

A New Method for Comparing Experiments and Measuring Information

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(ABSTRACT)

A statistic that summarizes an entire data set without losing any information about the family of distributions or the model is often called a sufficient statistic. Generally, one would like to use the statistic that contains the most information about the parameter space. Sometimes there are several sufficient statistics. At other times the only sufficient statistic is the entire data set. A large data set can be difficult to work with. In this case, can one use a statistic that, though not sufficient, does summarize the data set somewhat? How much information would be lost? How can one compare two statistics that aren't sufficient in terms of the amount of information each provides?

A new method for comparing experiments and measuring information is introduced. No assumptions are made and no conditions are required in order for this new method to measure the amount of information contained in almost any statistic. Several properties of this new method are discussed and a new characterization of sufficiency based on this new method is presented. The new method is used to evaluate the expected efficiency of a statistic in discriminating between any two values of the parameter as compared to a sufficient statistic. This new method can be self-calibrated to give this expected efficiency a meaningful scale. It is shown that this new method has some advantages over existing methods of measuring information.

This new method is applied to Casino Blackjack. Several card-counting statistics are compared by the amount of information each provides in discriminating between different deck compositions as compared to a sufficient statistic. This new method provides new insight about information in card-counting statistics by putting this information on a meaningful scale.

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