Chapter VI

Conclusions and Recommendations for Future Research

6.1 Summary and Conclusions

In this dissertation, we have been concerned with Euclidean location and capacitated location-allocation problems, denoted respectively as EMFLP and EDLAP. Both problems are plagued by the nondifferentiability of the objective function that precludes the use of many first and second-order derivative based nonlinear programming approaches. Moreover, the location-allocation problem poses a further challenge in that it is nonconvex and possesses local minima that differ from global optimal solutions. For the pure location problem, we have characterized the entire subdifferential (set of subgradients) and have developed two new approaches, both of which have been demonstrated to be competitive with the popular hyberboloid approximation procedure (HAP). The first of these overcomes the nondifferentiability handicap of the problem by reformulating it as an equivalent smooth, convex program. Two such formulations are developed and explored computationally. The first of these works directly in the primal space, while the second applies Lagrangian duality in concert with a special construct that involves the optimization of a linear function over a unit ball. In fact, the latter approach recovers Francis and Cabot's dual to the Euclidean location problem, hence providing a new derivation of this dual problem. Moreover, this approach settles an open question in the literature since 1972 regarding the recovery of a primal optimum from the solution to the dual problem by showing that a set of optimal Lagrange multipliers for this dual problem exist and that the entire set of such multipliers coincides with the set of optimal solutions to EMFLP. Using these two smooth reformulations, any standard nonlinear convex programming software (that is dual adequate) can be used to derive optimal solutions to EMFLP. We also explore theoretical results concerning the set of Karush-Kuhn-Tuker (KKT) solutions for these reformulations and their optimal solutions. We show via proof and counterexamples that while every KKT point is an optimal solution to EMFLP, the converse is not necessarily true, except under some established additional conditions. This provides insights into the anticipated performance of using standard software based on reduced gradient and Augmented Lagrangian penalty function approaches to solve these smooth reformulations. Actually, using trignometric functions, we derive a third equivalent smooth reformulation of EMFLP for which every optimal solution to EMFLP is indeed a KKT point for this problem. However, this third reformulation turns out to be nonconvex, and could possess non-optimal KKT solutions. Hence, it only serves to demonstrate that one can indeed capture the optimal solution set of EMFLP within the KKT solutions of an equivalent smooth reformulation of this problem, but unlike the foregoing two reformulations, we do not recommend this reformulation as a practical solution approach for EMFLP.

The second approach developed for solving EMFLP directly addresses the nondifferentiability issue by designing a conjugate or deflected subgradient method to be used in concert with specialized line-search procedures. There is considerable flexibility in designing such an approach and we have experimented with several possible strategies. Based on our results, we have proposed for implementation a particular variant of a variable target value conjugate subgradient approach that uses a combination of two types of subgradients at nondifferentiable points in order to promote the generation of good descent directions, and that uses a specialized inexact Newton based line-search technique whenever a descent direction is detected. This composite method is shown to be very efficient, numerically robust, and provides very accurate solutions with minimal effort.

Finally, we have dealt with the challenging nondifferentiable, nonconvex capacitated Euclidean location-allocation problem for which we have developed a first-ever global optimization method that has been implemented and tested. At the heart of this method is a lower bounding, smooth, convex relaxation that provides a tight outer approximation to the nonconvex problem. This relaxation is derived via a specialized design of the Reformulation-Linearization Technique (RLT) of Sherali and Tuncbilek

(1992), and is augmented by certain cut-set based inequalities in the allocation-space, and projected objective function based cuts derived in the location-space. An alternative projected location-space lower bound is also derived and is tested by itself as well as in conjunction with the RLT-based lower bound. These lower bounding techniques are embedded within a branch-and-bound framework along with suitable upper bounding and partitioning strategies that are proven to induce finite convergence to a global optimum for this problem. The developed methodology employs a special Lagrangian dual approach along with a conjugate subgradient technique to effectively solve the lower bounding RLT relaxation. This requires an elaborate, numerically stable implementation. We have applied this method to several randomly generated test problems. The results indicate that the proposed method offers a very promising, viable approach for determining global optima for this challenging class of problems. In particular, for two standard problems in the literature, our algorithm has discovered significantly improved solutions over those reported in the literature, solving these problems to global optimality for the first time.

6.2 Recommendations for Future Research

There are several investigations that we can pursue for further research based on the developed theory and algorithms in this dissertation.

In the context of the pure location problem, we have developed some equivalent smooth convex reformulations to which standard (dual adequate) convex nonlinear programming software can be applied. However, these generic methods fail to recognize and exploit the inherent special structures. One can develop more effective, robust, and numerically accurate specialized procedures that yield an improved performance by exploiting such special structures.

For the capacitated Euclidean location-allocation problem, we have developed tight relaxations using the RLT approach along with other valid inequalities. This can be enhanced in three ways. First, other types of cutting planes can be generated to further augment this relaxation (see Selim, 1979, and Sherali and Shetty, 1980, for example).

Second, as an alternative to the Lagrangian dual approach that we have designed to solve these RLT relaxations, other solution techniques could be explored, including the use of commercial software used as callable subroutines within the overall program. Furthermore, other lower bounding techniques can also be investigated. There is a need to enhance the solution capability of this class of challenging problem, and we hope that this work encourages further research along these lines. Third, the model itself can be extended to consider interactions between new facilities, and the methodology can be accordingly modified.

Finally, we suggest an extension of the developed methodology to solve the capacitated l_p distance location-allocation problem. In practice, neither the l_1 nor the l_2 norm is very accurate in measuring geographical distances (except in special cases), and empirically determined l_p norms are usually more appropriate metrics. The RLT method can be modified using the new results developed by Sherali (1996) to design suitable relaxations for this problem. Other constraints derived in Chapter 5, as well as the aforementioned extensions for the Euclidean location-allocation problem, can be brought to bear upon this class of problems for which no viable solution technique exists in the literature.