J Hirsch at the Texas Transportation Institute developed equations to determine  $R_w$  (kips), the total transverse resistance of a rail and  $L_c$  (ft), the critical wall length, given in NCHRP Project 12-33 "Development of a Comprehensive Bridge Specification and Commentary" (1993). It is noted that these equations were originally presented with different notation in "Analytical Evaluation of Texas Bridge Rails to Contain Buses and Trucks" (Hirsch, 1978). The equations with current notation are given below.

$$R_w = \frac{8M_b}{L_c - L_t/2} + \frac{8M_w H}{L_c - L_t/2} + \frac{M_c L_c^2}{H(L_c - L_t/2)}$$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_w H)}{M_c}}$$

$M_b$  = moment capacity of beam at top of wall, k ft

$M_c$  = flexural resistance of wall about horizontal axis, k ft/ft

$M_w$  = flexural resistance of wall about vertical axis, k ft/ft

$L_t$  = length of distribution of impact force, ft

$H$  = wall height, ft

Hirsch developed these equations by yield line analysis of the constant thickness parapet wall with a top beam shown in Figure 3.1. The parapet was analyzed using the yield line pattern shown in Figure 3.2.

### 3.2 VERIFICATION OF HIRSCH EQUATIONS

The first step in the development of equations for  $R_w$  and  $L_c$  is to calculate the external work. External work = deformed area times load. The deformed area is shown in Figure 3.3.

#### 1. Calculate external work

- a. Calculate  $x$  by proportion

$$x = \frac{\Delta(L_c - L_t)}{L_c}$$

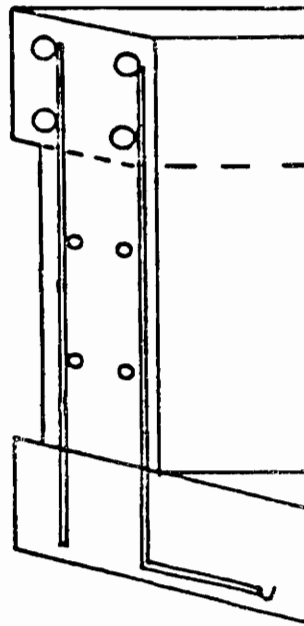
- b. Calculate deformed area under line load

$$\text{deformed area} = \frac{\Delta L_t(L_c - L_t / 2)}{L_c}$$

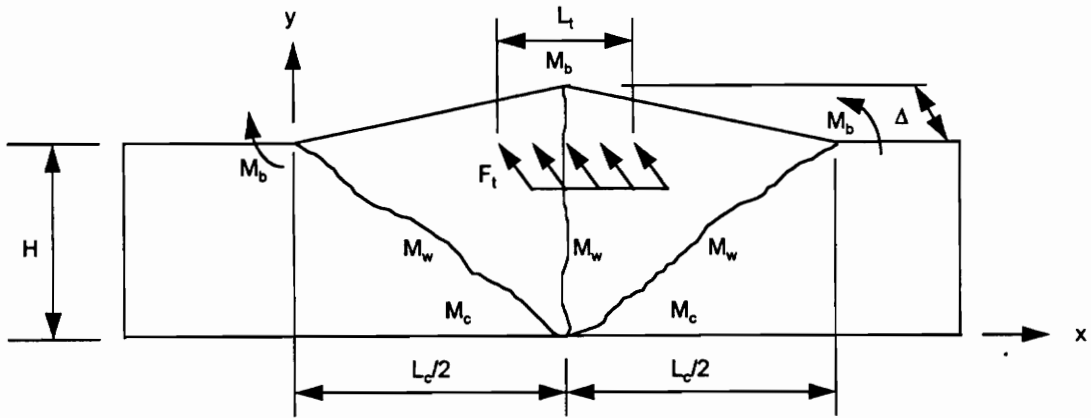
- c. Calculate work done by external forces

$$\text{External work} = \frac{F_t \Delta(L_c - L_t / 2)}{L_c}$$

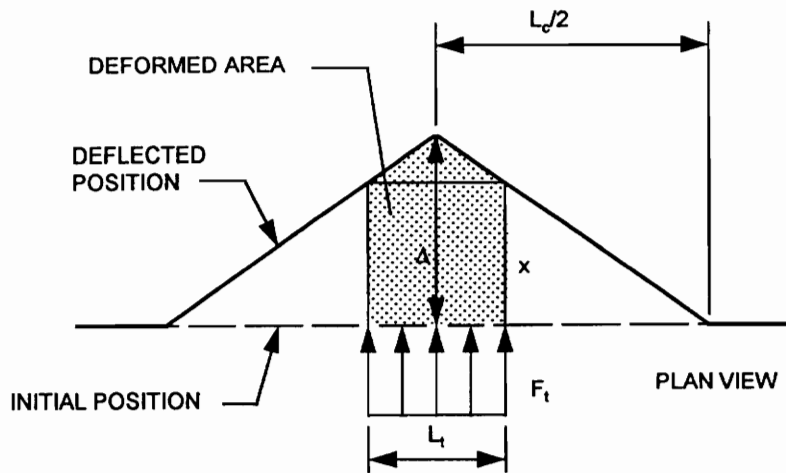
The second step is to calculate the internal work.



**FIGURE 3.1**  
**CONSTANT THICKNESS PARAPET WALL (Hirsch, 1978)**



**FIGURE 3.2**  
HIRSCH YIELD LINE PATTERN



**FIGURE 3.3**  
DEFORMED AREA

## 2. Calculate Internal Work

- a. Calculate internal work done by beam, see Figure 3.4.

$$\text{Beam internal work} = 4M_b\theta = \frac{8M_b\Delta}{L_c}$$

$$\text{(for small angles } \theta = \frac{\Delta}{L_c/2}\text{)}$$

- b. Calculate internal work done by horizontal wall reinforcement

$$\text{Internal work} = \frac{8M_wH\Delta}{L_c}$$

- c. Calculate internal work done by vertical wall reinforcement

$$\text{Internal work} = \frac{McL_c\Delta}{H}$$

d. Total internal work =  $\frac{8M_b\Delta}{L_c} + \frac{8M_wH\Delta}{L_c} + \frac{McL_c\Delta}{H}$

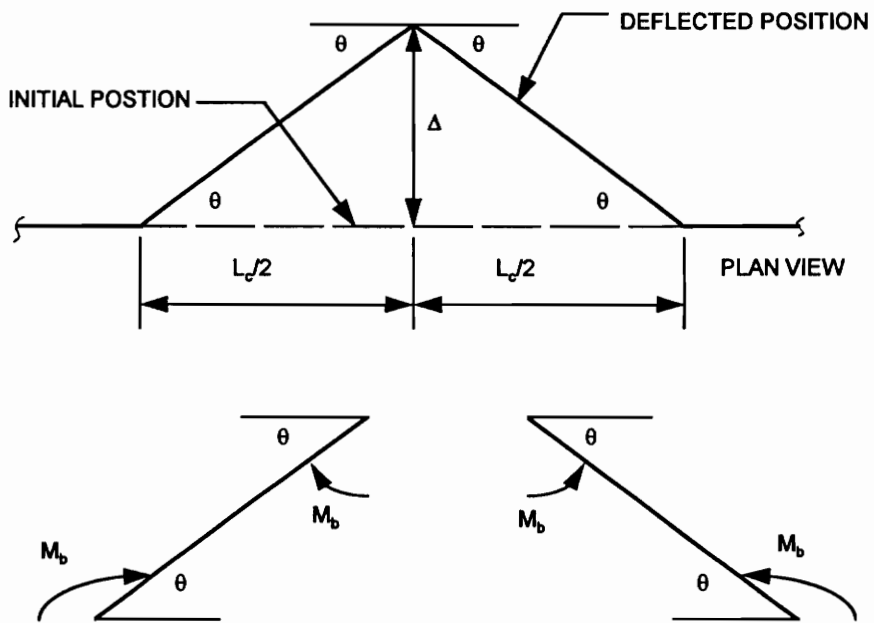
## 3. Solve For $F_t$ and $R_w$

- a. Equate external and internal work

$$\frac{F_t\Delta(L_c - L_t/2)}{L_c} = \frac{8M_b\Delta}{L_c} + \frac{8M_wH\Delta}{L_c} + \frac{McL_c\Delta}{H}$$

- b. Solve for  $F_t$

$$F_t = \frac{8M_b}{L_c - L_t/2} + \frac{8M_wH}{L_c - L_t/2} + \frac{McL_c^2}{H(L_c - L_t/2)}$$



**FIGURE 3.4**  
**BEAM INTERNAL WORK**

$$F_t \leq R_w$$

$$R_w = \frac{8M_b}{L_c - L_t/2} + \frac{8M_wH}{L_c - L_t/2} + \frac{M_c L_c^2}{H(L_c - L_t/2)}$$

The next step is to determine the length,  $L_c$ , that gives a minimum  $R_w$ . It is derived from  $R_w$  by taking  $dR_w/dL_c$  and setting it equal to zero.

#### 4. Solve For $L_c$

a. Take derivative  $\frac{d(R_w)}{dL_c} = 0$

$$\frac{d(R_w)}{dL_c} = L_c^2 - L_t L_c - H \frac{8M_b + 8M_wH}{M_c}$$

b. Solve the quadratic for  $L_c$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_wH)}{M_c}}$$

## CHAPTER 4

### ANALYSIS OF VARIABLE THICKNESS CONCRETE PARAPET WALL

#### 4.1 YIELD LINE ANALYSIS

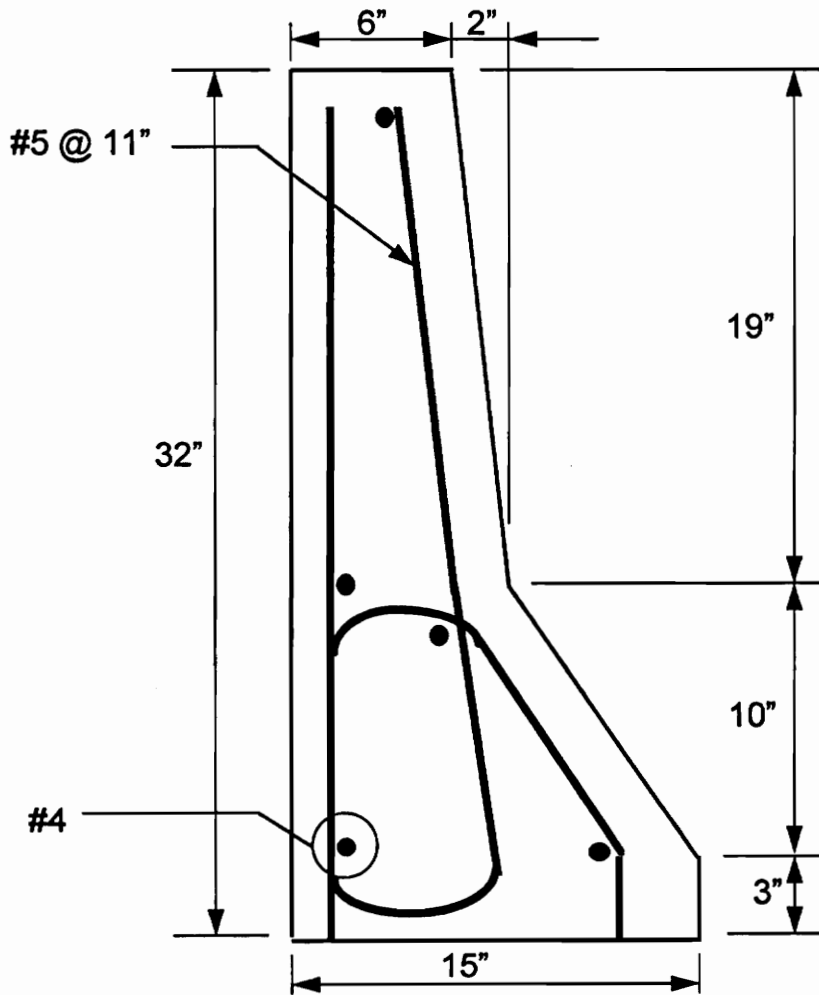
Equations for  $R_w$  (kips), total transverse resistance of a rail, and  $L_c$  (ft), the critical length of wall failure, are developed based on yield line analysis of the variable thickness AASHTO New Jersey concrete parapet wall shown in Figure 4.1. The parapet is reinforced with 60 ksi bars with a 2" clear cover and the concrete has a compression strength of 4500 psi. The yield line pattern analyzed is the same one that Hirsch (1978) used in his work and is shown in Figure 4.2. The first step in the analysis is to calculate the external work.

##### 4.1.1 External Work

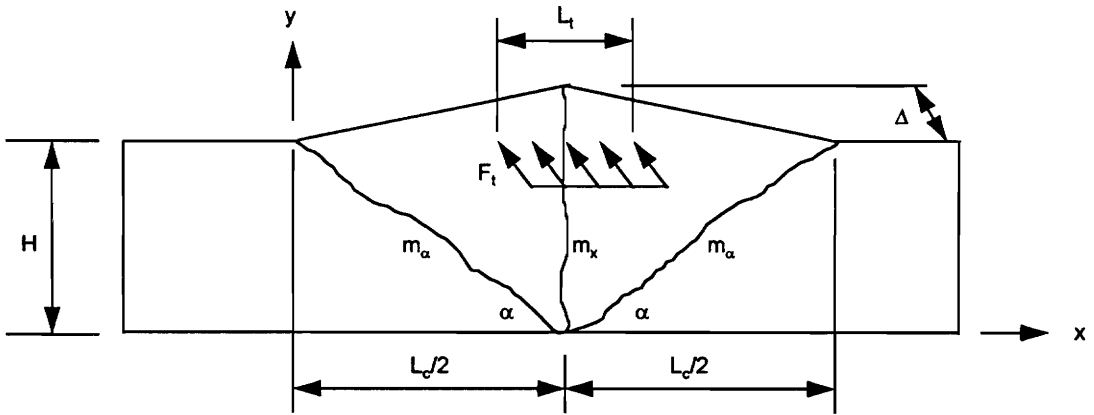
The external work is simply the deformed area times the load,  $F_t$ , per unit length,  $L_t$ . The deformed area and loading are shown in Figure 4.3.

$$\text{External Work} = \frac{F_t \Delta (L_c - L_t / 2)}{L_c} \quad (4-1)$$

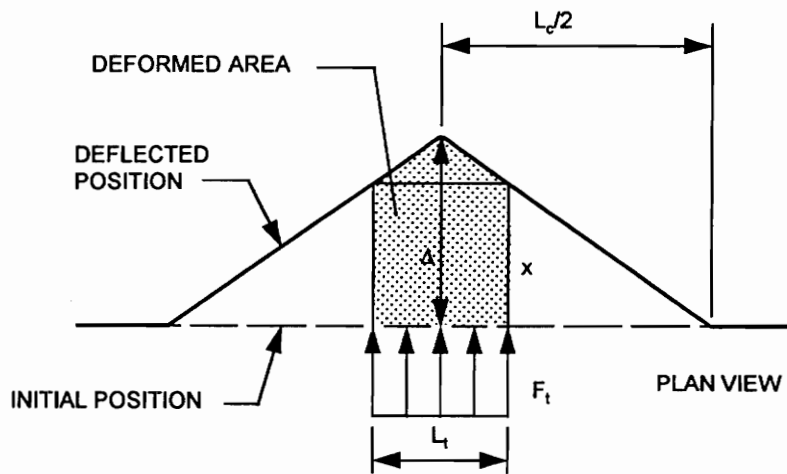
The next step in yield line analysis is to calculate the internal work done by the barrier as it deforms into the yield line pattern.



**FIGURE 4.1**  
**STANDARD AASHTO NEW JERSEY PARAPET WALL**



**FIGURE 4.2**  
YIELD LINE PATTERN



**FIGURE 4.3**  
**DEFORMED AREA AND LOADING**

### 4.1.2 Internal Work

To calculate the internal work, it is necessary to determine the moment resistance provided by the reinforcing in each direction. The moment capacity in negative bending (contoured face is in tension) provided by the y-direction (vertical) reinforcing is approximated by the equation  $m_y'(h)$ . (The moment capacity in positive bending in the vertical direction is not needed to analyze the given yield line pattern.) This equation is developed by calculating the moment capacity at various distances along the height of the wall, starting at a distance of 17.25 inches from the top of the wall (which is the development length for an epoxy coated #5 bar under the given conditions using AASHTO specifications). Using polynomial regression analysis, a parabolic curve is fitted to the data points (height, moment capacity) resulting in the equation  $m_y'(h)$  which approximates the moment resistance provided by the y-direction (vertical) reinforcing at a given height. Equation  $m_y'(h)$  is given below. The data points are given in Table 4.1.

$$m_y'(h) = 0.004h^2 + 0.888h - 1.02, \text{ in k/in} \quad (4-2)$$

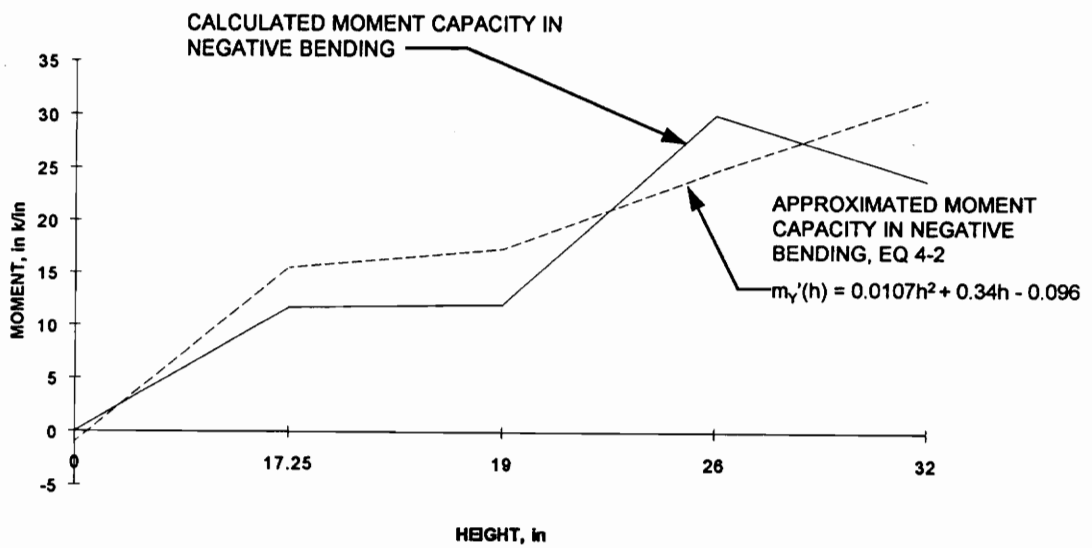
Shown in Figure 4.4 is a graph comparing the calculated moment capacity (vertical direction) vs height and the approximated moment capacity vs height.

The moment resistance for positive and negative bending provided by the x-direction (horizontal) reinforcing is calculated two ways. First, it is calculated as

**TABLE 4.1**

**CALCULATED MOMENT CAPACITY VS APPROXIMATED MOMENT  
CAPACITY (Y-DIRECTION REINFORCING)**

<b>HEIGHT, in</b>	<b>CALCULATED MOMENT CAPACITY, in k/ in</b>	<b>MOMENT CAPACITY APPROXIMATED BY EQ 4-2, in k/ in</b>
0	0	-1.02
17.25	11.72	15.488
19	12.03	17.296
23	25.95	21.52
26	30.00	24.77
29	34.10	28.096
32	23.87	31.49



**FIGURE 4.4**

CALCULATED MOMENT CAPACITY IN NEGATIVE BENDING PROVIDED BY Y-DIRECTION (VERTICAL) REINFORCEMENT VS THE APPROXIMATED MOMENT CAPACITY

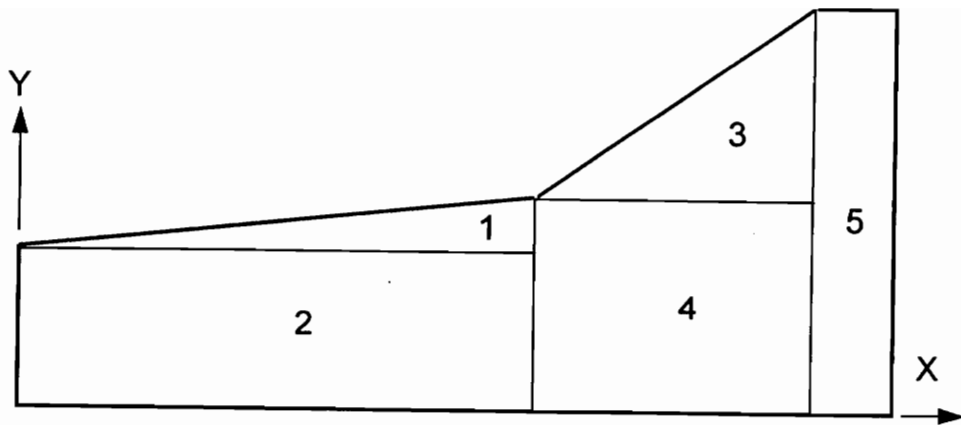
a constant value per parapet cross section ( $m_x, m_x'$ ) and secondly, as a function of the parapet height ( $m_x(h), m_x'(h)$ ).

In order to calculate the moment resistance,  $m_x, m_x'$ , provided by the x-direction (horizontal) reinforcing in positive and negative bending, the neutral axis for the parapet cross section must be located. Gere and Timoshenko (1984) state that the neutral axis of a cross section is the same as the principal centroidal axis if the moment vector is directed along one of the principal centroidal axes. Characteristics of principal centroidal axes are:

1. Moments of inertia are maximum and minimum for the axes
2. The product of inertia is zero
3. Axes of symmetry are principal axes

The origin of the principal centroidal axes is at the centroid of the cross section and the axes are oriented at the angle  $\theta_p$  determined by Equation 4-3.

The centroid,  $I_x, I_y,$  and  $I_{xy}$  of the parapet wall cross section are determined. The parapet wall cross-section, shown in Figure 4.5, is divided into 5 sections consisting of 3 rectangles and 2 triangles. The moments of inertia are calculated for each section and are then determined for the uncracked parapet wall cross section using parallel axis theorems. Equation 4-3 is used to calculate the orientation of the principal centroidal axes, angle  $\theta_p$ .



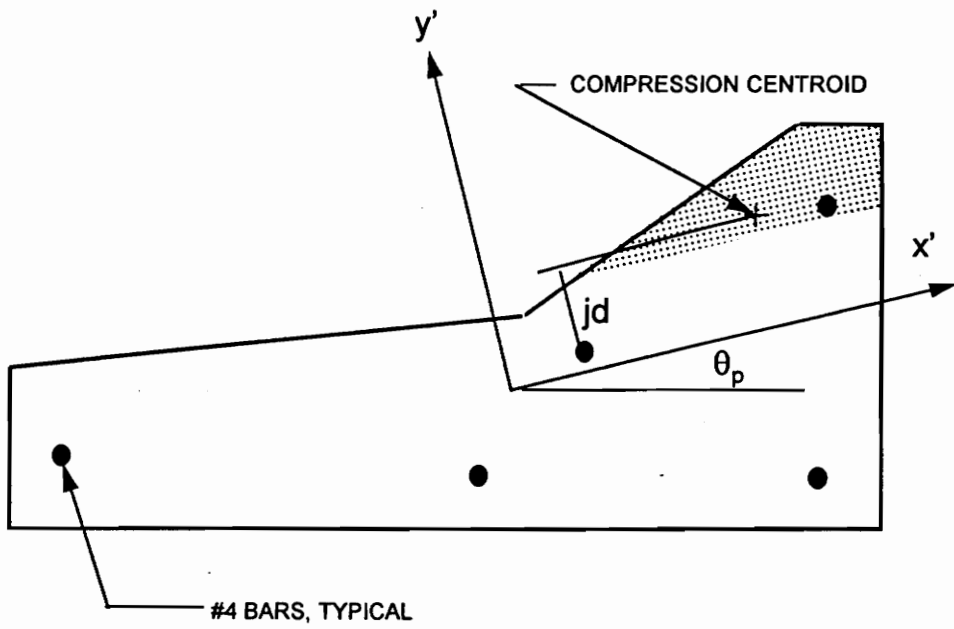
**FIGURE 4.5**  
**PARAPET WALL CROSS SECTION**

$$\tan 2\theta_p = \frac{-2I_{xy}}{I_x - I_y} \quad (4-3)$$

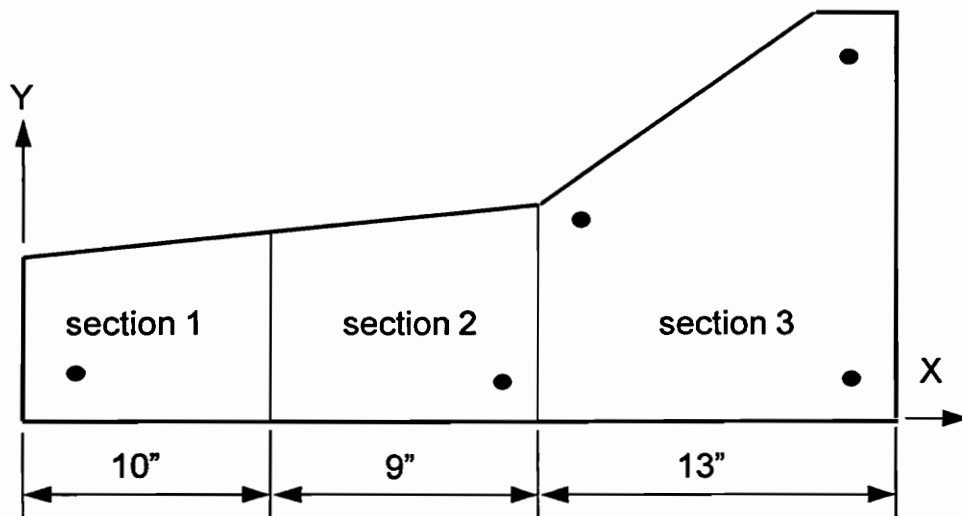
For purposes of calculating the moment capacity, the moment vector is assumed to be applied along the x'-principal centroidal axis and it is assumed that the neutral axis is parallel to the x'-principal centroidal axes. Using algebraic expressions, the moment of the concrete compression area that balances the moment of the tensile steel forces is determined. The moment capacity of the section is calculated by summing moments about the compression area centroid for each tensile steel force. The moment arm, jd, for the tensile forces is measured along the y'-principal centroidal axes. (See Figure 4.6.) The constant moment capacities provided by x-direction steel are given below.

$$m_x = 11.18 \text{ in k/in}, \quad m_x' = 10.94 \text{ in k/in}$$

The moment resistance in positive and negative bending provided by the x-direction (horizontal) reinforcing is also approximated as a function of the height. The parapet is divided into 3 sections shown in Figure 4.7. The sections are 10 inches, 9 inches and 13 inches long. The moment capacity provided by the x-direction bars is calculated for each section using the same methodology that was used to calculate the moment capacity of the entire parapet cross section. Data points consisting of the height at the middle of each section and the moment capacity of that section are fitted to parabolic curves,  $m_x(h)$  (positive bending) and  $m_x'(h)$  (negative bending). The data points are given in Table 4.2 and the equations are given below. Graphs of the moment equations are shown in Figure 4.8.



**FIGURE 4.6**  
PRINCIPAL CENTROIDAL AXES OF PARAPET WALL CROSS SECTION

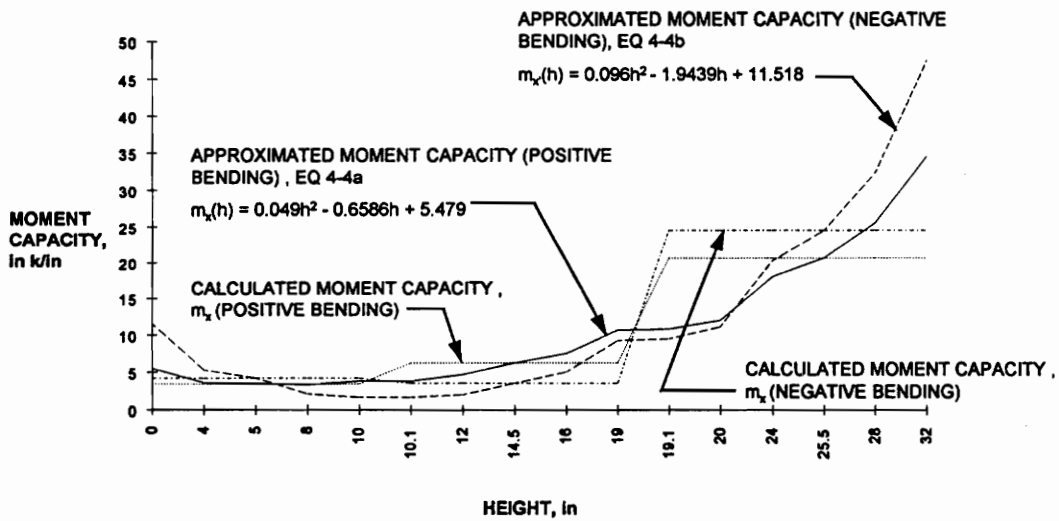


**FIGURE 4.7**

PARAPET WALL SECTIONS USED TO CALCULATE  $m_X(h)$ ,  $m_X'(h)$

**TABLE 4.2**  
**CALCULATED MOMENT CAPACITY VS APPROXIMATED MOMENT**  
**CAPACITY (X-DIRECTION REINFORCING)**

HEIGHT, in	CALCULATED MOMENT CAPACITY, in k/ in		MOMENT CAPACITY APPROXIMATED BY EQS 4-4a AND 4-4b, in k/ in	
	$m_x$	$m_x'$	$m_x$	$m_x'$
5	3.40	4.20	3.41	4.20
14.5	6.22	3.53	6.23	3.52
25.5	20.55	24.42	20.54	24.37



**FIGURE 4.8**

CALCULATED MOMENT CAPACITY IN POSITIVE,  $m_x$ , AND NEGATIVE,  $m_x'$ , BENDING PROVIDED BY X-DIRECTION (HORIZONTAL) REINFORCING VS APPROXIMATED MOMENT CAPACITY

$$m_x(h) = 0.0491h^2 - 0.6586h + 5.749 \text{ , in k/in} \quad (4-4a)$$

$$m_x'(h) = 0.096h^2 - 1.9439h + 11.518 \text{ , in k/in} \quad (4-4b)$$

The next step after approximating the moment resistance provided by the horizontal and vertical reinforcing is to calculate the internal work done by yield lines. The yield line pattern and its corresponding moment resistances are shown in Figure 4.2. The internal work is given by Equation 4-5.

$$\text{Internal work} = m_n l \theta_n \quad (4-5)$$

$m_n$  = normal moment/unit length along a yield line

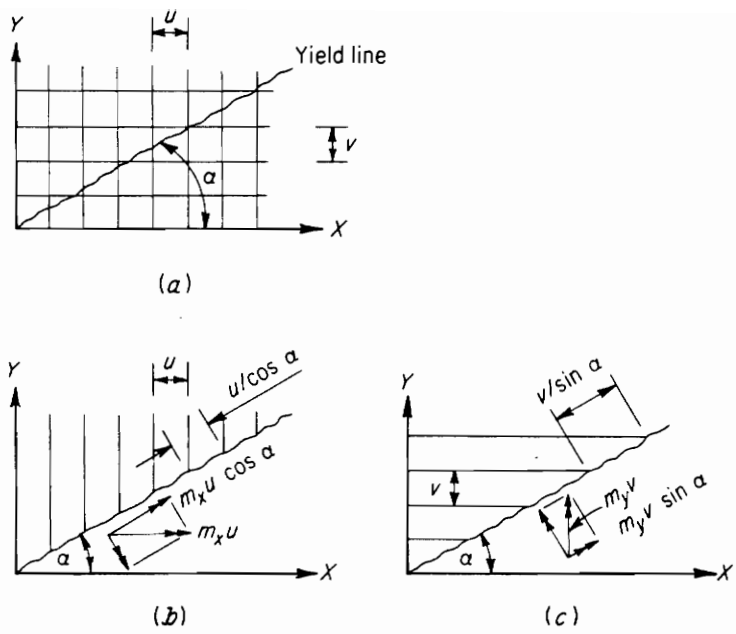
$\theta_n$  = normal rotation of the yield line

$l$  = length of the yield line (Jones and Wood, 1967)

The internal work for the vertical yield line is  $\theta \int_0^H m_x dh$ . However, the moment capacity,  $m_\alpha$ , per unit length along the diagonal line yield line is a combination of the moment resistance provided by both the vertical and horizontal reinforcing.

#### 4.1.2.1 Development of $m_\alpha$

Nilson and Winter (1991) derive an expression for  $m_\alpha$ . In Figure 4.9a is an orthogonal grid of reinforcement with a diagonal yield line at angle  $\alpha$ . The X direction bars are spaced at  $v$  unit length and provide moment resistance,  $m_y$ , per unit length about the Y axis. The Y direction bars are at unit spacing,  $u$ , and provide moment resistance,  $m_x$ , per unit length about the X axis.



**FIGURE 4.9**

DERIVATION OF  $m_\alpha$  (Nilson and Winter, 1991)

The resisting moment per Y direction bar about the X axis is  $m_x u$ . The component of  $m_x u$  along the diagonal yield line is  $m_x u \cos \alpha$ . See Figure 4.9b. The moment capacity provided by the Y direction bars along the diagonal yield line is

$$m_{\alpha y} = \frac{m_x u \cos \alpha}{u / \cos \alpha} = m_x \cos^2 \alpha \quad (4-6)$$

Similarly, the resisting moment per unit length along the  $\alpha$  axis provided by the X direction bar is calculated. See Figure 4.9c. It is

$$m_{\alpha x} = \frac{m_y v \sin \alpha}{v / \sin \alpha} = m_y \sin^2 \alpha \quad (4-7)$$

The resisting moment,  $m_{\alpha}$ , per unit length along the  $\alpha$  axis is the sum of  $m_{\alpha y}$  and  $m_{\alpha x}$ .

$$m_{\alpha} = m_x \cos^2 \alpha + m_y \sin^2 \alpha \quad (4-8)$$

For this study, due to the differences in notation,

$$m_{\alpha} = m_y'(h) \cos^2 \alpha + m_x'(h) \sin^2 \alpha \quad (4-9)$$

#### 4.1.2.2 Integration of $m_\alpha$

Since  $m_\alpha$  is a function of the variable  $h$  it is necessary to integrate along the diagonal yield line to calculate the internal work. The internal work for the diagonal yield line is

$$\frac{1}{e} \int_0^s m_\alpha ds \quad (4-10)$$

$\frac{1}{e}$  is the normal angle of rotation. See Figure 4.10 of yield line pattern defining  $e$  and  $s$ . The equation  $m_\alpha$  is parameterized to integrate over the distance  $s$ . The parametric representation of  $m_\alpha$  (using parameter  $t$ ) is

$$x = \frac{L_c}{2}t = g(t) \quad (4-11a)$$

$$y = Ht = h(t) \quad (4-11b)$$

( $y$  = variable 'h' in moment equations)

$$\frac{dx}{dt} = \frac{L_c}{2} = g'(t) \quad (4-12a)$$

$$\frac{dy}{dt} = H = h'(t) \quad (4-12b)$$

Equation  $m_\alpha$  is smooth since it's parametric representation is such that

$$x = g(t)$$

$$y = h(t)$$

Where  $a \leq t \leq b$  such that  $g'$  and  $h'$  are continuous and not simultaneously 0 on  $[a, b]$  (Swokowski, 1983). Since equation  $m_\alpha$  is smooth, the line integral of  $f$  along  $C$  from  $A$  to  $B$  is

$$\int_C f(x, y) ds \quad (4-13)$$

Where  $A$  and  $B$  are points on  $C$  determined by parameter values  $a$  and  $b$ . According to Theorem 18.10 (Swokowski, 1983), the line integral is equal to the following integrand

$$\int_C f(x, y) ds = \int_a^b f(g(t), h(t)) \sqrt{[g'(t)]^2 + [h'(t)]^2} dt \quad (4-14)$$

where  $a \leq t \leq b$ . The variables  $a$  and  $b$  are parameter  $t$  values at the beginning and end of the line integral, respectively.

The total internal work done by the yield line pattern in Figure 4.10 is

$$2 \left[ \theta \int_0^H m_x(h) + \frac{1}{e} \int_0^1 m_\alpha(t) \sqrt{\left(\frac{L_c}{2}\right)^2 + H^2} dt \right] \quad (4-15)$$



### 4.1.3 Equate External and Internal Work

The evaluated internal work, with  $H = 32$  inches, is

$$\frac{3175.42}{L_c} + 0.4538L_c \quad (4-16)$$

The next step is to equate the internal work to the external work, resulting in

$$F_t = \frac{0.4538L_c^2 + 3175.42}{L_c - \frac{L_t}{2}} = R_w, \text{ kips} \quad (4-17a)$$

or

$$R_w = \frac{0.4538L_c^2 + 3175.42}{L_c - \frac{L_t}{2}} \quad (4-17b)$$

Where  $L_c$  and  $L_t$  are in inches.

$\frac{dR_w}{dL_c} = 0$  gives the equation for  $L_c$ , the critical length of wall failure.

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 6997.40}, \text{ inches} \quad (4-18)$$

Equations 4-17 and 4-18 incorporate the varying moment capacities in both x and y-directions and are solely applicable to the standard AASHTO New Jersey parapet wall shown in Figure 4.1. Given below are equations developed in the same manner as the previous ones. However, these are based on constant

moment capacities in the x and y-directions instead of varying moment capacities. Equations 4-19 and 4-20 were developed using an average  $m_y'$ .

Where the average  $m_y' = \frac{\int_0^H m_y'(h) dh}{H} = 14.55$  in k/in.

$$R_w = \frac{0.4546L_c^2 + 1445.04}{L_c - \frac{L_t}{2}}, \text{ kips} \quad (4-19)$$

Where  $L_c$  and  $L_t$  are in inches.

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 3178.70}, \text{ inches} \quad (4-20)$$

Equations 4-21 and 4-22 were developed using  $m_y'$  calculated at the base of the parapet wall and not as an average value used in the previous two equations (which corresponds to Hirsch's  $M_c$ .)

$$F_t = \frac{0.7459L_c^2 + 1445.04}{L_c - \frac{L_t}{2}} = R_w, \text{ kips} \quad (4-21)$$

Where  $L_c$  and  $L_t$  are in inches.

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 1937.31}, \text{ inches} \quad (4-22)$$

## CHAPTER 5

### DISCUSSION

#### 5.1 EXAMPLES

In the following examples, the total transverse resistance of a rail,  $R_W$ , and the critical length of wall failure,  $L_C$ , are calculated for the standard AASHTO New Jersey parapet wall analyzed in this study and shown again in Figure 5.1.  $R_W$  and  $L_C$  will be determined using equations developed in this report as well as by those developed by T J Hirsch.

##### Example 1

$L_C$  and  $R_W$  calculated by Equations 4-17 and 4-18 based on varying moment capacities.

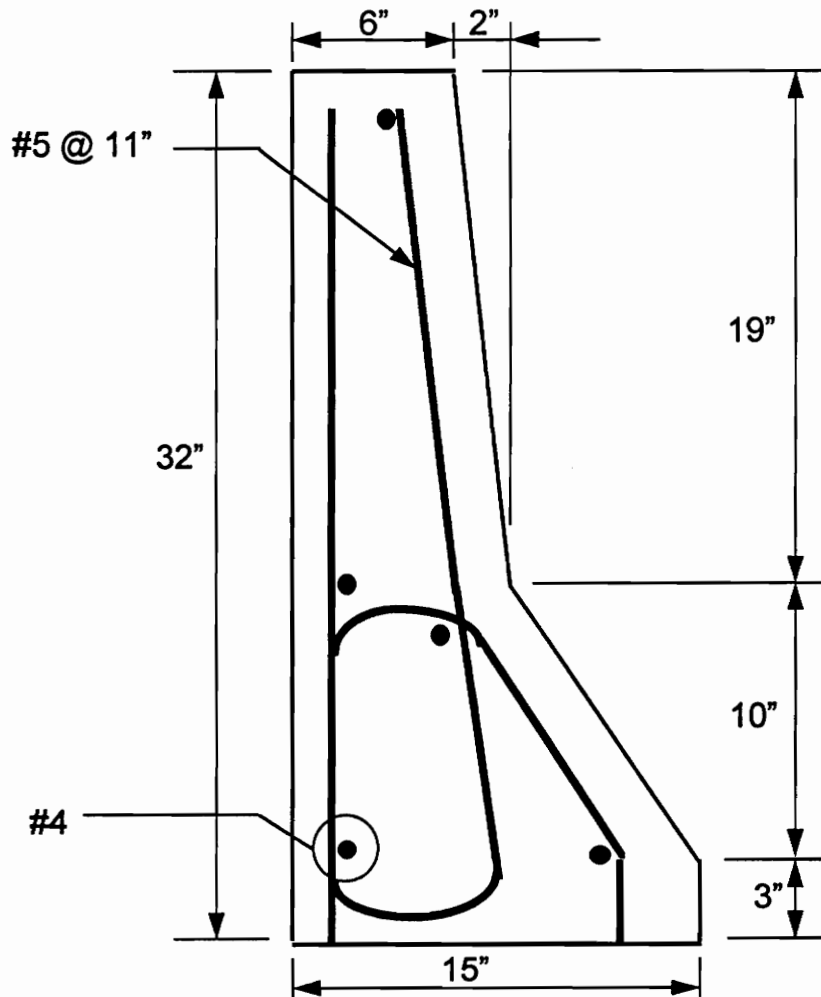
1. Determine  $L_C$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 6997.40}$$

$$L_t = 3.5' = 42'' \text{ (impact length for performance level 2)}$$

$$L_c = \frac{42}{2} + \sqrt{\left(\frac{42}{2}\right)^2 + 6997.40}$$

$$L_c = 21 + \sqrt{7438.4} = 107.246'' (8.937')$$



**FIGURE 5.1**

**AASHTO NEW JERSEY CONCRETE PARAPET WALL**

2. Determine  $R_w$

$$R_w = \frac{0.4538L_c^2 + 3175.42}{L_c - \frac{L_t}{2}}$$

$$R_w = \frac{0.4538(107.246)^2 + 3175.42}{107.246 - \frac{42}{2}}$$

$$R_w = \frac{8394.91}{86.246} = 97.33 \text{ kips}$$

### Example 2

$L_c$  and  $R_w$  calculated with Equations 4-19 and 4-20 based on constant moment capacities where  $m_y'$  is averaged.

1. Calculate  $L_c$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 3178.70}$$

$$L_c = \frac{42}{2} + \sqrt{\left(\frac{42}{2}\right)^2 + 3178.70}$$

$$L_c = 21 + \sqrt{3619.7} = 81.16'' (6.76')$$

2. Calculate  $R_w$

$$R_w = \frac{0.4546L_c^2 + 1445.04}{L_c - \frac{L_t}{2}}$$

$$R_w = \frac{0.4546(81.16)^2 + 1445.04}{81.16 - \frac{42}{2}}$$

$$R_w = \frac{4439.465}{60.16} = 73.79 \text{ kips}$$

### Example 3

$L_c$  and  $R_w$  determined using  $m_y'$  calculated at the base of the parapet wall (which corresponds to Hirsch's  $M_c$ ) and not an averaged value (Equations 4-21 and 4-22.)

1. Calculate  $L_c$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + 1937.31}$$

$$L_c = \frac{42}{2} + \sqrt{\left(\frac{42}{2}\right)^2 + 1937.31}$$

$$L_c = 21 + \sqrt{2378.31} = 69.767'' (5.81')$$

2. Calculate  $R_w$

$$R_w = \frac{0.7459L_c^2 + 1445.04}{L_c - \frac{L_t}{2}}$$

$$R_w = \frac{0.7459(69.767)^2 + 1445.04}{69.767 - \frac{42}{2}}$$

$$R_w = \frac{5075.66}{48.767} = 104.00 \text{ kips}$$

#### Example 4

$L_c$  and  $R_w$  calculated using Hirsch equations with an average value for  $M_c$ .

1. Calculate  $L_c$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_wH)}{M_c}}$$

$$L_c = \frac{3.5}{2} + \sqrt{\left(\frac{3.5}{2}\right)^2 + \frac{8(2.67)(0 + 11.06(2.67))}{14.55}}$$

$$L_c = 8.56'$$

2. Calculate  $R_w$

$$R_w = \frac{8M_b}{L_c - L_t/2} + \frac{8M_wH}{L_c - L_t/2} + \frac{M_c L_c^2}{H(L_c - L_t/2)}$$

$$R_w = \frac{8(0)}{8.56 - 3.5/2} + \frac{8(11.06)2.67}{8.56 - 3.5/2} + \frac{14.55(8.56)^2}{2.67(8.56 - 3.5/2)}$$

$$R_w = 93.3 \text{ kips}$$

### Example 5

$L_c$  and  $R_w$  calculated using Hirsch equations with  $M_c$  calculated at the base of the parapet wall and not averaged. (Note: This is the standard way to calculate  $M_c$ .)

1. Calculate  $L_c$

$$L_c = \frac{L_t}{2} + \sqrt{\left(\frac{L_t}{2}\right)^2 + \frac{8H(M_b + M_wH)}{M_c}}$$

$$L_c = \frac{3.5}{2} + \sqrt{\left(\frac{3.5}{2}\right)^2 + \frac{8(2.67)(0 + 11.06(2.67))}{23.87}}$$

$$L_c = 7.18'$$

2. Calculate  $R_w$

$$R_w = \frac{8M_b}{L_c - L_t/2} + \frac{8M_wH}{L_c - L_t/2} + \frac{M_c L_c^2}{H(L_c - L_t/2)}$$

$$R_w = \frac{8(0)}{7.18 - 3.5/2} + \frac{8(11.06)2.67}{7.18 - 3.5/2} + \frac{23.87(7.18)^2}{2.67(7.18 - 3.5/2)}$$

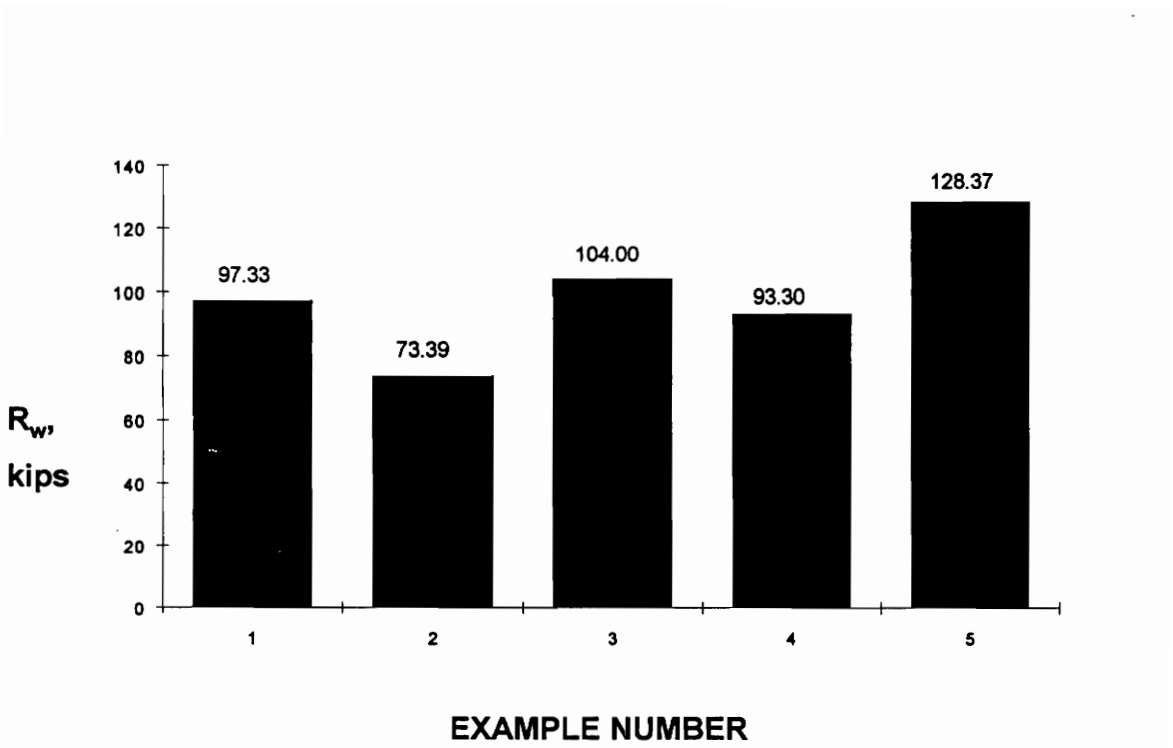
$$R_w = 128.37 \text{ kips}$$

## 5.2 COMPARISON OF RESULTS

The values for  $R_w$  and  $L_c$  calculated by the equations developed within and by Hirsch equations are given in Table 5.1. The equations based on variable moment capacities (Example 1) give a value for  $R_w$  that is 24% less than that calculated by Hirsch (Example 5.) However,  $R_w$  calculated in Example 1 is only 4% more than Hirsch values when an average value is used for  $M_c$  (Example 4.) The results from equations based on constant moment capacities (using an average value for  $m_y'$ , Example 2) give results 21% and 40% less than Hirsch equations in Examples 4 and 5, respectively. Equations (Example 3) based on constant moment capacities ( $M_c = m_y'$ ) give  $R_w$  values 19% less than Hirsch (Example 5) and 10% more than Hirsch equations with an average  $M_c$  (Example 4.) A graphical representation of these results is shown in Figure 5.2.

**TABLE 5.1**  
COMPARISON OF  $L_c$  AND  $R_w$

EXAMPLE NUMBER	EQUATION DESCRIPTION	EQUATION NUMBER	$L_c$ , ft	$R_w$ , kips
1	Based on variable moment capacities	4-17, 4-18	8.973	97.33
2	Based on constant moment capacities (average $m'_y$ )	4-19, 4-20	6.76	73.79
3	Based on constant moment capacities	4-21, 4-22	5.81	104.00
4	Hirsch equations (averaged $M_c$ )		8.56	93.30
5	Hirsch equations		7.18	128.37



**FIGURE 5.2**  
COMPARISON OF  $R_w$

## CHAPTER 6

### CONCLUSIONS AND RECOMMENDATIONS

#### 6.1 Conclusions and Recommendations

By comparing values for  $R_W$ , the total transverse resistance of a barrier rail, it is obvious that the Hirsch equations, used as intended (without an averaged  $M_C$ .) over estimate  $R_W$ . This is because intrinsically the Hirsch equations assume that  $M_C$ , the flexural resistance of the wall about horizontal axis, ( $m_y'$  in this study) is constant. However, for the New Jersey parapet wall analyzed this value is much greater at the base (where  $M_C$  is calculated) than it is for the rest of the wall. This results in an inflated value for  $R_W$  which is nonconservative.

#### 6.2 RECOMMENDATIONS

When an average value for  $M_C$  is used in Hirsch equations the results are very close (4% less) to those determined by the equations based on variable moment capacities which are the most accurate of the equations developed. Based on this fact, if  $M_C$  varies significantly along the height of a parapet wall, it is recommended that an average value for  $M_C$  be used in the Hirsch equations instead of  $M_C$  calculated at the base of the parapet wall.

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