subject to [constraints (5.26e) - (5.26k)]. (5.27b)

Given any $\lambda = (\lambda^1, \lambda^2, \lambda^3)$, where $\lambda^1 \geq 0$, $\lambda^2 \geq 0$, we can evaluate $v(\lambda) = \nu(\lambda^1, \lambda^2, \lambda^3)$ as described in the next section. Let $(\bar{w}, \bar{\alpha}, \bar{x}, \bar{y}, \bar{\theta}, \bar{\phi}, \bar{\gamma})$ solve this problem. Then the components of the subgradient $g = (g^1, g^2, g^3)$ of ν at this λ, corresponding to the components of the associated Lagrange multiplier vector $(\lambda^1, \lambda^2, \lambda^3)$ are given in Table 7, where we need to substitute $(w, \alpha, x, y, \theta, \phi, \gamma) \equiv (\bar{w}, \bar{\alpha}, \bar{x}, \bar{y}, \bar{\theta}, \bar{\phi}, \bar{\gamma})$.

Table 7. Lagrangian Dual Variables and their Corresponding Subgradient Components.

Lagrangian Dual Variable	Associated Constraint	Subgradient Component
λ_{ij}^{111}	l.h.s. of inequality (5.25b)	$u_{\alpha_{ij}} w_{ij} + \alpha_{ij} u_{ij} - u_{\alpha_{ij}} u_{ij} - \gamma_{ij}$
λ_{ij}^{112}	r.h.s. of inequality (5.25b)	$\gamma_{ij} - u_{\alpha_{ij}} w_{ij} - \alpha_{ij} l_{ij} + u_{\alpha_{ij}} l_{ij}$
λ^{13}	inequality (5.25c)	$\nu^* - \sum_i \sum_j c_{ij} w_{ij}^* \alpha_{ij}$
λ^{14}	inequality (5.25d)	$\nu_1 - \sum_i \sum_j \alpha_{ij}$
λ_{ij}^{151}	1st l.h.s. inequality of (5.25f)	$\theta_{ij} - w_{ij} a_j + l_{ij}(\alpha_{ij} - x_i + a_j) - \gamma_{ij}$
λ_{ij}^{152}	2nd l.h.s. inequality of (5.25f)	$w_{ij} a_j - \theta_{ij} + l_{ij}(\alpha_{ij} + x_i - a_j) - \gamma_{ij}$
λ_{ij}^{153}	3rd l.h.s. inequality of (5.25f)	$\phi_{ij} - w_{ij} b_j + l_{ij}(\alpha_{ij} - y_i + b_j) - \gamma_{ij}$
λ_{ij}^{154}	4th l.h.s. inequality of (5.25f)	$w_{ij} b_j - \phi_{ij} + l_{ij}(\alpha_{ij} + y_i - b_j) - \gamma_{ij}$
λ_{ij}^{155}	1st r.h.s. inequality of (5.25f)	$\gamma_{ij} - \theta_{ij} + w_{ij} a_j - u_{ij}(\alpha_{ij} - x_i + a_j)$
λ_{ij}^{156}	2nd r.h.s. inequality of (5.25f)	$\gamma_{ij} - w_{ij} a_j + \theta_{ij} - u_{ij}(\alpha_{ij} + x_i - a_j)$
λ_{ij}^{157}	3rd r.h.s. inequality of (5.25f)	$\gamma_{ij} - \phi_{ij} + w_{ij} b_j - u_{ij}(\alpha_{ij} - y_i + b_j)$
λ_{ij}^{158}	4th r.h.s. inequality of (5.25f)	$\gamma_{ij} - w_{ij} b_j + \phi_{ij} - u_{ij}(\alpha_{ij} + y_i - b_j)$
λ_{ijk}^{161}	1st l.h.s. inequality of (5.25g)	$\theta_{ij} + \phi_{ij} - w_{ij}(a_j + b_j) + l_{ij}(\sqrt{2}\,\alpha_{ij} - x_i + a_j - y_i + b_j) - \sqrt{2}\,\gamma_{ij}$

λ^{162}_{ijk}	2nd l.h.s. inequality of (5.25g)	$\theta_{ij}-\phi_{ij}+w_i(-a_j+b_j)+l_{ij}(\sqrt{2}\,\alpha_{ij}-x_i+a_j+y_i-b_j)-\sqrt{2}\,\gamma_j$
λ^{163}_{ij}	3rd l.h.s. inequality of (5.25g)	$\phi_{ij}-\theta_{ij}+w_{ij}(a_j-b_j)+l_{ij}(\sqrt{2}\,\alpha_{ij}+x_i-a_j-y_i+b_j)-\sqrt{2}\,\gamma_j$
λ^{164}_{ij}	4th l.h.s. inequality of (5.25g)	$-\phi_{ij}-\theta_{ij}+w_{ij}(a_j+b_j)+l_{ij}(\sqrt{2}\,\alpha_{ij}+x_i-a_j+y_i-b_j)-\sqrt{2}\,\gamma_j$
λ^{165}_{ij}	1st r.h.s. inequality of (5.25g)	$\sqrt{2}\,\gamma_j-\theta_{ij}-\phi_{ij}+w_{ij}(a_j+b_j)-u_{ij}(\sqrt{2}\,\alpha_{ij}-x_i+a_j-y_i+b_j)$
λ^{166}_{ij}	2nd r.h.s. inequality of (5.25g)	$\sqrt{2}\,\gamma_j-\theta_{ij}+\phi_{ij}-w_{ij}(-a_j+b_j)-u_{ij}(\sqrt{2}\,\alpha_{ij}-x_i+a_j+y_i-b_j)$
λ^{167}_{ij}	3rd r.h.s. inequality of (5.25g)	$\sqrt{2}\,\gamma_j-\phi_{ij}+\theta_{ij}-w_{ij}(a_j-b_j)-u_{ij}(\sqrt{2}\,\alpha_{ij}+x_i-a_j-y_i+b_j)$
λ^{168}_{ij}	4th r.h.s. inequality of (5.25g)	$\sqrt{2}\,\gamma_j+\phi_{ij}+\theta_{ij}-w_{ij}(a_j+b_j)-u_{ij}(\sqrt{2}\,\alpha_{ij}+x_i-a_j+y_i-b_j)$
λ^{171}_{ijk}	l.h.s. inequality of (5.25j)	‘ $w_{ij}\Psi_{0k}-u_{ij}(\Psi_{0k}-\Psi_{1k}x_i-\Psi_{2k}y_i)-\Psi_{1k}\theta_{ij}-\Psi_{2k}\phi_{ij}$
λ^{172}_{ijk}	r.h.s. inequality of (5.25j)	$-w_{ij}\Psi_{0k}+l_{ij}(\Psi_{0k}-\Psi_{1k}x_i-\Psi_{2k}y_i)+\Psi_{1k}\theta_{ij}+\Psi_{2k}\phi_{ij}$
λ^{2}_{ij}	constraint (5.25k)	$(x_i-a_j)^2+(y_i-b_j)^2-u\alpha_{ij}\alpha_{ij}$
λ^{3}_{j}	constraint (5.25l)	$d_j-\sum_i w_{ij}$

5.2.1 Evaluation of the Lagrangian Dual Objective Function.

5.2.1.A. Form of the Objective Function ν (λ)

The objective function in (5.27) can be written in the following form.

$$\lambda^1 h+\lambda^3 d+\min\,[\,\bar{c}_w w+\bar{c}_\theta\theta+\bar{c}_\phi\phi+\bar{c}_\gamma\gamma+\bar{c}_x x+\bar{c}_y y+\bar{c}_\alpha\alpha$$
$$+\sum_i\sum_j\lambda^2_{ij}\,[(x_i-a_j)^2+(y_i-b_j)^2] \qquad (5.28)$$

where the coefficients $\bar{c}_w$, $\bar{c}_\theta$, $\bar{c}_\phi$, $\bar{c}_\gamma$, $\bar{c}_x$, $\bar{c}_y$, and $\bar{c}_\alpha$ are given in Table 8.