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CALL FOR COMMENTS: A PROPOSAL FOR STANDARD NOTATION AND TERMINOLOGY IN MODAL ANALYSIS

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ABSTRACT

As the quantity and diversity of work that employs modal analysis increases, it seems that the use of notation and terminology also seems to follow a similarly disparate trend. The net result of this is that readers of publications related to modal analysis and testing are often distanced from the thesis of the work as they first have to interpret some of the basic terms. This paper seeks to demonstrate how the adoption of a standard form of notation and terminology could improve the communication of material to all workers in modal analysis by making papers more readily 'approachable'. In addition to providing examples of how this can be achieved, a chart of appropriate notation is given.

AUTHORS NOTE

The proposed notation is still in the process of review and revision, prior to being "frozen" for a 2-year trial period. Comments from readers of this article are invited and should be submitted to arrive no later than 31 May 1992 to: Prof D. J. Ewins, Mechanical Engineering Department, Imperial College, London SW7 2BX, UK. Fax: 44/0-71-584-1560. Email: dje @ me. ic. ac. uk

1. Introduction

The use of modal analysis as a design and research tool has increased manyfold over the last decade, as is reflected in the numerous papers in journals and conferences relating to this field. As the application of modal analysis has diversified, so has the background of the workers involved - now encompassing for example, control, structural dynamics, stress analysis etc. This has led to the adoption of many different forms of terminology and nomenclature, all of which are valid, but which nevertheless have the effect of distancing all but a select few of the readers from the goals set out in the published work.

2. Terminology and Notation

Frequently, terminology is applied in a general way so that the correct technical definition is inappropriate. For instance, one of the most used (possibly the most abused) terms in modal analysis is 'modal shape', which is used to describe anything from a forced vibration (operating) displacement shape

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to the vector describing a mode of free vibration. In the same area, the term 'normal mode' is often interpreted as being the description of an undamped mode of free vibration-whereas it is also used, perhaps correctly, to encompass real *and* complex free modes of vibration. This type of inconsistency, of which there are many examples, can all too easily mislead other workers.

It is not only terminology which is applied in a variety of ways: a baffling assortment of notation is also frequently used. To demonstrate this point, it is appropriate to include an example of how the simplest formulation can be expressed in a variety of different ways. Taking the basic formulation of the generalized forced damped equation of motion, it can be seen that although each of the alternative descriptions is recognizable, they vary considerably in their notations [1-6], as follows:

$$A\ddot{q} + B\dot{q} + Cq = Q(t)$$

$$\underset{\sim}{M}\,\underline{\ddot{x}} + \underset{\sim}{D}\,\underline{\dot{x}} + \underset{\sim}{K}\,\underline{x} = \underline{F}(t)$$

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t)$$

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = p(t)$$

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}$$

$$\underline{M}\,\underline{\ddot{r}} + \underline{C}\,\underline{\dot{r}} + \underline{K}\,\underline{r} = \underline{R}(t)$$

Not only do these equations differ in their notation for mass, stiffness and damping, but also they often give little indication as to which of the quantities are in matrix, vector or scalar form. Thus, the subsequent derivation from these equations can lead to a loss of clarity if the reader's own choice of notation does not coincide exactly with that of the author's. Accordingly, one of the aims of this paper is to propose a basic standard notation specific to modal analysis applications, which has been segmented into the following six categories:

(1) Basic Terms, Dimensions and Subscripts;
(2) Matrices, Vectors and Scalars;
(3) Spatial Properties;
(4) Modal Properties;
(5) Response Properties; and
(6) Standard Abbreviations.

These sections only provide a basic definition of the most commonly used terms. Therefore, there is scope for development of this appendix by individual users to accommodate their own specific applications.

3. Concluding Remarks

Although every effort has been made to make the notation consistent, comprehensive and representative of the authors' experiences, inevitably there will be additions and modifications which users will feel appropriate to make. It is hoped that the Appendix provided will be used as a tool which will facilitate increased understanding and communication of ideas in the modal analysis community at large.

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(1) Basic Terms, Dimensions and Subscripts

x, y, z	translational degrees of freedom/coordinates
θ_x, θ_y, θ_z	rotational degrees of freedom/coordinates
N	total number of degrees of freedom/coordinates
N_o	number of nodes
n	number of primary/master/measured DOFs (also denoted by subscript 1)
s	number of secondary/slave/unmeasured DOFs (also denoted by subscript 2)
m	number of included/effective modes
L	number of correlated mode pairs
r	current mode number *or* matrix rank
j, k, l	integers
q	generalized coordinate
p	principal/modal coordinate
ω, f	frequency in rads^{-1}/Hz
Ω	rotation speed
i	$\sqrt{-1}$

(2) Matrices, Vectors and Scalars

[ ]	matrix
{ }	column vector

$(\ )$	single element (of matrix or vector)
$\begin{bmatrix} \diagdown & \\ & \diagdown \end{bmatrix}$	diagonal matrix
$[\ ]^T; \{\ \}^T$	transpose of a matrix; vector (i.e. row vector)
$[\ ]^H$	complex conjugate (Hermitian) transpose of a matrix
$\begin{bmatrix} \diagdown & & \\ & I & \\ & & \diagdown \end{bmatrix}$	identity matrix
$[0]$	null matrix
$[\ ]^{-1}$	inverse of a matrix
$[\ ]^{+}$	generalized/pseudo inverse of a matrix
$[\ ]^*; \{\ \}^*; (\ )^*$	complex conjugate of matrix; vector; single element
$[U]$, $[V]$	matrices of left and right singular vectors
$[\Sigma]$	rectangular matrix of singular values (where σ_j is the jth singular value)
$[T]$	transformation matrix
$[A^R]; [A^E]$	reduced, expanded matrix
$\Vert\ \Vert_p$	p-norm of a matrix/vector
ε	value of a norm/error/perturbation

(3) Spatial Properties

$[M]; [K]; [C]; [D]$	mass; stiffness; viscous damping; structural (hysteretic) damping matrices
$[M_A]; \ldots$	analytical/theoretical/predicted/FE mass; ... matrix
$[M_X]; \ldots$	experimentally-derived/test mass; ... matrix
$[\Delta M]=[M_X]-[M_A]; \ldots$	mass; ... error/modification matrix
$[M_U]=[M_A]+[\Delta M]; \ldots$	updated/refined/improved mass; ... matrix
$\left[\begin{array}{c:c} M_{11} & M_{12} \\ \hdashline M_{21} & M_{22} \end{array}\right]; \ldots$	partitioned mass; ... matrix

(4) Modal Properties

ω_r	natural frequency for rth mode (rad/s)
ζ_r	viscous damping ratio of rth mode
η_r	structural damping loss factor of rth mode
m_r	modal/effective mass of rth mode
k_r	modal/effective stiffness of rth mode
c_r	modal/effective viscous damping of rth mode (proportional damping)
Q	Q factor
δ	logarithmic decrement

$\begin{bmatrix} \ddots & & \\ & \lambda_r & \\ & & \ddots \end{bmatrix}$ eigenvalue matrix

$[\Psi]$ mode shape/eigenvector matrix

$[\phi]$ mass-normalized mode shape/eigenvector matrix

$\{\psi\}_j$; $\{\phi\}_j$ rth mode shape/eigenvector

$(\psi_r)_j$; $(\phi_r)_j$ jth element of rth mode shape/eigenvector

$$[\Theta]_{2N\times 2N} = \left[\begin{array}{c:c} [\Psi] & [\Psi]^* \\ \hdashline [\Psi]\begin{bmatrix} \ddots & & \\ & \lambda_r & \\ & & \ddots \end{bmatrix} & [\Psi]^*\begin{bmatrix} \ddots & & \\ & \lambda_r & \\ & & \ddots \end{bmatrix}^* \end{array}\right]$$

eigenvector for viscously-damped system

$$\{\theta\}_{r(2N\times 1)} = \left\{\begin{array}{c} \{\psi\}_r \\ \hdashline \lambda_r\{\psi\}_r \end{array}\right\}$$

rth mode shape/eigenvector for viscously-damped system

$[S]$ sensitivity matrix

(5) Frequency Response Properties

TABLE 1 FREQUENCY RESPONSE PROPERTIES DUE TO INPUT FORCE(S) F(ω).

Response (R)	$\frac{R}{F}$	$\frac{F}{R}$
Displacement	Receptance/Dynamic Compliance $[\alpha(\omega)]$	Dynamic Stiffness
Velocity	Mobility $[Y(\omega)]$	Mechanical Impedance
Acceleration	Accelerance/Inertance $[A(\omega)]$	Apparent/Effective Mass

$[H(\omega)]$ Frequency Response Function (FRF) matrix

$H_{jk}(\omega) = \left(\frac{x_j}{f_k}\right)_{f_l=0;l=1,n;l\neq jk}$ individual FRF element between coordinates j and k

${}_rA_{jk}=(\phi_r)_j\,(\phi_r)_k$ modal constant/residue

$[h(t)]$ Impulse Response Function matrix

$[R]$ residual matrix

$$R_{jk} = \sum_{r=m+1} \frac{{}_rA_{jk}}{\omega_r^2}$$ high-frequency residual for FRF between coordinates j and k

(6) Standard Abbreviations

CMP	Correlated Mode Pair
DOF(s)	Degree(s) of Freedom
FE	Finite Element
FRF	Frequency Response Function
IRF	Impulse Response Function
MAC	Modal Assurance Criterion
COMAC	Coordinate Modal Assurance Criterion
SVD	Singular Value Decomposition
FD	Frequency Domain
TD	Time Domain