

**Appendix A**

Procedure for Calculating Cavity Expansion Pressure  
(Yu and Houlsby's Model)

This procedure is abstracted from the paper by Yu and Houlsby.

- (i) Calculate initial stress  $p_0$  that is the pressure corresponding to the initial cavity with radius  $a_0$ .
- (ii) For a given cavity pressure  $p$ , greater than initial yielding cavity pressure  $p_1$ , calculate  $R$  from the following equation.

$$R = \frac{(m + \alpha)[Y + (\alpha - 1) \cdot p]}{\alpha(1 + m)[Y + (\alpha - 1) \cdot p_0]}$$

where,  $m = 1$  (cylindrical cavity) &  $2$  (spherical cavity),  $\alpha = \frac{1 + \sin \phi}{1 - \sin \phi}$ ,  $Y = \frac{2 \cdot c \cdot \cos \phi}{1 - \sin \phi}$

- (iii) Evaluate  $\Lambda_1$  from the equation – only a few terms are sufficient.

$$\Lambda_1(x, y) = \sum_{n=0}^{\infty} A_n^1$$

where,  $A_n^1 = \frac{y^n}{n!} \ln x$ , if  $n = \gamma$  ( $\gamma = \frac{\alpha(\beta + m)}{m(\alpha - 1)\beta}$ ,  $\beta = \frac{1 + \sin \psi}{1 - \sin \psi}$ )

$$A_n^1 = \frac{y^n}{n!(n - \gamma)} [x^{n-\gamma} - 1], \text{ otherwise}$$

- (iv) Evaluate  $a/a_0$  from the following and from that the cavity pressure can be determined.

$$\frac{a}{a_0} = \left\{ \frac{R^{-\gamma}}{(1 - \delta)^{(\beta+m)/\beta} - (\gamma/\eta)\Lambda_1(R, \xi)} \right\}^{\beta/(\beta+m)}$$

where,  $\delta = \frac{Y + (\alpha - 1)p_0}{2(m + \alpha)G}$ , ( $G = \frac{E}{2(1 + \nu)}$ )

$$\eta = \exp\left\{ \frac{(\beta + m) \cdot (1 - 2\nu) \cdot [Y + (\alpha - 1) \cdot p_0] \cdot [1 + (2 - m) \cdot \nu]}{E \cdot (\alpha - 1) \cdot \beta} \right\},$$

$$\xi = \frac{[1 - \nu^2 \cdot (2 - m)] \cdot (1 + m) \cdot \delta}{(1 + \nu) \cdot (\alpha - 1) \cdot \beta} \times \left[ \alpha \cdot \beta + m \cdot (1 - 2\nu) + 2\nu - \frac{m \cdot \nu \cdot (\alpha + \beta)}{1 - \nu \cdot (2 - m)} \right]$$