The International Journal of Analytical and Experimental Modal Analysis v 7 n 3 p 157-168 Jul 1992

[redacted]

by N.G. Nalitolela, J.E.T. Penny and M.I. Friswell, Aston Univeristy

ABSTRACT

Most methods used to update dynamic structural models use either frequency response data or both eigenvalues and mode shape data. This paper presents a technique to adjust the parameters using eigenvalues alone. Eigenvalues may be measured very accurately whereas mode shapes often contain substantial errors. The structure and its theoretical model are perturbed by adding mass or stiffness. The measured eigenvalues before and after each mass or stiffness addition are used to update the parameters by sensitivity analysis. It is shown that with error-free data and a proper choice of the perturbing coordinates, exact parameters can be identified from eigenvalues alone using unconstrained optimization. Due to measurement errors and possible inaccuracies in the structure of the model matrices, the parameters of a real structure are adjusted by incorporating a constraint of minimum changes from the initial estimates using a Bayesian estimator.

1. Introduction

The adjustment of the parameters of a structural dynamic model using experimental data has received considerable interest recently and a number of techniques have been proposed. However, there is no generally acceptable method due to a number of difficulties mainly (i) mismatch between the number of measurement degrees of freedom (DOF) and model DOF, (ii) an incomplete number of measured or excited modes and/or (iii) experimental errors in the measured data.

There are two basic procedures that may be used; either frequency domain methods or methods that use both eigenvalues and mode shapes data. The latter group of methods can be subdivided into three main sub-groups. The first is of non-iterative methods which seek to derive the formula for the updated mass and stiffness matrices which are close to the theoretical ones subject to some constraint condition, usually orthogonality with the measured modes [1-3]. They require full length mode shapes and therefore it is necessary to estimate the mode shape displacements of the unmeasured DOF. Due to the incomplete number of measured modes and the errors in the estimated and measured modal data, the structure of the updated matrices is distorted. It is often difficult to relate the elements in the mass and stiffness matrices to the FE model or the physical structure. The second group are based on the equation error approach and uses the homogeneous equation of motion and the orthogonality equations, with measured modal data, to formulate the coefficients for the unknown mass and stiffness parameters [3,4].

Because the individual parameters are determined, rather than whole matrices, the structure of the

Dr. N. G. Nalitolela (SEM member), Dept. of Mechanical Engineering, University of Dar es Salaam, P. O. Box 35131 Dar es Salaam, Tanzania. Dr. J. E. T. Penny (SEM member) and Dr. M. I. Friswell (SEM member), Dynamics, Control and Vibration Research Group, Department of Mechanical and Electrical Engineering, Aston University, Birmingham B4 7ET, U.K.

Final manuscript received: January 28, 1992

model matrices is maintained. However, full length mode shapes are also required and the unmeasured mode shape data has to be estimated. Since the contaminated modal data is used to formulate the coefficients of the mass and stiffness parameters, the updated parameters are biased. The third group is of methods based on the sensitivities of the eigenvalues and eigenvectors to derive equations linear in the unknown parameters [5,6]. In these methods, bias problems associated with the contaminated data in the coefficients of the unknown mass and stiffness parameters are avoided. Also the equations relating the parameters and the experimental data can be formulated even if mode shape data of some coordinates is missing. However, in order to generate at least as many equations as the number of unknown parameters, mode shape measurement at many coordinates is required because the number of eigenvalues measured is usually small.

Although mode shape measurements are generally less accurate than natural frequency measurements, no technique can identify the parameters using eigenvalues of the structure alone, because there is an infinite set of parameters that can reproduce a given set of eigenvalues. This paper describes a technique that circumvents this limitation. Selected structural parameters are updated without the use of mode shapes by using eigenvalues of both the system and the system perturbed in a controlled manner.

2. Updating Procedure

This parameter updating procedure, previously presented [7], is best described with reference to Fig. 1. One or more FRFs of the structure are measured and the eigenvalues obtained from the FRFs by modal analysis. The FRFs of the structure with a mass or stiffness added at one or more locations are then measured and the eigenvalues are again obtained from the FRFs by modal analysis. The analytical eigenvalues of the corresponding structures are derived from FE models using best estimates for the parameters to be adjusted.

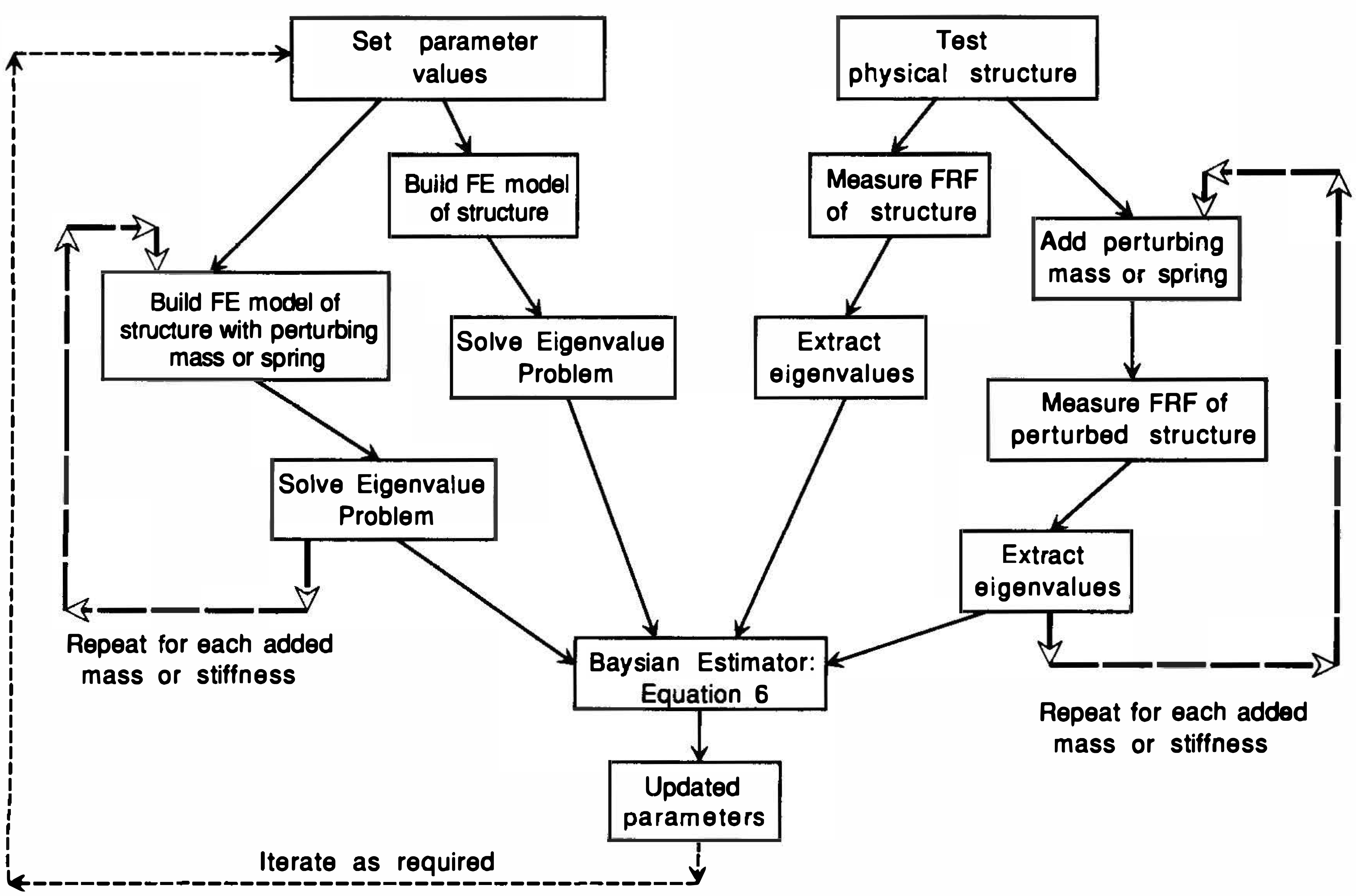


Fig. 1 Schematic of updating procedure

The corresponding measured and analytical eigenvalues are then compared. With error-free data and no mismatch between the model and the structure, structural parameters can be adjusted as follows. Suppose we measure the eigenvalues of m modes in both the original and in r perturbed systems. We then have $m(r+1)$ system eigenvalues and we can compute the corresponding model eigenvalues. The sensitivity of each of these eigenvalues can be determined with respect to each unknown system parameter using the method described by Fox and Kapoor [8]. If there are p unknown system parameters then a sensitivity matrix, $[J]$, can be formed with p rows and $m(r+1)$ columns. Thus

$$[J]\{\delta s\} = \{\delta\lambda\} \tag{1}$$

where $\{\delta\lambda\}$ is a vector of differences between the measured eigenvalues of the original and perturbed systems and the eigenvalues of the corresponding current analytical models and $\{\delta s\}$ is a vector of the differences between the current and the updated parameter estimates. The eigenvalues are nonlinear functions of the parameters to a greater or lesser extent and they must be linearized by using first order approximations. Thus Eq. (1) is used iteratively to reduce the difference between the analytical and experimental eigenvalues to zero, thereby determining the converged parameter estimates.

In practice the measured data will include measurement errors and there is likely to be a mismatch between the FE model and the real structure. Under these circumstances the parameters of the structure are updated iteratively using a Baysian approach with the constraint of minimum changes from the initial parameter estimates.

We will now examine how a proper choice is made of the coordinates at which mass or stiffness is added. Only mass addition will be explored in depth as the addition of grounded stiffnesses is analogous.

3. Choice of Coordinates to Perturb

A general rule to apply when choosing which coordinates to perturb is that the more coordinates that are perturbed, the more information that is available to update the parameters and so the better the quality of the parameter estimates. In view of this we will begin by considering the situation that arises when every degree of freedom of the model is perturbed in sequence to establish whether all the unknown parameters in the model can be identified. Let $[\Delta M_j]$ be the perturbation of the mass matrix due to the added mass at degree of freedom j. Then the measured frequencies correspond to the eigenvalues of the model given by

$$\det\left([K] - \lambda\left([M] + [\Delta M_j]\right)\right) = 0 \tag{2}$$

where $[M]$ and $[K]$ are the unperturbed mass and stiffness matrices. Since only the frequencies are measured a system with the same eigenvalues may be obtained be pre and post multiplying the matrices in Eq. (2) by any nonsingular matrices. Thus we obtain a new system , with the same eigenvalues, given by

$$\det\left([K_x] - \lambda\left([M_x] + [\Delta M_j]\right)\right) = 0 \tag{3}$$

where
$$\begin{aligned} [M_x] &= [R][M][T] \\ [K_x] &= [R][K][T] \\ [\Delta M_j] &= [R][\Delta M_j][T] \qquad \text{for } j = 1,..,n \end{aligned}$$

for any non singular matrices $[R]$ and $[T]$. The constraint involving $[\Delta M_j]$ implies that the position and value of the mass added to the structure is known. Hence the part of the mass matrix due to the mass addition must be known. Symmetry of the mass and stiffness matrices implies that $[T] = [R]^T$. Thus

$$\left[\Delta M_j\right] = [R]\left[\Delta M_j\right][R]^T \qquad \text{for } j = 1,..,n \tag{4}$$

Consider the case, without loss of generality, when a unit mass is added at coordinate j. Then Eq. (4) becomes

$$\left[Z_j\right] = \left\{R_j\right\}\left\{R_j\right\}^T \tag{5}$$

where $\{R_j\}$ is the jth column of $[R]$ and $[Z_j]$ is a matrix of zeros with a 1 in the (j,j) position. Thus, from Eq. (5), $\{R_j\}$ is a vector of zeros except ± 1 in the jth position. Since all the degrees of freedom are perturbed the matrix $[R]$ is diagonal, with ± 1 on the diagonal.

Matrix $[R]$ will give a number of different mass and stiffness matrices depending on the sign of the diagonal terms. The matrices nearest to the initial analytical matrices will occur when matrix $[R]$ is the identity matrix. Then the mass and stiffness matrices are unique. This usually implies that the parameters to be identified are unique but in some cases different sets of parameters could conceivably lead to the same mass and stiffness matrices. In this case no dynamic test will be able to distinguish between sets of parameters producing the same mass and stiffness matrices.

Assuming unique unknown parameters, then sufficient frequencies, derived from several perturbations to the system, are required to produce enough independent equations to successfully update the parameters. Simply counting the number of measured frequencies and checking that this is greater than the number of unknown parameters is not a reliable criteria to use. The only sensible method is to check the linear independence of the equations used to calculate the parameters, i.e., to calculate the condition of the sensitivity matrix $[J]$ given in Eq. (1). Equation (1) is usually solved by premultiplying by $[J]^T$, or by a singular value decomposition (SVD) technique. The first method relies on $[J]^T[J]$ being well conditioned. The condition of $[J]^T[J]$ will be approximately the square of the condition of $[J]$. Applying SVD techniques also squares the condition number of the coefficient matrix. There are methods to solve Eq. (1) without this degredation in condition [9] but these methods are not considered further. The usual method to check the condition of a matrix is to calculate the ratio of the largest to smallest singular value of the matrix.

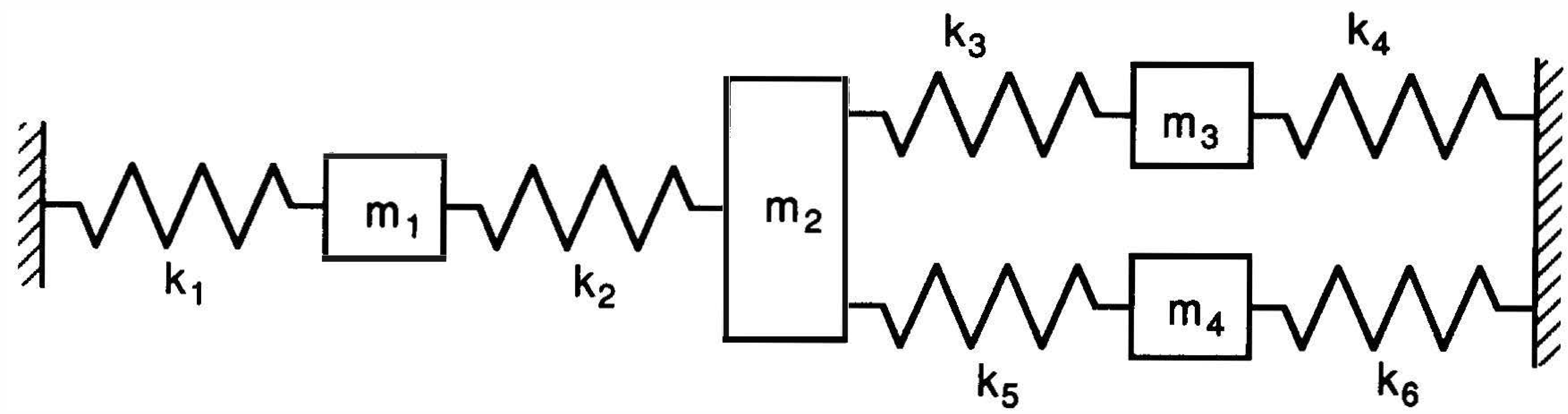
Checking the adequacy of the perturbed coordinates in this way requires the calculation of the sensitivity matrix but does not require any measured data. Thus the check may be performed before the measurements are taken. The proposed check is easily implemented since calculating the sensitivity matrix is an integral part of the updating algorithm. The current values of the unknown parameters change during the updating process so it is possible that the sensitivity matrix could become ill-conditioned. Provided the changes in the unknown parameters are small this is unlikely to occur.

What are the problems that might prevent us from choosing a correct distribution of perturbed coordinates and an adequate number of frequencies and how might the number of frequencies be increased? Considering the first question, we may not be able to make a free choice of modeled degrees of freedom of a structure to perturb because in a real structure some degrees of freedom will be inaccessible. On the question of increasing the number of natural frequencies, thereby increasing the number of equations, perturbing each available coordinate with two or more different masses would appear to increase the number of frequencies. While this technique will increase the data available it may not increase the number of independent equations. Relatively small masses added at the same coordinate will give changes in the eigenvalues that are approximately linearly related to the changes in mass. Thus the number of independent equations is not increased above the single mass case. To produce independent equations would require significant mass to be added so that the frequencies change in a nonlinear fashion with the added mass, as illustrated in the following section. If more natural frequencies are required there is no alternative but to measure the FRFs of the structure over a wider frequency band in order to encompass and identify more natural frequencies.

4. Examples of Perturbing Coordinate Selection

Three simple mass-spring systems will be used to demonstrate important features in the choice of coordinates to perturb. All calculations are performed in MATLAB which has a 17 digit accuracy. The parameters have been normalized so their initial value is unity to improve the condition of the parameter estimation. Figure 2 shows the first example which has four degrees of freedom and 10 unknown parameters (six stiffnesses and four masses). Table 1 shows the ability of the algorithm to update the unknown parameters for a number of different perturbed coordinates and measured frequencies. The algorithm will be successful if the condition of $[J]^T[J]$ is low and the rank of $[J]^T[J]$ equals the number of unknown parameters. The effective rank of $[J]^T[J]$ in Table 1 is based on the floating point arithmetic accuracy of MATLAB. It is seen that when two of the four degrees of freedom are perturbed only nine parameters may be identified independently. Perturbing three coordinates is sufficient to identify all the parameters. Figure 3 shows a similar system with the same number of unknown parameters and degrees of freedom. Table 2 shows that in this case perturbing three or even four coordinates with a mass of 0.35 kg is not necessarily sufficient to identify the 10 parameters. These two examples illustrate that the condition of $[J]$ does not only depend on the number of unknown parameters and the number of measured frequencies but also on how the parameters are distributed in the structure.

Suppose the system shown in Fig. 3 is perturbed with two or more different masses at each coordinate. For this system, Fig. 4 shows the change in the eigenvalues against the mass added to each coordinate. The added masses are given as a percentage of the mass at the perturbed coordinate. The eigenvalues, especially the lower eigenvalues, are approximately linear functions of the added mass over a wide range. Table 2



Initial value of parameters

m_1 = 9.7 kg $\quad$ m_2 = 9.7 kg $\quad$ m_3 = 4.7 kg

m_4 = 4.7 kg

$k_1 = 1.2 \times 10^6$ N/m $\quad$ $k_2 = 1.2 \times 10^6$ N/m $\quad$ $k_3 = 1.2 \times 10^6$ N/m

$k_4 = 0.95 \times 10^6$ N/m $\quad$ $k_5 = 0.96 \times 10^6$ N/m $\quad$ $k_6 = 0.6 \times 10^6$ N/m

Fig. 2 Simple 4 DOF spring mass system

shows the ability of the updating algorithm to update the 10 unknown parameters with different combinations of perturbed coordinates and measured frequencies. Using two different perturbation masses helps significantly in only one case. When all the coordinates are perturbed but only two frequencies are measured the algorithm will be unsuccessful using one perturbing mass, but is successful with the sets of two perturbing masses tried. The lack of condition in the sensitivity matrix in this case arises from rounding errors, compounded by there being only 10 equations to update 10 parameters, rather than a non-uniqueness of the parameters.

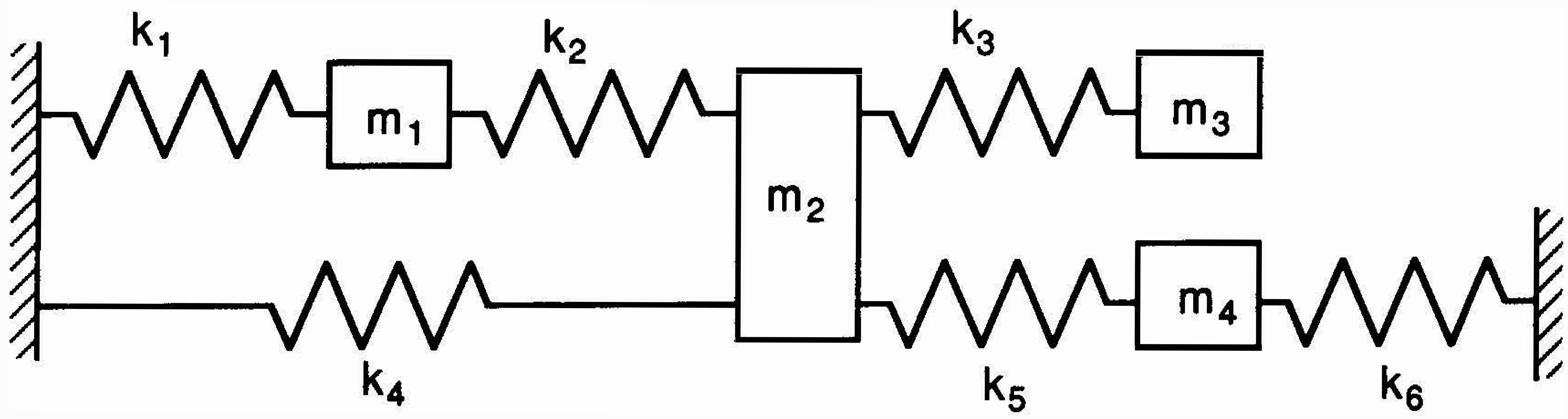
The system shown in Fig. 5 demonstrates a different problem. If a parameter has very little influence on the measured frequencies then it will be difficult to obtain any information about it. In this system it is assumed that the two masses are known and the two spring parameters are unknown. Table 3 shows the condition of $[J]$ and the condition and rank of $[J]^T[J]$ when different degrees of freedom are perturbed. If only the lowest natural frequency is measured then little information is obtained about k_1, in this case because of the difference in stiffness magnitudes. This problem may be identified using the singular value decomposition of the matrix $[J]^T[J]$ as the smallest singular value will be associated with the single problem parameter.

TABLE 1 THE CONDITION OF THE SENSITIVITY MATRIX FOR THE SYSTEM SHOWN IN FIG. 2

# of modes	perturbed coordinates	condition of $[J]^T[J]$	# of Eqs	rank of $[J]^T[J]$
4	1,2,3,4	1.63×10^7	20	10
3	1,2,3,4	1.49×10^7	15	10
2	1,2,3,4	5.41×10^7	10	10
4	1,2,3	2.57×10^7	16	10
4	1,2,4	4.29×10^7	16	10
3	1,2,3	9.72×10^8	12	10
3	1,2,4	1.51×10^8	12	10
4	1,2	2.86×10^{18}	12	9
4	2,3	1.53×10^{19}	12	9

TABLE 2 THE CONDITION OF THE SENSITIVITY MATRIX FOR THE SYSTEM SHOWN IN FIG. 3

		perturbing mass 0.35 kg			perturbing masses 0.35 & 0.7 kg			perturbing masses 0.35 & 5.0 kg		
# of modes	pertrubed coordiantes	condition of$[J]^T[J]$	# of Eqs	rank $[J]^T[J]$	condition of $[J]^T[J]$	# of Eqs	rank $[J]^T[J]$	condition of $[J]^T[J]$	# of Eqs	rank $[J]^T[J]$
4	1,2,3,4	2.08×10^7	20	10	8.09×10^6	36	10	9.32×10^5	36	10
3	1,2,3,4	1.50×10^8	15	10	2.97×10^7	27	10	9.92×10^5	27	10
2	1,2,3,4	1.66×10^{15}	10	9	6.59×10^{10}	18	10	1.38×10^8	18	10
4	1,2,3	3.02×10^7	16	10	1.12×10^7	28	10	7.79×10^5	28	10
4	1,2,4	9.18×10^{18}	16	9	7.80×10^{17}	28	9	6.67×10^{17}	28	9
3	1,2,3	8.04×10^8	12	10	1.46×10^8	21	10	2.69×10^6	21	10
3	1,2,4	3.38×10^{18}	12	9	6.54×10^{17}	21	9	1.10×10^{18}	21	9
4	1,2	6.48×10^{18}	12	9	2.58×10^{18}	20	9	1.81×10^{18}	20	9
4	2,3	2.37×10^{18}	12	9	1.96×10^{18}	20	9	1.01×10^{18}	20	9



Initial value of parameters

$m_1 = 9.7$ kg $\quad m_2 = 9.7$ kg $\quad m_3 = 4.7$ kg

$m_4 = 4.7$ kg

$k_1 = 1.2 \times 10^6$ N/m $\quad k_2 = 1.2 \times 10^6$ N/m $\quad k_3 = 0.96 \times 10^6$ N/m

$k_4 = 0.6 \times 10^6$ N/m $\quad k_5 = 1.2 \times 10^6$ N/m $\quad k_6 = 0.95 \times 10^6$ N/m

Fig. 3 Simple 4 DOF spring mass system

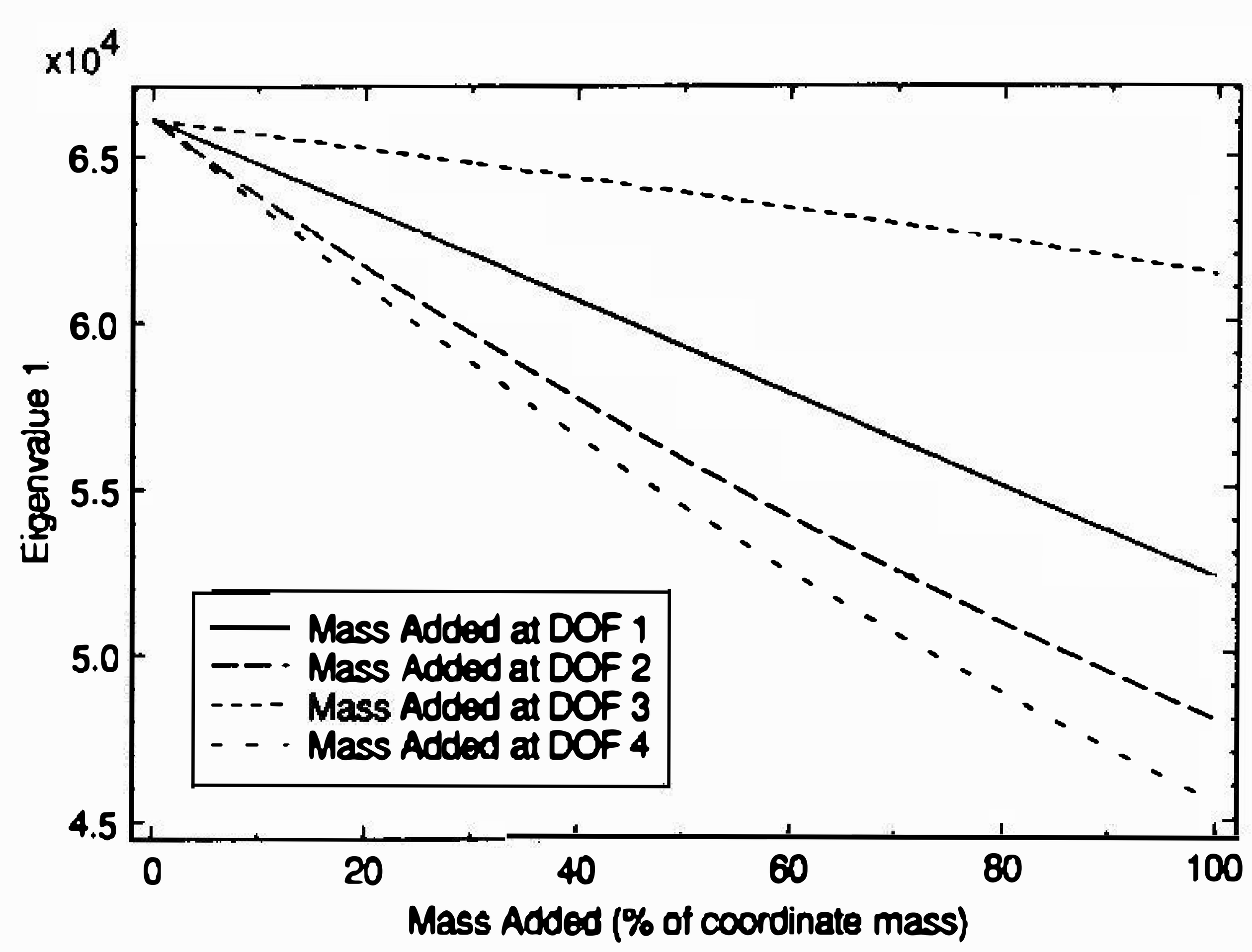


Fig. 4a Variation of the lowest eigenvalue with added mass

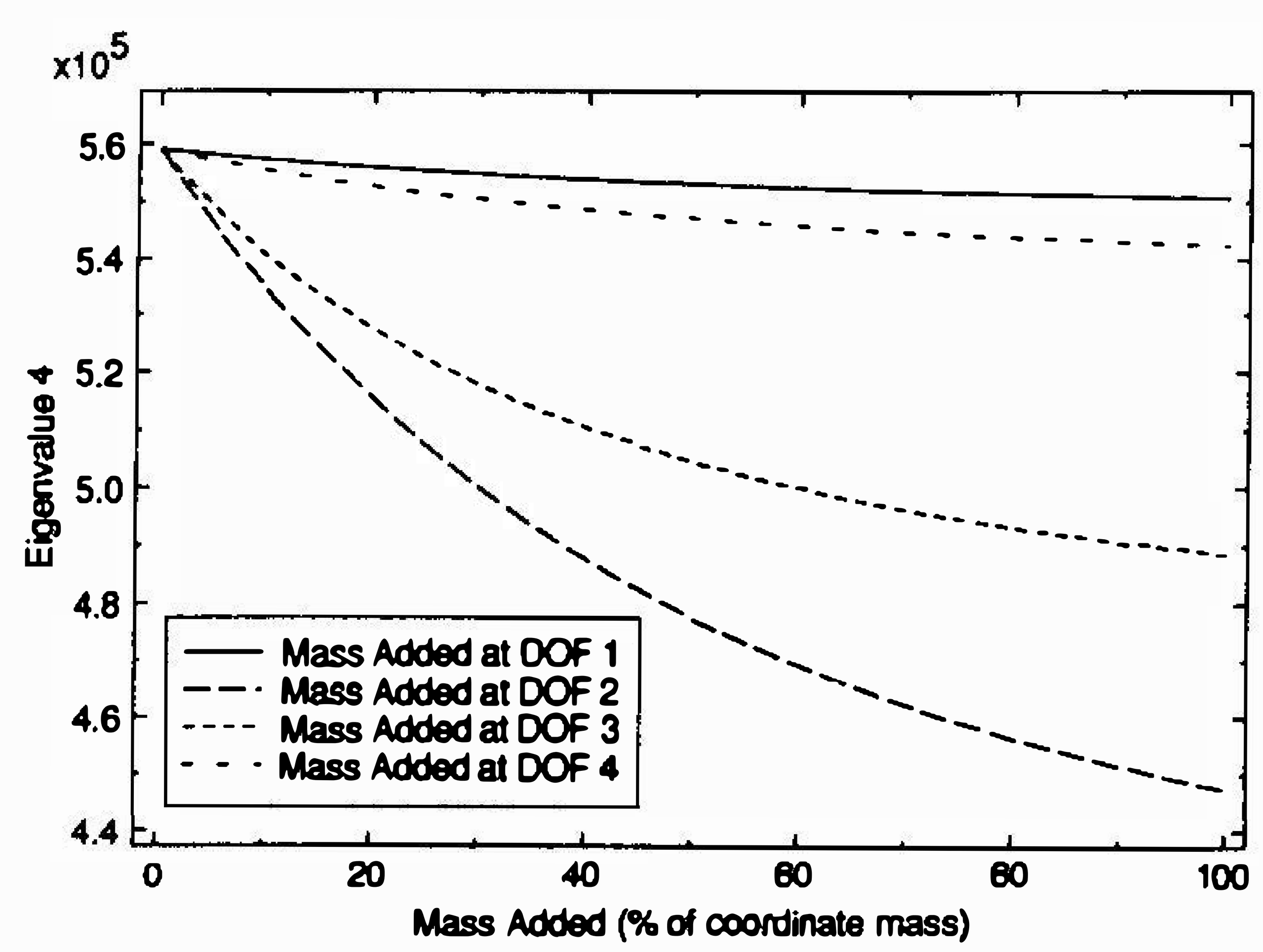
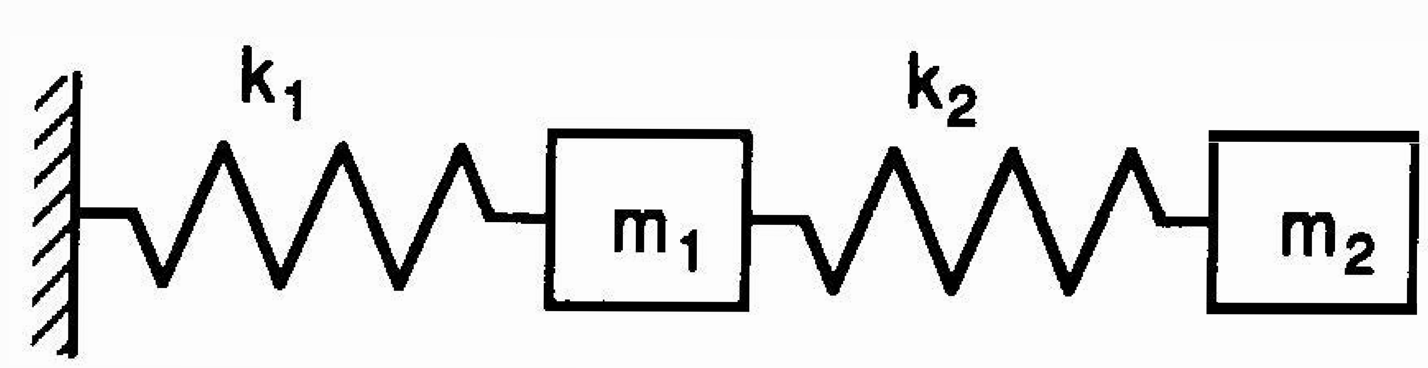


Fig. 4b Variation of the highest eigenvalue with added mass

TABLE 3 THE CONDITION OF THE SENSITIVITY MATRIX FOR THE SYSTEM SHOWN IN FIG. 5

# of modes	perturbed coordinates	condition of [J]	condition of $[J]^T[J]$	# of Eqs	rank of $[J]^T[J]$
2	1,2	1.00×10^4	1.00×10^8	6	2
1	1,2	4.47×10^8	2.00×10^{17}	3	1
2	1	9.37×10^3	8.78×10^7	4	2
2	2	1.07×10^4	1.14×10^8	4	2
1	1	6.67×10^8	4.51×10^{17}	2	1
1	2	7.74×10^8	6.03×10^{17}	2	1



Initial value of parameters

$m_1 = 1$ kg $\quad m_2 = 1$ kg

$k_1 = 1$ N/m $\quad k_1 = 10^{-4}$ N/m

Fig. 5 Simple 2 DOF spring mass system

5. Simulated Updating Example

The four DOF spring-mass system shown in Fig. 2 will be used to test the updating algorithm. The system is simulated using the mass and stiffness parameters given in Table 4 which results in natural frequencies at 42.87, 76.33, 94.29 and 117.66 Hz. An initial analytical model is assumed whose parameters are given in Table 4 and which produces natural frequencies at 46.46, 84.02, 98.90 and 126.63 Hz. Consider updating by adding a mass of 0.35 kg, in turn, at coordinates 1, 2 and 4 and using eigenvalues of the first three modes of the unperturbed and the perturbed systems in the updating process. The previous section showed that this selection of perturbing coordinates and natural frequencies is able to update all 10 parameters. The natural frequencies of the simulated and the analytical model, with and without the added mass, are given in Table 5. The parameters were updated, using this eigendata, by a least squares solution of Eq. (1). The parameters converged to their correct values and Table 4 shows the convergence at each iteration step.

TABLE 4 CONVERGENCE OF THE PARAMETERS OF THE SYSTEM SHOWN IN FIG. 2

	simulation values	initial estimates	updated estimates-iteration steps			
			1	2	3	4
k_1 ($\times 10^6$ N/m)	1.00	1.20	0.9258	0.9978	1.0000	1.0000
k_2	1.00	1.20	1.0217	1.0007	1.0000	1.0000
k_3	1.00	1.20	1.0346	0.9952	1.0000	1.0000
k_4	1.00	0.95	1.0572	1.0063	1.0000	1.0000
k_5	1.00	0.96	0.9981	0.9996	1.0000	1.0000
k_6	0.50	0.60	0.4673	0.4984	0.5000	0.5000
m_1 (kg)	10.00	9.70	9.8027	9.9954	10.0000	10.0000
m_2	10.00	9.70	9.7228	10.0224	10.0001	10.0000
m_3	5.00	4.70	5.1634	5.0086	4.9999	5.0000
m_4	5.00	4.70	4.9098	4.9945	5.0000	5.0000

TABLE 5 SIMULATED AND ANALYTICAL NATURAL FREQUENCIES FOR THE SYSTEM SHOWN IN FIG. 2

0.35kg added @ coordinate	simulated natural frequency			analytical natural frequency		
	mode 1	mode 2	mode 3	mode 1	mode 2	mode3
None	42.87	76.33	94.29	46.46	84.02	98.90
1	42.65	75.47	94.25	46.24	83.09	98.82
2	42.53	76.25	94.19	46.08	83.96	98.78
4	42.60	75.66	92.23	46.19	83.07	97.38

6. Updating the Parameters of Real Structures

So far it has been assumed that the measured eigenvalues are error-free and the structure of the model matrices is exact. In practice this is hardly the case and a practical updating algorithm has to take into account these facts. We will assume the eigendata is contaminated by random errors with an expected mean of zero and standard deviations which can be reasonably estimated. In this case, the weighted least squares solution method can be used to find parameter updates on the current analytical model. The weighted least squares method results in unbiased estimates and, in theory, the result will approach the correct parameters as the data sample becomes infinitely large. In practice measured data is finite and the model and system are not identical. Therefore the 'exact' parameters, if they exist, cannot be identified. Numerical study with the weighted least squares method [10] shows that with a practical size of the data sample, random errors can producc inaccurate parameters. The accuracy of the updated parameters is considerably degraded by insensitivity of the eigenvalues to changes in some parameters. This leads to ill-conditioning of the parameter estimation process already demonstrated in section 4. As a rule, stiffness parameters of elements undergoing little flexure and mass parameters of elements undergoing little displacements for the modes used in the updating are the least accurate. In some cases small errors in the eigenvalues can result in a considerable degradation in the accuracy of the updated parameters.

Many authors have updated the parameters by incorporating a constraint of minimum changes of the parameters from either the current or initial analytical model, usually applied as a Bayesian updating algorithm [5,6]. Giving this weight to the current or initial parameters makes the updating processes well conditioned even when the measured eigenvalues provide insufficient information on their own. In this paper the Bayesian approach will be used, incorporating a constraint of minimum changes on the parameters of the initial model, $\{s_i\}$, rather than the parameters of the current model, $\{s_c\}$. The updated parameter estimates, using this approach, are given by Eq. (6). In this equation $\{\delta s\}=(\{s\}-\{s_c\})$ where $\{s\}$ are the updated parameters, $[W_\lambda]$ and $[W_s]$ are the weighting matrices for the measured eigenvalues and the parameters of the initial analytical model.

$$\{\delta s\} = \left([J]^T[W_\lambda][J]+[W_s]\right)^{-1}\left\{[J]^T[W_\lambda]\{\delta\lambda\}+[W_s]\{s_i - s_c\}\right\} \qquad (6)$$

The unbiased estimator, Eq. (6), has been used to update the parameters of a simple but real structure by the mass addition method. The results for a free beam are given in the following section.

A potential difficulty that can arise when applying the mass addition technique to real structures is that adding inertia to a translatory degree of freedom may also add inertia to a coincident rotary degree of freedom and vice-versa. The difficulty can be resolved by either including both translational and rotational inertia terms in the perturbed mass matrix or ensuring that the perturbing inertia has a negligible radius of gyration and a significant mass or a significant radius of gyration and a negligible mass. The fact that we can create added inertias with a range of translation to rotation inertia ratios can be advantageous in generating more independent equations in the sensitivity matrix.

7. Experimental Example

An aluminum beam of size 25×50×800 mm long was supported, horizontally, by two sets of relatively soft springs to simulate an unrestrained configuration. The supports were situated at 90 mm and 110 mm from the opposite ends of the beam. The beam was set with the wider side of the cross-section horizontal. An electrodynamic shaker was used to excite the beam at 490 mm from one end, using random excitation.

Eigenvalues were extracted from the measured FRFs using Dobson's modal analysis algorithm [11] assuming hysteretic damping. The measurement frequency range was from 0 to 1600 Hz at 2 Hz resolution.

The beam was also modeled by an 8 element 18 DOF FE model as shown in Fig. 6. A Young's modulus of 70 kN/mm^2 was used for aluminum and a mass loading (m_L) of the shaker-beam interface was included in the modeling, assuming an initial value of zero. With this data and from the dimensions and mass of the beam, the parameters used in the FE model and their confidence, expressed by their standard deviation estimates are given in Table 6.

The experimental natural frequencies in the measurement frequency range were 200.1, 548.4 and 1075.0 Hz and the corresponding analytical natural frequencies were 203.7, 561.8 and 1103.0 Hz.

The analytical model was updated by mass addition using eigenvalues of the first two elastic modes before and after each mass addition. To simulate a system with many parameters, the parameters for the eight beam elements were treated as independent. Thus, there were a total of 17 parameters to update. The added mass was 0.2 kg at each of the translational coordinates. It added a negligible inertia to the corresponding rotational coordinate. Thus 20 frequencies were measured. In this case, the condition number of $[J]^T[J]$ was 3.82×10^9, implying that the coordinates chosen were adequate to identify the exact parameters if the data were error-free and the structure of the model matrices exact. Due to the measurement errors, however, exact parameters could not be identified. The parameters were updated for an optimum model using the Bayesian estimator. The experimental and analytical natural frequencies of the first two

TABLE 6 INITIAL AND UPDATED PARAMETERS OF THE BEAM SHOWN IN FIG. 6 (ESTIMATED STANDARD DEVIATION IN PARENTHESES)

element number	stiffness (Nm^2) initial value		stiffness (Nm^2) updated	mass (kg/m) initial value		mass (kg/m) updated
1	4557	(150)	4520	3.4	(0.1)	3.22
2	4557	(150)	4337	3.4	(0.1)	3.50
3	4557	(150)	4384	3.4	(0.1)	3.52
4	4557	(150)	4517	3.4	(0.1)	3.42
5	4557	(150)	4450	3.4	(0.1)	3.41
6	4557	(150)	4426	3.4	(0.1)	3.41
7	4557	(150)	4459	3.4	(0.1)	3.43
8	4557	(150)	4529	3.4	(0.1)	3.37
m_L(kg)				0.0	(0.1)	0.076

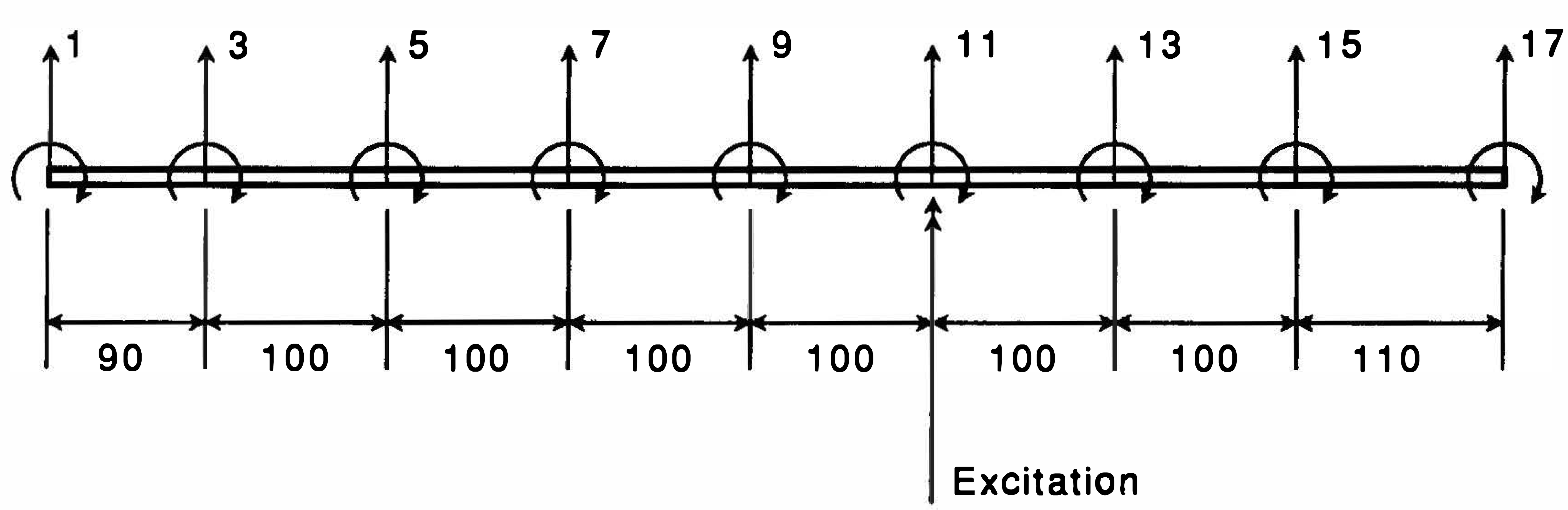


Fig. 6 Dimensions and measurement locations of the beam (dimensions in mm)

modes after each mass addition are shown in Table 7. Standard deviations of 0.5 Hz and 1.0 Hz were assumed for the natural frequencies for all the first and all the second modes respectively. The updated parameters, after four iterations, are given in Table 6. The convergence of the parameters is rapid and essentially complete after two or three iterations.

The updated model was used to predict the FRFs of the beam at coordinates 1 and 15 for an excitation at coordinate 11. The predicted receptances at these coordinates are compared with measured and initial model receptances in Fig. 7. The updated model has reasonably reproduced the FRFs of the beam over the whole frequency range (0-1600 Hz) although the highest natural frequency used in the updating algorithm was 545.6 Hz and the frequency response data was not used in the updating process.

8. Conclusions

The mass or stiffness addition technique to update the parameters of a dynamic model has been presented. The technique uses eigenvalues of the structure before and after the structure is perturbed by adding mass or stiffness. With error-free data and exact structure of the model matrices, correct mass and stiffness parameters can be identified by a proper choice of the perturbing coordinates. The adequacy of

TABLE 7 EXPERIMENTAL AND ANALYTICAL NATURAL FREQUENCIES OF THE BEAM SHOWN IN FIG. 6.

0.2 kg added @ coordinate	experimental natural frequency (Hz) mode 1	mode 2	analytical natural frequency (Hz) mode 1	mode 2
None	200.1	548.4	203.7	561.8
1	179.3	493.6	183.3	514.1
3	194.1	541.2	198.2	560.6
5	200.1	525.2	203.6	540.4
7	195.2	521.2	198.4	537.6
9	190.8	548.5	194.2	561.5
11	193.9	528.6	197.1	542.8
13	199.7	522.5	203.1	535.3
15	196.2	544.9	200.4	561.7
17	179.1	500.9	183.3	514.0

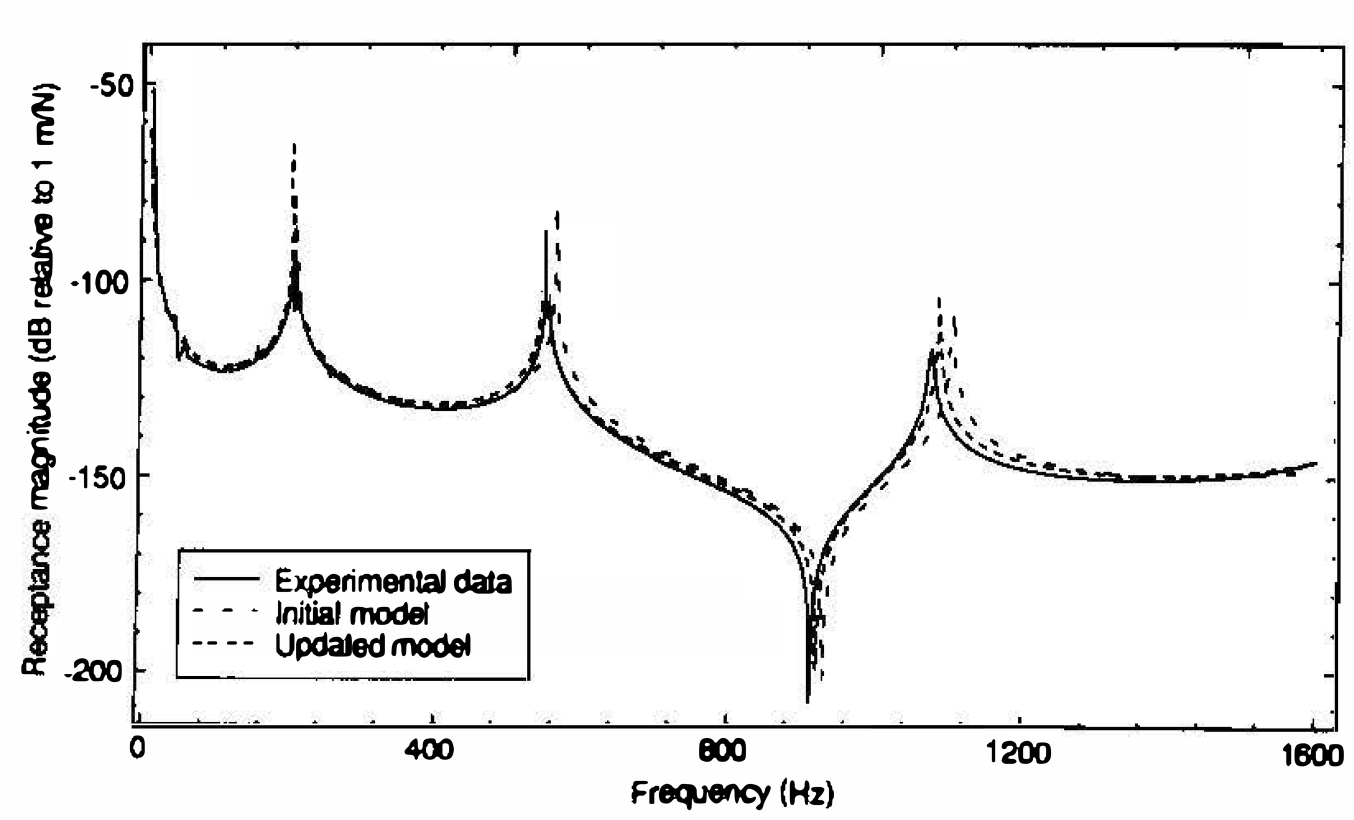


Fig. 7a FRF for the experimental beam example (response at coordinate 1)

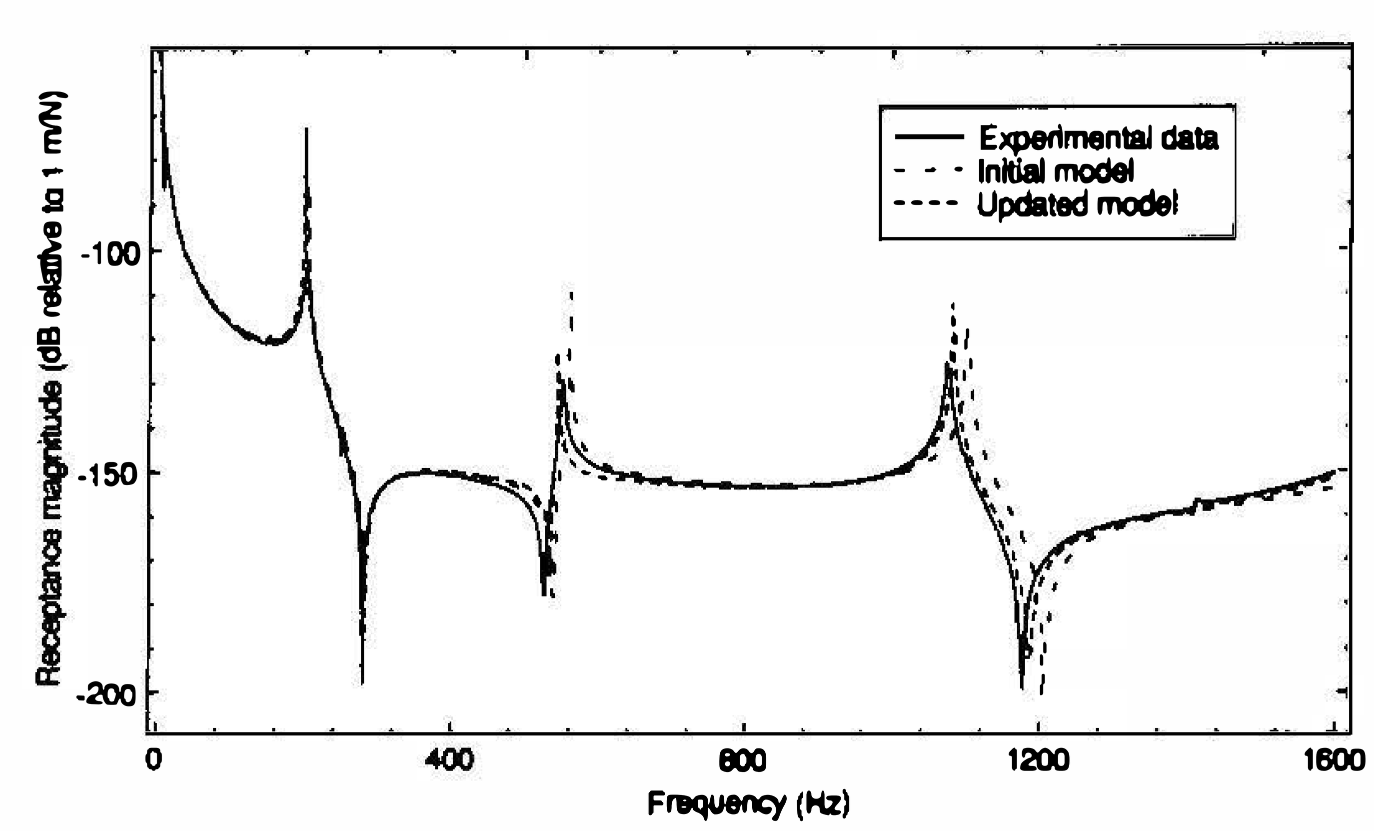


Fig. 7b FRF for the experimental beam example (response at coordinate 15)

the perturbing coordinates is established by considering the condition number of the sensitivity matrix. This adequacy not only depends on the number of unknown parameters but also on their distribution in the structure. The success of the technique depends on sufficient perturbation of the structure so that significant natural frequency changes are measured. For this reason the mass addition technique may not be suitable for a large structure because the size of the additional mass necessary to produce even a small natural frequency change may be impractical.

References

1. Baruch, M.; Bar-Itzhack, I.Y. "Optimal weighted orthogonalization of measured modes." *AIAA J* v 16 p 346-351 1978.
2. Berman, A.; Nagy, E.J. "Improvement of a large analytical model using modal test data." *AIAA J* v 21 p 1168-1173 1983.
3. Tlusty, J.; Ismail, F. "Dynamic structural identification: tasks and methods." *Ann CIRP* v 29 p 251-255 1980.
4. Ibrahim, S.R. "Dynamic modeling of structures from measured complex modes." *AIAA J* v 21 p 893-901 1983.
5. Collins, J.D.; Hart, C.G.; Hesselman, T.K.; Kennedy, B."Statistical identification of structures." *AIAA J* v 12 p 185-190 1974.
6. Friswell, M.I. "The adjustment of structural parameters using a minimum variance estimator." *Mech Syst Signal Process* v 3 p 143-155 1989.
7. Nalitolela, N.G.; Penny J.E.T.; Friswell, M.I. "Updating structural parameters of a finite element model by adding mass or stiffness to the system." *Proceedings of the 8th International Modal Analysis Conference,* Kissimmee, FL, Jan 29-Feb 1, 1990. v 2 p 836-842
8. Fox, R.L.; Kapoor, M.P. "Rates of change of eigenvalues and eigenvectors." *AIAA J* v 6 p 2426-2429 1968.
9. Rothwell, E.; Drachman, B. "A unified approach to solving ill-conditioned matrix problems." *Int J Numer Methods Eng* v 28 p 609-620 1989.
10. Nalitolela, N.G. *System Identification of Linear Vibrating Structures.* PhD Thesis, University of Aston in Birmingham, UK, 1991.
11. Dobson, B.J. "A straight line technique for extracting modal properties from frequency response data." *Mech Syst Signal Process* v 1 p 29-40 1987.