

# **Error Visualization in Comparison of B-Spline Surfaces**

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(ABSTRACT)

Geometric trimming of surfaces results in a new mathematical description of the matching surface. This matching surface is required to closely resemble the remaining portion of the original surface. Typically, the approximation error in such cases is measured with a view to minimize it. The data associated with the error between two matching surfaces is large and needs to be filtered into meaningful information. This research looks at suitable norms for achieving this data reduction or abstraction with a view to provide quantitative feedback about the approximation error. Also, the differences between geometric shapes are easily discerned by the human eye but are difficult to characterize or describe. Error visualization tools have been developed to provide effective visual inputs that the designer can interpret into meaningful information.

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