APPENDIX D
DERIVATION OF EQUATION FOR AVERAGE DIMENSIONLESS SHEAR STRESS OVER THE FLAT-BED REGION

The average dimensionless shear stress over the flat-bed region can be determined by integrating the expression for dimensionless flat-bed stress-depth over the width of the flat-bed region, and then dividing this by the flat-bed width.

$$d_{ave}^{*} = \frac{2\int_{0}^{\frac{B_f^*}{2}} d^{*}\left(y^{*}\right)dy^{*}}{B_f^{*}} \tag{D1}$$

Due to symmetry of the stress-depth distribution about the center of the channel, integration may be performed over the channel's half-width and the result multiplied by 2. The dimensionless stress-depth at any point along the flat-bed is given by

$$d^{*}\left(y^{*}\right) = 1 + \frac{\left(d_{cr}^{*} - 1\right)}{\cosh\left(\frac{B_f^{*}}{2\sqrt{y_0}}\right)}\cosh\left(\frac{y^{*}}{\sqrt{y_0}}\right) \tag{D2}$$

where

$$y_0 = \left(\tfrac{1}{12}\ln\left(\frac{30}{k^{*}}\right) - \tfrac{5}{72}\right)\left(1 + \frac{1}{2\ln\left(30/k^{*}\right) - \tfrac{17}{3}}\right)$$

Substituting this expression into Equation D1 and integrating the numerator results in the following equation:

$$d_{ave}^{*} = \frac{2\left.\left(y^{*} + \frac{\left(d_{cr}^{*} - 1\right)}{\cosh\left(\frac{B_f^{*}}{2\sqrt{y_o}}\right)}\sqrt{y_o}\,\sinh\left(\frac{y^{*}}{\sqrt{y_o}}\right)\right)\right|_{0}^{\frac{B_f^*}{2}}}{B_f^{*}} \tag{D3}$$

Evaluating the right-hand side of Equation D3 at the junction point ($y^{*} = B_f^{*}/2$) and at the center of the channel ($y^{*} = 0$), then subtracting the result of the latter from that of the former, yields the expression for the average dimensionless stress-depth over the flat-bed region is obtained.

$$d_{ave}^{*} = 1 + \frac{2\left(d_{cr}^{*} - 1\right)\sqrt{y_o}}{B_f^{*}}\tanh\left(\frac{B_f^{*}}{2\sqrt{y_o}}\right) \tag{D4}$$

The following expressions define dimensionless shear stress τ^{*} and dimensionless stress-depth δ^{*}:

$$t^* = \frac{t}{(r_s - r)gd_{50}} \tag{D5}$$

$$d^* = \frac{t}{rgSD_c} \tag{D6}$$

Dividing Equation D5 by Equation D6,

$$\frac{t^*}{d^*} = \frac{SD_c}{R_S d_{50}} \tag{D7}$$

where

$$R_S = \frac{(r_s - r)}{r}$$

It should be noted that for a given channel, the right-hand side of Equation D7 is constant. This means that the ratios of all dimensionless shear stresses to their corresponding dimensionless stress-depths are the same for a given channel. Therefore,

$$\frac{t^*_{ave}}{d^*_{ave}} = \frac{t^*_{cr}}{d^*_{cr}} \tag{D8}$$

Multiplying both sides of Equation D8 by d^*_{cr} and plugging in Equation D4 yields the equation for dimensionless average shear stress for the flat-bed region.

$$t^*_{ave} = \left(\frac{t^*_{cr}}{d^*_{cr}}\right)\left[1 + \frac{2(d^*_{cr} - 1)\sqrt{y_o}}{B^*_f}\tanh\left(\frac{B^*_f}{2\sqrt{y_o}}\right)\right] \tag{D9}$$