Decentralized Formation Selection Mechanisms Inspired by Foraging Bottlenose Dolphins

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1 Introduction

Formation control is an important sub-problem in multi-agent robotics and a number of decentralized control strategies have been developed to solve the formation control problem, e.g., [1], [2], [3]. However, there is still no underlying theory that governs the selection of a particular formation. In other words, if an agent has a choice between two formations, which one should it choose? In this paper, we would like to answer this question by drawing inspiration from foraging bottlenose dolphins, *Tursiops truncatus*. In fact, the networked control community has drawn significant inspiration from interaction-rules in social animals and insects [4], [5], [6]. In particular, the widely used nearest-neighbor-based interaction rules, used for example for formation control [7], [8], [9], consensus [3], [10], and coverage control [11], [12], has a direct biological counterpart, as shown in [4].

Bottlenose dolphins employ an unusual foraging technique known as the horizontal carousal; here the dolphins, after locating a sizeable amount of prey, form a large circle to trap the prey inside that circle. The prey is usually a school of fish and the dolphins slowly tighten the encirclement to restrict the movement of the fish. Eventually, the circle becomes small enough and the dolphins then take turns to charge through the fish while maintaining the integrity of the circle.

In this paper, we model the first phase of this method of capturing prey, and offer a choice of two formations for each agent — Large and Small Circle and by using

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tools from hybrid systems and decentralized networked control, we propose three strategies that enable the agents to switch between these formations. In Strategy 1, formation switches are based on instantaneous local error information; whereas, in Strategy 2, they are based on a global performance estimate propagated through the network. A more elaborate approach to switching is introduced in Strategy 3 where, through the help of dynamic average estimators, global performance estimates are improved by injecting new data in the form of the individual agents' local formation error.

The outline of the paper is as follows: In Section 2 we describe the social behavior of bottlenose dolphins with particular focus on their foraging techniques. In Section 3 we describe the network characteristics and in Section 4, we develop the three switching strategies for the horizontal carousal method. Simulation results comparing the three proposed hybrid control strategies are provided in Section 5, followed by the conclusions in Section 6.

2 Bottlenose Dolphins

Cetaceans (to which bottlenose dolphins belong) are complex and intelligent animals. The Encephalization Quotient (E.Q.), which is the brain to body mass ratio, is a factor often used to judge intelligence, and dolphins rate 2nd in E.Q., just behind human beings. In particular, the bottlenose dolphins, *Tursiops truncatus*, display complex social behaviors such as living in so-called *fission-fusion* societies [13]. This means that the group can break up into subgroups and later rejoin the main group to share food or participate in other activities (see [14] for details). The bottlenose dolphins, found off the coast of Florida and Western Australia, also form alliances during mating [15] and have a well defined social hierarchy [16]. They also exhibit cooperative behavior while advertising resources, defending other members, and while foraging and capturing prey - the cooperative behavior we study in this paper.

While searching for food, the dolphins are most often spotted in groups and they maintain very specific formations. Some common types of formations are: front, double front, team, echelon, tight group, and line [17]. Team and line formations require a leader and this role usually goes to the largest male, which is again due to the well-defined social hierarchy of the dolphins. Dolphin formations are in general highly adaptive. Groups of male bottlenose dolphins are observed to move in a single file formation near the shore and maintain front formation farther from shore during foraging. In safe territories, dolphins also perform a diffused search pattern where individual members scatter off into different directions [17].

Once the dolphins find their prey, they mainly use two methods to actually catch the fish. One method is to form a front formation and drive the fish against the shore. The Wall method, as it is known, is not very interesting from a networked control point of view since the dolphins simply drive their prey against the shore and capture them from the foam of returning water [17]. The other method is known as the horizontal carousal and this will serve as inspiration for our mechanism to select formations. The particulars of this method, together with a mathematical model of the foraging dolphins, is the topic of the next section.

3 Network Characteristics

3.1 Topology

Let us consider a pod with N^d bottlenose dolphins and a school of N^f fish, with the corresponding index sets $\mathbf{N}^d = \{1, \ldots, N^d\}$, $\mathbf{N}^f = \{1, \ldots, N^f\}$. We have used the convention that the superscripts d and f refer to "bottlenose dolphins" and "fish" respectively. We assume that the dolphin and fish states take on values in a n-dimensional space (in a kinematic dolphin model, n would typically be 3), i.e., that $x_i^d \in \Re^n, \ \forall\, i \in \mathbf{N}^d$ and $x_i^f \in \Re^n, \ \forall\, i \in \mathbf{N}^f$.

Before we proceed, let us characterize the inter-dolphin interactions, and in [18] the movement of dolphins in a formation is found to be such that each dolphin has a "bubble" around it that other dolphins do not intrude. This implies that every dolphin is aware of the position of its neighbors. This observation leads us to postulate the nearest-neighbor rule for inter-agent interactions. Furthermore, dolphins perceive their environment and share information through their limited range echolocation system since they are primarily audial creatures using both sonar and a series of "chatters" to communicate with one another [13, 19]. Due to the limited interaction range over which the dolphins can detect each other, we can define a dolphin proximity graph $\mathbf{G}^d(t) = \mathbf{N}^d \times \mathbf{E}^d(t)$, where, for two distinct dolphins $(i,j) \in \mathbf{E}^d(t) \ \Leftrightarrow \ \|x_i^d(t) - x_j^d(t)\| \leq \Delta$, for some critical interaction distance Δ. This construction ensures that the interaction graph is simple (no self-loops) and undirected. In fact, it is a so-called Δ-disk proximity graph, as defined for example in [8], [11].

We would now like to apply a decentralized control strategy over the set of dolphins in such a way that the control law is only allowed to contain references to the relative displacements between a dolphin and its neighbors in the interaction graph.

3.2 Dolphin Dynamics

As mentioned before, we are interested in the horizontal carousal method of foraging. As seen Figure 1, the dolphins encircle the fish and gradually tighten the encirclement by forming smaller and smaller circles [13]. At one point, when the circles are small enough, dolphins take turns to charge through the fish and feed.

Dolphins aside, the overarching objective of this paper is to develop decentralized formation selection mechanisms. And, we will assume that the two formations available to the dolphins are Large and Small Circle. We will also assume that the fish are stationary. Small fish generally maintain a constant "inter-individual distance" [20] and as the dolphins form smaller circles, the fish are increasingly constrained to move in smaller bubbles. In essence, the dolphins herd their prey and restrict their movement; and if we assume that the fish are stationary, then the centroid of the school of fish becomes the point of interest for the dolphins and the feeding fish will be represented by a single node in the graph. The centroid of the

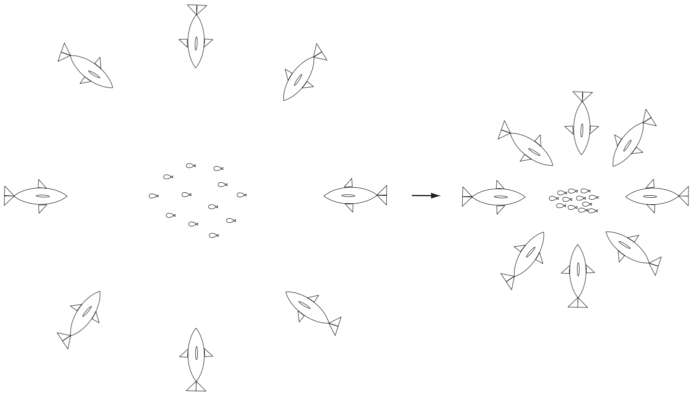


Figure 1. *Bottlenose dolphins tightening the encirclements around a school of fish.*

fish, ρ, is given by $\rho = \frac{1}{N^f}\sum_{i=1}^{N^f} x_i^f$.

Here x_i^f represents the state of fish i and N^f is the number of fish in the school. Thus, in order to encircle the fish, the dolphins need to stay a distance r_j, $j = 1, 2$, from the centroid of the fish (r_1 for Large Circle and r_2 for Small Circle) and consequently, maintain a distance $K_j(r_j)$ from its single-hop neighbors ($K_1(r_1)$ for Large Circle and $K_2(r_2)$ for Small Circle), given by $K_j(r_j) = 2r_j sin(\frac{\theta}{2})$, where θ is $\frac{2\pi}{N^d}$.

One standard choice for locally enforcing (e.g. [23]), if feasible, a certain agent separation (in our case, K_j) is given by

$$\dot{x}_i(t) = -\sum_{k|(i,k)\in \mathbf{E}^d(t)} (\| x_i(t) - x_k(t) \| - K_j)(x_i(t) - x_j(t)),\ \forall i \in \mathbf{N}^d,\ \forall j = \{1, 2\}.$$

We will modify this so that the distance between a dolphin and the centroid of the fish, r_j, is enforced as well. As a result, the new dynamics is given by

$$\begin{aligned}\dot{x}_i(t) = -\sum_{k|(i,k)\in \mathbf{E}^d(t)} & ((\| x_i(t) - x_k(t) \| - K_j)(x_i(t) - x_j(t)) \\ & + (\| x_i(t) - \rho \| - r_j)(x_i(t) - \rho)),\ \forall i \in \mathbf{N}^d,\ \forall j = \{1, 2\}.\end{aligned}$$

We have dropped the subscript "d" from the dolphin states since we have already assumed that the fish are stationary. Hence, the dynamics used for performing Large and Small Circle are governed by the choice of r_j. Once r_j is specified for the two formations, then the dynamics $\dot{x}_i(t) = f_1$ will denote that agent i is performing Large Circle and $\dot{x}_i(t) = f_2$ will denote that it is performing Small Circle. Now that we have the established the network characteristics, in the following section we will propose the three selection strategies for the horizontal carousal problem, simulate, and evaluate their relative merits and demerits.

4 Selecting Formations

In essence, any formation selection mechanism is a hybrid control strategy. What we propose in this paper, is to let the selection be driven by the agents' estimates of their errors associated with each formation, and consequently select the formation with the least error. Here we are simply exploring three different strategies and each has its own advantages and disadvantages in terms of complexity, communication, and extraction of global properties. We will now describe these strategies in detail.

4.1 Strategy 1: Local Instantaneous Errors

In this strategy, an agent calculates the instantaneous local error associated with each formation. Let $E_{i,j}(t)$ be agent i's local error associated with formation j, where $j \in \{1,2\}$. We will assign $E_{i,1}(t)$ to be the error of performing Large Circle and assign $E_{i,2}(t)$ to be the error of performing Small Circle. As mentioned before, to form the encirclement, each agents needs to keep track of the distance from its neighbors, K_j, and distance from the centroid of the fish, r_j; as a result, for each agent, we can simply assign the instantaneous local error for each formation to be the mean squared error of these distances, given by

$$E_{i,j}(t)^2 \;=\; \sum_{k|(i,k)\in\mathbf{E}^d(t)} ((\| x_i(t) - x_k(t) \| - K_j)^2 + (\| x_i(t) - \rho \| - r_j)^2),$$

$\forall i \in \mathbf{N}^d$, $\forall j = \{1,2\}$. The selection mechanism we propose is to let agent i selects the dynamics $\dot{x}_i(t) = f_1$ over $\dot{x}_i(t) = f_2$ if $E_{i,1} < E_{i,2}$ and vice versa as shown in Figure 2.

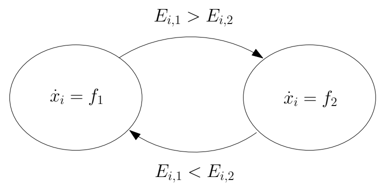


Figure 2. *The hybrid automaton implementing Strategy 1. Agent i selects the dynamics $\dot{x}_i(t) = f_1$ (form Large Circle) over $\dot{x}_i(t) = f_2$ (form Small Circle) if $E_{i,1} < E_{i,2}$.*

The underlying disadvantage of this strategy is that decisions are made based on local properties and the global performance of the network is not taken into account by the agents. An agent switches based on its instantaneous error measurements but does not consider what the other agents are trying to do. It is a reactive behavior which results in a lot of switching back and forth between formations. To address this issue, we propose Strategy 2, which is further categorized into (a) and (b), based on the nature of the transition rules.

4.2 Strategy 2a: Instantaneous Averaged Initial Errors

In this strategy, formation switching by each agent is based upon its global performance estimate. Let $\xi_{i,j}(t)$ be agent i's global error estimate of formation j, then $\xi_{i,j}(t)$ is determined as follows:

$$\dot{\xi}_{i,j}(t) = - \sum_{k \ | \ (i,k) \in \mathbf{E}^d(t)} (\xi_{i,j}(t) - \xi_{k,j}(t)), \ \forall i \in \mathbf{N}^d, \ \forall j = \{1, 2\} \tag{1}$$

where $\xi_{i,j}(0) = E_{i,j}(0)$. Hence, the global error estimate of an agent is actually the average of the initial local errors of itself and its neighbors and we propose that agent i executes $\dot{x}_i(t) = f_1$ if $\xi_{i,1}(t) < \xi_{i,2}(t)$ and $\dot{x}_i(t) = f_2$ if $\xi_{i,1}(t) > \xi_{i,2}(t)$. But there is a caveat with this strategy: agents must be able to communicate with each other. This was not a requirement in Strategy 1 since agents only measured their positions with respect to their neighbors and the centroid of the fish. Here neighbors must share their initial local errors to initialize the estimation process.

A problem with both Strategy 1 and Strategy 2a is that agents may potentially undergo a lot of switches before finally settling on a formation. It might be more desirable to develop strategies where the agents spend more time "thinking" than "moving," which leads us to Strategy 2b.

4.3 Strategy 2b: Delayed Averaged Initial Errors

This is similar to Strategy 2a except for one crucial difference: switches are not instantaneous. Agents select a particular formation once Equation (1) "converges," which can be defined as settling below some threshold. Thus, the agents must wait and process information and this waiting time is given by the convergence time, T_{conv}, of Equation (1). Now, as long as the network is connected, the rate of convergence depends on the algebraic connectivity [21] of the graph and we will use this fact to solve for T_{conv} as follows:

Equation (1) can be rewritten in matrix form as $\dot{\xi}(t) = -\mathbf{L}(t)\xi(t)$, where $\mathbf{L}(t)$ is the graph laplacian. Now $\xi(t)$ asymptotically approaches $\bar{\xi}\mathbf{1}$, where, $\bar{\xi} = \frac{1}{N^d}\sum_{k=1}^{N^d} \xi_k(0)$ and $\mathbf{1} = (1, \ldots, 1)^T$ is the vector with ones along each components [21]. If we assume that the graph is static, then, $\| \xi(t) - \bar{\xi}\mathbf{1} \| \leq \| \xi(0) - \bar{\xi}\mathbf{1} \| e^{-\lambda_2(\mathbf{L}(t))t}$, where, $\lambda_2(\mathbf{L}(t))$ is the second smallest eigenvalue of $\mathbf{L}(t)$, also known as the algebraic connectivity. If $\xi(0)$ is bounded as $\| \xi(0) - \bar{\xi}\mathbf{1} \| \geq M$ and we define successful convergence as $\| \xi(t) - \bar{\xi}\mathbf{1} \| \leq \varepsilon$, then $T_{conv} \geq \frac{1}{\lambda_2 \ln(\frac{M}{\varepsilon})}$.

As in Strategy 2a, agent i undergoes the dynamics $\dot{x}_i(t) = f_1$ if $\xi_{i,1}(t) < \xi_{i,2}(t)$ and $\dot{x}_i(t) = f_2$ if $\xi_{i,1}(t) > \xi_{i,2}(t)$ as seen in Figure 3. This strategy was introduced to reduce the amount of switching that arises from implementing Strategy 2a, but in the process, we have introduced the notion that the agents must somehow keep track of time. Agents are required to wait for T_{conv} and then select a formation. From Figure 7 it can be seen that Strategy 2b was an obvious improvement over Strategy 1 and 2a in terms of the total number of switches.

Agents executing Strategy 2a and 2b took into account "what everyone else was doing" before committing to a particular formation; however, the global performance estimates were based on initial instantaneous local errors and this is a drawback since the estimation failed to incorporate any new information that might be available in the network. This leads us to develop Strategy 3a and 3b, which incorporate new data into the global performance estimate of the network.

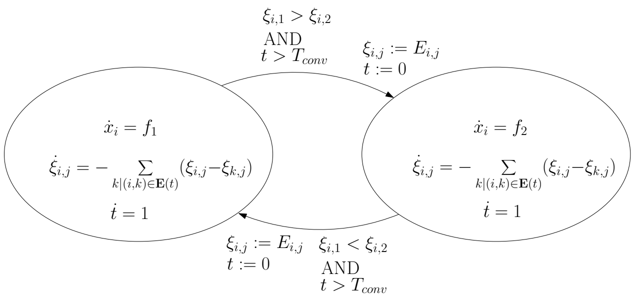


Figure 3. *Automaton of Strategy 2b.*

4.4 Strategy 3a: Proportional Dynamic Averaged Errors

Strategy 2 can be characterized as a static average consensus, as opposed to a dynamic average consensus, where estimates are driven by injection of new information [22]. For us, this new information may simply arrive in the form of an agent's instantaneous local formation error and intuitively, it makes sense that a strategy that combines both Strategy 1 (using current information) and Strategy 2 (using a global performance estimate) may be a step in the right direction in our search for an improved method of selecting formations. Thus, we would like to modify Strategy 2 as follows:

$$\dot{\xi}_{i,j}(t) = -\sum_{k \ | \ (i,k)\in\mathbf{E}^d(t)} ((\xi_{i,j}(t) - \xi_{k,j}(t)) + F(E_{i,j}(t) - \xi_{i,j}(t)))$$

$\forall i \in \mathbf{N}^d, \ \forall j = \{1, 2\}$, where $F(E_{i,j}(t) - \xi_{i,j}(t))$ is an insertion of the instantaneous local error, $E_{i,j}(t)$, of agent i with respect to formation j. This is the idea behind Strategy 3 as seen in Figure 4.

But inserting new information into a consensus estimation is not trivial. [22] presents two estimation algorithms: proportional and proportional-integral algorithms and they will form the basis for Strategy 3a and 3b, respectively. Strategy 3a uses a proportional dynamic consensus estimator as follows:

$$\dot{w}_{i,j}(t) = -\gamma w_{i,j}(t) - \sum_{k \ | \ (i,k)\in\mathbf{E}^d(t)} (\xi_{i,j}(t) - \xi_{k,j}(t))$$

$$\xi_{i,j}(t) = w_{i,j}(t) + E_{i,j}(t), \ \forall i \in \mathbf{N}^d, \ \forall \ j = 1, 2$$

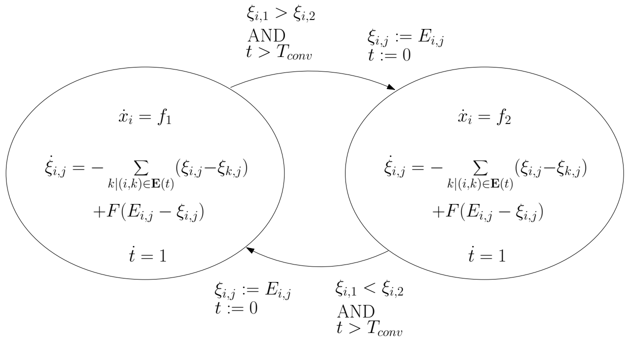


Figure 4. *Automaton of Strategy 3. As opposed to Strategy 2b, Strategy 3 incorporates new data in the form of the instantaneous local error $E_{i,j}(t)$ into the global performance estimate of the network.*

where, $w_{i,j}(t)$ is the estimator state, and $\gamma \geq 0$ is the rate at which new information is introduced. The algorithm requires us to assume that the input $E_{i,j}(t)$ and its derivative is bounded [22]. Again, $\xi_{i,j}(t)$ is agent i's estimate of the global error of performing formation j and it is initialized as before by $\xi_{i,j}(0) = E_{i,j}(0)$.

Figures 7(d) shows that this improved global error estimate decreases the number of switches made by the agents; however, this is computationally more complex than all the previous strategies. In particular, compared to Strategy 1, we now require agents to have the ability to communicate with their neighbors, keep track of time, and implement a dynamic consensus estimator.

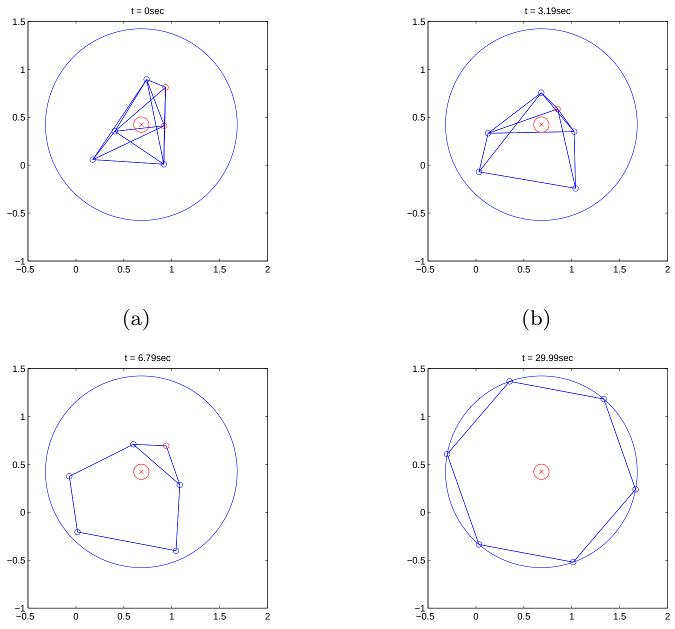
4.5 Strategy 3b: Proportional-Integral Dynamic Averaged Errors

In this final strategy, the new information is introduced into the global estimate using a proportional-integral dynamic consensus estimator as follows:

$$\begin{aligned}
\dot{\xi}_{i,j}(t) &= \gamma(E_{i,j}(t) - \xi_{i,j}(t)) - \sum_{k \ | \ (i,k)\in\mathbf{E}^d(t)} (\xi_{i,j}(t) - \xi_{k,j}(t)) \\
&\quad + \sum_{k \ | \ (i,k)\in\mathbf{E}^d(t)} (w_{i,j}(t) - w_{k,j}(t)) \\
\dot{w}_{i,j}(t) &= - \sum_{k \ | \ (i,k)\in\mathbf{E}^d(t)} (\xi_{i,j}(t) - \xi_{k,j}(t)), \ \forall i \in \mathbf{N}^d \ \forall \ j = 1, 2
\end{aligned}$$

where $w_{i,j}(t)$ is the estimator state for agent i implementing formation j and as before, the estimator is initialized by $\xi_{i,j}(0) = E_{i,j}(0)$.

As mentioned in [22], the advantage of this estimator lies in the fact that the input $E_{i,j}(t)$ does not directly affect $\xi_{i,j}$ and hence, it provides better filtering of noisy inputs.



(a) (b) (c) (d)

Figure 5. *Simulation of six agents executing Strategy 1. The centroid of the fish is shown with a '$\times$'. The circles around the centroid represent the orbits where the agents must lie for each formation.*

5 Simulations

To compare the strategies, we simulate each one under the same initial positions for all the agents. Some agents undergo a lot of switching and a plot of the local error of agent 5 is provided in Figure 6. All the strategies deliver a successful formation if the graph is connected and we ensure this by initially placing the agents sufficiently close to each other [23].

6 Conclusions

In this paper, we address the issue of switching between formations. In particular, we draw inspiration from bottlenose dolphins, the *Tursiops truncatus*, and we model these dolphins as networked, first-order systems in which dolphin interactions are defined through spatial proximity. Using tools from hybrid systems and decentralized networked control we propose three strategies that enable us to switch between formations. In Strategy 1, formation switches are based on instantaneous local error information; whereas in Strategy 2, they are based on a global performance estimate propagated through the network. In Strategy 3 where, through the help of dynamic

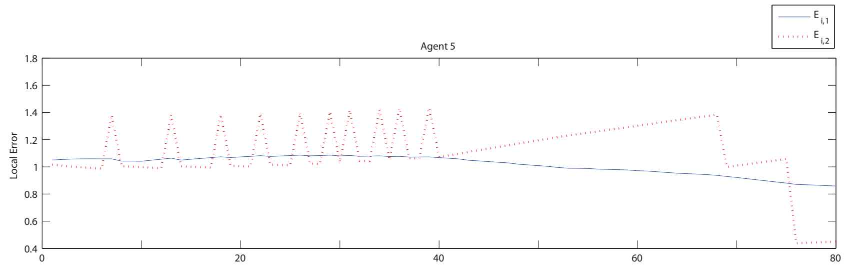
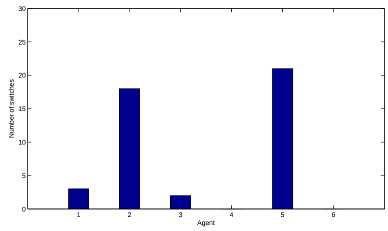
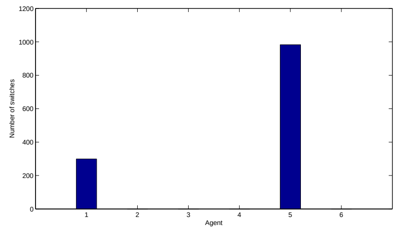


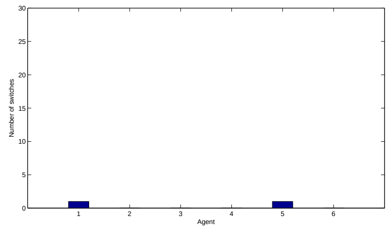
Figure 6. *Instantaneous local errors of agents* 5 *using Strategy 1.* $E_{i,1}$ *is the instantaneous local error of Large Circle and* $E_{i,2}$ *is the instantaneous local error of Small Circle. This agent also undergoes the most switching.*



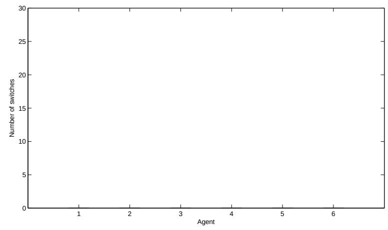
(a) Strategy 1



(b) Strategy 2a



(c) Strategy 2b



(d) Strategy 3a and 3b

Figure 7. *Number of switches made by agents under each strategy and identical initial positions.*

average estimators, global performance estimates are improved by injecting new data in the form of an agents local formation error. Finally, simulation results are provided to compare the three proposed hybrid control strategies.

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