APPENDIX A

SAMPLE CALCULATIONS

Example Calculation of Predicted Natural Frequency

Given: Spring constant, $k = 395\ lb/in.$

Calculate:

Effective Vertical Floor Mass Weight = $W_{eff,\,vert}$

$$W_{eff,\,vert} = W_{blocks} + W_{slab}$$

$$W_{eff,vert} = 9(60\ lb) + 150\ {}^{lb}\!/_{ft^3}\ (12\ ft)(12\ ft)\left(5.5\ in./12\ {}^{in.}\!/_{ft}\right)\left(\frac{1\ air\ spring}{4\ total\ air\ springs}\right)$$

$W_{eff,\,vert} = 3015\ lb$

$m = W_{eff,\,vert} / g$

Predicted Natural Frequency = $f_{n,pred}$

$$f_{n,\,pred} = \frac{1}{2p}\sqrt{k/m}$$

$$f_{n,\,pred} = \frac{1}{2p}\sqrt{\frac{395\ {}^{lb}\!/_{in.}}{3015\ lb/386.4\ {}^{in.}\!/_{s^2}}}$$

$f_{n,\,pred} = 1.13\ hz$

Example Calculation of Predicted Natural Frequency for Corner Rotation Mode of Oscillation (Approximate Method)

Given: Axis of rotation illustrated in Figure 2.7 and discussion of Equations (2.2)-(2.4) in Chapter 2.

Calculate:

$$W_{eff,vert} = W_b + \frac{wL_s^{\ 2}}{2}$$

$W_b = 9(60\ lb.) = 540\ lb$

$$w = 150\ {}^{lb}\!/_{ft^3}\left(\frac{5.5 in.}{12\ {}^{in.}\!/_{ft}}\right) = 68.75 psf$$

$L_s = 12.0\ ft$

$$W_{eff,vert} = 540\ lb + \frac{(68.75 psf)(12.0\,ft)^2}{2}$$

$$W_{eff,vert} = 5490\ lb$$

$$W_{eff,rotate} = W_b + wL\int_0^L y^2 dx$$

$y = \dfrac{x}{aL}$ = *modal displacement of effective section*

x = distance from axis of rotation to spring location (corner of slab)

a =(distance from axis of rotation to air spring) / (length of effective section)

L = length of effective section

$$W_{eff,rotate} = W_b + wL\int_0^L \left(\frac{x}{aL}\right)^2 dx$$

$$W_{eff,rotate} = W_b + \left[\frac{wL}{a^2L^2}\right]\left[\frac{x^3}{3}\right]_0^L = W_b + \frac{wL^2}{3a^2}$$

$$a = \frac{94\ in.}{102\ in.} = 0.922$$

L = 8.5 ft

$$W_{eff,rotate} = 540\ lb + \frac{(68.75\ psf)(8.5\ ft)^2}{3\ (0.922)^2}$$

$$W_{eff,rotate} = 2488\ lb$$

$$f_{n,rotate} = f_{n,vert}\sqrt{W_{eff,vert}/W_{eff,rotate}}$$

$$f_{n,vert} = 1.13\ hz$$

$$f_{n,rotate} = (1.13\ hz)\sqrt{5490\ lb/2488\ lb}$$

$$f_{n,rotate} = 1.68\ hz$$

Example Calculation of Predicted Natural Frequency for Side Rotation Mode of Oscillation (Approximate Method)

Given: Axis of rotation illustrated in Figure 2.8 and discussion of equations (2.2)-(2.4) in Chapter 2.

Calculate:

$$W_{eff,vert} = W_b + wL_s^{\ 2}$$

W_b = 9(60 lb) = 540 lb

$$w = 150\ {}^{lb}/_{ft^3}\left(\frac{5.5\ in.}{12\ {}^{in.}/_{ft}}\right) = 68.75\ psf$$

L_s = 6.0 ft

$$W_{eff,vert} = 540\ lb + (68.75\ psf)(6.0\ ft)^2$$

$$W_{eff,vert} = 3015\ lb$$

$$W_{eff,rotate} = W_b + wL\int_0^L y^2 dx$$

$$y = \frac{x}{aL}$$

x = distance from axis of rotation to spring location (side of slab)
a=(distance from axis of rotation to air spring) / (length of effective section)
L = length of effective section

$$W_{eff,rotate} = W_b + wL\int_0^L \left(\frac{x}{aL}\right)^2 dx$$

$$W_{eff,rotate} = W_b + \left[\frac{wL}{a^2 L^2}\right]\left[\frac{x^3}{3}\right]_0^L = W_b + \frac{wL^2}{3a^2}$$

$$a = \frac{65\ in.}{72\ in.} = 0.903$$

L = 6.0 ft

$$W_{eff,rotate} = 540\ lb + \frac{(68.75\ psf)(6.0\ ft)^2}{3\ (0.922)^2}$$

$$W_{eff,rotate} = 1552\ lb$$

$$f_{n,rotate} = f_{n,vert}\sqrt{W_{eff,vert} / W_{eff,rotate}}$$

$$f_{n,vert} = 1.13\ hz$$

$$f_{n,rotate} = (1.13\ hz)\sqrt{3015\ lb / 1552\ lb}$$

$$f_{n,rotate} = 1.57\ hz$$

Example Calculation of Predicted Natural Frequency for Corner Rotation Mode of Oscillation (Exact Method)

Given: Axis of rotation illustrated in Figure 2.7 and Equations (2.5) - (2.9) in Chapter 2.

Calculate: Using the parallel axis theorem, calculate the mass moment of inertia of the blocks added to the corner of the floor slab:

$$I_{blocks} = \bar{I} + md^2$$

$\bar{I} = 0$ (relative to total I_{blocks})

$$m = \frac{2\ (9)\ (60\ lb)}{32.2\ ft/s^2} = 33.54\ \frac{lb - s^2}{ft}$$

$$d = 8.485\text{ ft} - \frac{9.9\text{ in.}}{12\ \text{in.}/\text{ft}} = 7.66\text{ ft}$$

$$I_{blocks} = 33.54\ \frac{\text{lb - s}^2}{\text{ft}}\ (7.66\text{ ft})^2$$

$$I_{blocks} = 1968\text{ lb - ft - s}^2$$

$$I_{slab} = \bar{I} + md^2$$

d = 0 (slab rotates about centroidal axis)

$$\bar{I}_{slab} = \int y^2\, dm = r\, t \int y^2\, dA = r\, t\, I_x$$

$$r = 150\ \text{lb}/\text{ft}^3 \left(\frac{1\text{ s}^2}{32.2\text{ ft}}\right)$$

$$t = \frac{5.5\text{ in.}}{12\ \text{in.}/\text{ft}} = 0.458\text{ ft}$$

$$I_x = \frac{1}{12}\ bh^3$$

$$I_x = 2\left[\frac{1}{12}\ (2)(8.485\text{ ft})(8.485\text{ ft})^3\right] = 1728\text{ ft}^4$$

$$I_{slab} = \left[\left(150\ \text{lb}/\text{ft}^3\right)\left(\frac{1\text{ s}^2}{32.2\text{ ft}}\right)\right](0.458\text{ ft})\left(1728\text{ ft}^4\right)$$

$$I_{slab} = 3689\text{ lb - ft - s}^2$$

$$I_{total} = I_{blocks} + I_{slab} = (1968 + 3689)\text{ lb - ft - s}^2$$

$$I_{total} = 5657\text{ lb - ft - s}^2$$

$$M = 2\ k\ (d)(d)\ \Theta$$

$$M = 2\left(395\ \text{lb}/\text{in.}^3\right)\left(7.66\text{ ft x }12\ \text{in.}/\text{ft}\right)(7.66\text{ ft})\ \Theta$$

$$M = 566{,}244.7\ \Theta\ \text{(lb - ft)}$$

Substitute into Equation (2.5):

$$M + I\,(d^2\Theta/dt^2) = 0$$

$$566{,}244.7\ \Theta + 5657\ d^2\Theta/dt^2 = 0$$

$$w = \sqrt{\frac{2\ k\ d^2}{I}} = \sqrt{\frac{566{,}244.7\text{ lb - ft}}{5657\text{ lb - ft - s}^2}} = 9.92\ \text{rad}/\text{s}$$

$$f_{n,rotate} = \frac{1}{2p}\ w = \frac{1}{2p}\left(9.92\ \text{rad}/\text{s}\right)$$

$$f_{n,rotate} = 1.58\text{ hz}$$

Example Calculation of Predicted Natural Frequency for Side Rotation Mode of Oscillation (Exact Method)

Given: Axis of rotation illustrated in Figure and Equations (2.5) - (2.9) in Chapter 2.

Calculate: Using the parallel axis theorem, calculate the mass moment of inertia of the blocks added to the side of the floor slab:

$$I_{blocks} = \bar{I} + md^2$$

$$\bar{I} = 0 \quad \text{(relative to total } I_{blocks}\text{)}$$

$$m = \frac{4\ (9)\ (60\ \text{lb})}{32.2\ {}^{\text{ft}}/_{\text{s}^2}} = 67.08\ \frac{\text{lb - s}^2}{\text{ft}}$$

$$d = 6.0\ \text{ft} - \frac{7.0\ \text{in.}}{12\ {}^{\text{in.}}/_{\text{ft}}} = 5.42\ \text{ft}$$

$$I_{blocks} = 67.08\ \frac{\text{lb - s}^2}{\text{ft}}\ (5.42\ \text{ft})^2$$

$$I_{blocks} = 1968\ \text{lb - ft - s}^2$$

$$I_{slab} = \bar{I} + md^2$$

d = 0 (slab rotates about centroidal axis)

$$\bar{I}_{slab} = \int y^2\ dm = r\ t \int y^2\ dA = r\ t\ I_x$$

$$r = 150\ {}^{\text{lb}}/_{\text{ft}^3} \left(\frac{1\ \text{s}^2}{32.2\ \text{ft}} \right)$$

$$t = \frac{5.5\ \text{in.}}{12\ {}^{\text{in.}}/_{\text{ft}}} = 0.458\ \text{ft}$$

$$I_x = \frac{1}{12}\ bh^3$$

$$I_x = \frac{1}{12}\ (12\ \text{ft})(12\ \text{ft})^3 = 1728\ \text{ft}^4$$

$$I_{slab} = \left[\left(150\ {}^{\text{lb}}/_{\text{ft}^3} \right) \left(\frac{1\ \text{s}^2}{32.2\ \text{ft}} \right) \right] (0.458\ \text{ft})(1728\ \text{ft}^4)$$

$$I_{slab} = 3689\ \text{lb - ft - s}^2$$

$$I_{total} = I_{blocks} + I_{slab} = (1968 + 3689)\ \text{lb - ft - s}^2$$

$I_{total} = 5657 \text{ lb - ft - s}^2$

$M = 4\,k\,(d)(d)\,\Theta$

$$M = 4\left(395 \text{ lb}/\text{in.}^3\right)\left(5.42 \text{ ft x } 12 \text{ in.}/\text{ft}\right)\left(5.42 \text{ ft}\right)\Theta$$

$$M = 566{,}292.7\,\Theta \;(\text{lb - ft})$$

Substitute into Equation (2.5):

$M + I\,(d^2\Theta/dt^2) = 0$

$566{,}292.7\,\Theta + 5657\, d^2\Theta/dt^2 = 0$

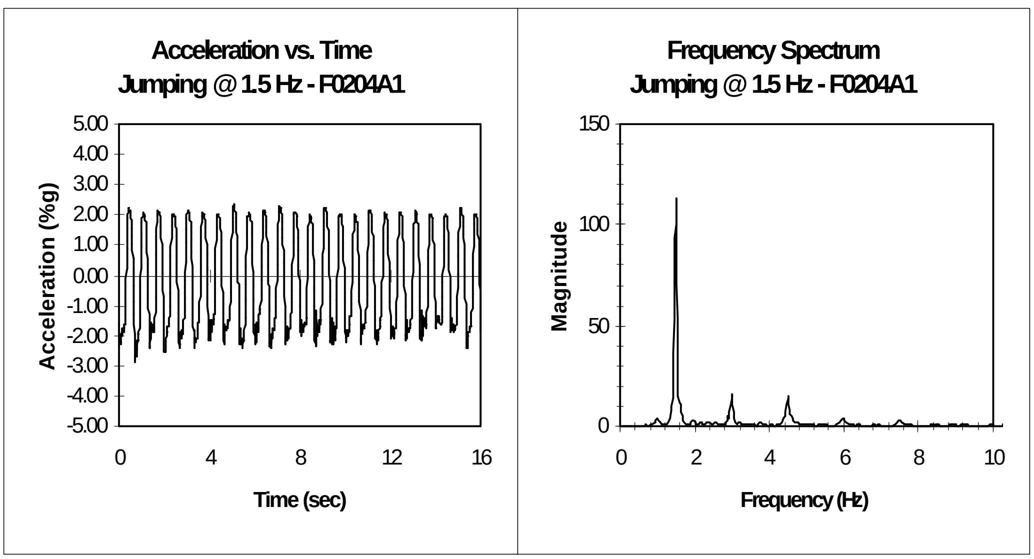
$$w = \sqrt{\frac{2\,k\,d^2}{I}} = \sqrt{\frac{566{,}292.7 \text{ lb - ft}}{5657 \text{ lb - ft - s}^2}} = 9.92 \text{ rad}/\text{s}$$

$$f_{n,rotate} = \frac{1}{2p}\,w = \frac{1}{2p}\left(9.92 \text{ rad}/\text{s}\right)$$

$$f_{n,rotate} = 1.58 \text{ hz}$$

Example Calculation of Measured Acceleration Response

Given: Frequency spectrum of acceleration response due to jumping at 1.5 hz
Valve Configuration = None



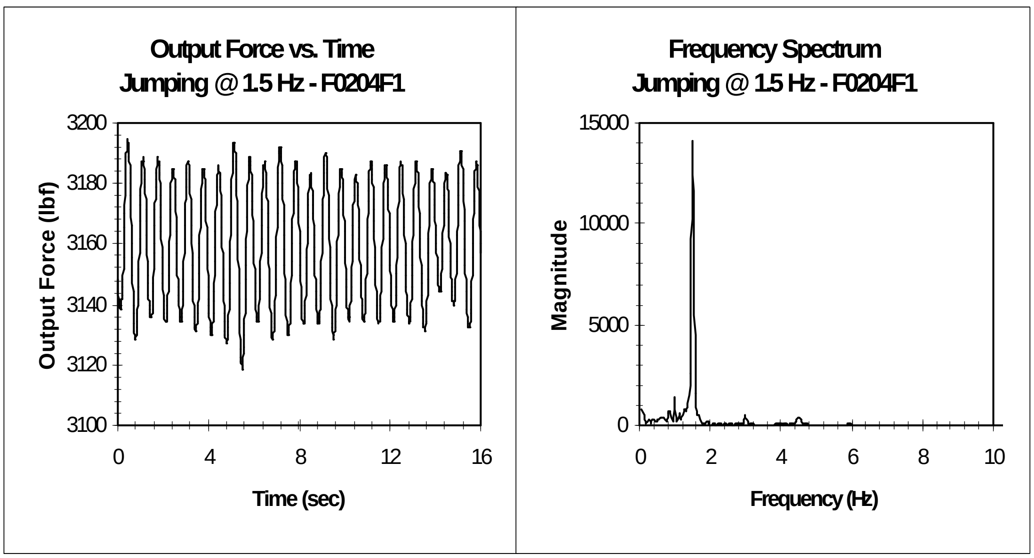
Measured Input Average Peak Acceleration = 2.24 %g
Magnitude of Frequency Spectrum at First Harmonic = 113.14
Magnitude of Frequency Spectrum at Second Harmonic = 16.32
Magnitude of Frequency Spectrum at Third Harmonic = 14.81

Calculate: Ratio of Frequency Spectrum Magnitudes

Second / First Harmonic = 16.32 / 113.14 = 0.14
Third / First Harmonic = 14.81 / 113.14 = 0.13
Measured Input Average Peak Acceleration at Second Harmonic =
(2.24 %g) x (0.14) = 0.32 %g
Measured Input Average Peak Acceleration at Third Harmonic =
(2.24 %g) x (0.13) = 0.29 %g

Example Calculation of Measured Force Transmission

Given: Frequency response of measured input and output force response due to jumping at 1.5 hz
Valve Configuration = None



Input force initial force plate reading = 205.7 lb
Output force initial force plate reading = 3161.6 lb
The following charts show the peak negative (-) and positive (+) readings from the input and output force response plots minus initial force plate reading
The average impact value is the average net impact for ten cycles.
For example, for the first cycle of the input force:
Plot (-) peak = 11.48 lbf
Plot (+) peak = 312.52 lbf

Chart (-) peak = Plot (-) peak - Initial force plate reading
= 11.48 lbf - 205.7 lbf
= - 194.22 lbf

Chart (+) peak = Plot (+) peak - Initial force plate reading
= 312.52 lbf - 205.7 lbf
= 106.82 lbf

Net Impact = Average (-) and (+) impact

$$= \frac{|-194.22| + |106.82|}{2}$$

= 150.52 lbf

These calculations were completed for each jumping frequency and each valve configuration. The results are listed in Table 3.7.

Input Force Chart

(-) (lbf)	***(+) (lbf)***	***Net Impact (lbf)***
194.22	106.82	150.52
196.16	110.71	153.435
196.16	102.94	149.55
196.16	114.59	155.375
194.22	108.77	151.495
196.16	106.82	151.49
196.16	161.2	178.68
194.22	112.65	153.435
194.22	196.16	195.19
196.16	102.94	149.55
	Average Impact:	**158.87**

Output Force Chart

(-) (lbf)	***(+) (lbf)***	***Net Impact (lbf)***
23.25	33.13	28.19
33.05	25.77	29.41
25.70	27.00	26.35
26.92	23.32	25.12
26.92	25.77	26.35
30.60	23.32	26.96
31.82	24.55	28.19
34.28	31.90	33.09
42.85	27.00	34.93
26.92	25.77	26.35
	Average Impact:	**28.49**

The average output force is the force transmitted at one spring. The total output force transmitted to the ground floor is as follows:

Total Output Force = Output Force at One Spring x 4
= 28.49 lbf x 4
= 113.96 lbf

Measured Force Transmission at First Harmonic
= Total Output Force / Input Force
= (113.96 lbf / 158.87 lbf) x 100
= 71.87 %

Measured Force Transmission at Second and Third Harmonic

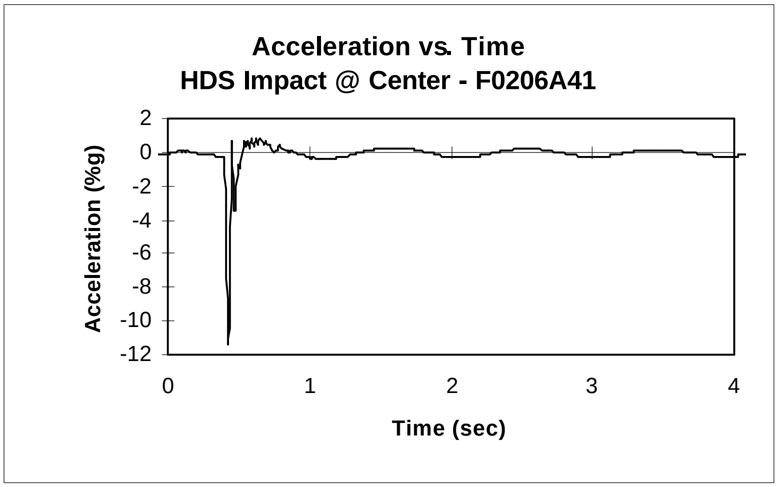
In a manner similar to the peak acceleration calculations, the ratios of the peak magnitudes of the frequency spectrum for the input and output force responses were calculated. The ratio of the first to second harmonic magnitudes was multiplied by the measured input and output forces at the first harmonic to determine the measured forces at the second harmonic. For example:

Input Force
Measured Input Force at First Harmonic = 158.87 lbf
Magnitude of Frequency Spectrum at First Harmonic = 77657
Magnitude of Frequency Spectrum at Second Harmonic = 15153
Magnitude of Frequency Spectrum at Third Harmonic = 8818
Ratio of Frequency Spectrum Magnitudes
Second / First Harmonic = 15153 / 77657 = 0.195
Third / First Harmonic = 8818 / 77657 = 0.113
Measured Input Force at Second Harmonic =
(158.87 lbf) x (0.195) = 30.94 lbf
Measured Input Force at Third Harmonic =
(158.87 lbf) x (0.113) = 18.01 lbf
Measured Force Transmission at Second Harmonic
= Total Output Force / Input Force
= (4.32 lbf / 30.94 lbf) x 100
= 13.95 %
Measured Force Transmission at Third Harmonic
= Total Output Force / Input Force
= (3.41 lbf / 18.01 lbf) x 100
= 18.95 %

These calculations were completed for each jumping frequency and each valve configuration. The results are listed in Table 3.8 and 3.9.

Example Calculation of Damping Ratio Using Logarithmic Decrement Method

Given: The acceleration response peaks due to HDS impact at the center of the floor slab. A typical acceleration response and corresponding peak acceleration magnitudes are shown below.



Peak (-) Acceleration (%g)	*Peak (+) Acceleration (%g)*	*Average Peak Acceleration (%g)*
0.44	0.82	0.63
0.35	0.23	0.29
0.30	0.18	0.24
0.25	0.13	0.19
0.22	0.08	0.15
0.20	0.06	0.13

Calculate: Determine the logarithmic decrement and damping ratio using Equation (2.10).

$d = logarithmic\ decrement$

$z = damping\ ratio$

$$d = \frac{2pz}{\sqrt{1-z^2}} = \frac{1}{n}\ln\left(\frac{x_o}{x_n}\right)$$

$n = 5$

$$d = \frac{1}{5}\ln\left(\frac{0.63}{0.13}\right) = 0.319$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = \frac{0.319}{\sqrt{4\pi^2 + (0.319)^2}}$$
$$\zeta = 0.0507 = 5.07\%$$

This calculation was repeated for five of the ten HDS impacts at the center of the slab for each valve configuration. The damping ratios were averaged to determine the reported damping ratio for each valve configuration, as shown in Table 3.1.

Example Calculation of Predicted Force Transmission

Given: Equation 2.12
Vibration characteristics for Valve Configuration = None
$\zeta = 0.0518$; $f_n = 1.0625$ hz

Calculate: **First Harmonic of Jumping Frequency f = 1.5 hz**

$$t_r = \sqrt{\left[\frac{1 + \left[2\zeta\left(f/f_n\right)\right]^2}{\left[1 - \left(f/f_n\right)^2\right]^2 + \left[2\zeta\left(f/f_n\right)\right]^2}\right]} x100$$

$$t_r = \sqrt{\left[\frac{1 + \left[2(0.0518)\left(1.5/1.0625\right)\right]^2}{\left[1 - \left(1.5/1.0625\right)^2\right]^2 + \left[2(0.0518)\left(1.5/1.0625\right)\right]^2}\right]} x100$$

$$t_r = 100.56\%$$

Second Harmonic of Jumping Frequency f = 3.0 hz

$$t_r = \sqrt{\left[\frac{1 + \left[2(0.0518)\left(3.0/1.0625\right)\right]^2}{\left[1 - \left(3.0/1.0625\right)^2\right]^2 + \left[2(0.0518)\left(3.0/1.0625\right)\right]^2}\right]} x100$$

$$t_r = 19.86\%$$

Third Harmonic of Jumping Frequency f = 4.5 hz

$$t_r = \sqrt{\left[\frac{1+\left[2(0.0518)\left(4.5/1.0625\right)\right]^2}{\left[1-\left(4.5/1.0625\right)^2\right]^2+\left[2(0.0518)\left(4.5/1.0625\right)\right]^2}\right]} x100$$

$$t_r = 10.44\%$$