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ABSTRACT

The measurement and utilization of rotational degrees of freedom for experimental modal analysis applications is presented in this work. The experimental data is obtained with a novel noncontacting measurement approach capable of simultaneously sensing dynamic translation (y) and two dynamic rotations (θ_x, θ_z) of a vibrating surface. The paper briefly discusses transducer operation and its performance. The transducer is utilized for the modal analysis of two structures (a cantilever beam and a printed circuit board) providing estimates of both the translation and the rotation modal residues at each measurement test position. Several methods to combine the estimated translation and the rotation modal residues to refine the description of the mode shapes are presented and compared. The integration focuses on the use of polynomial curve fitting methods incorporating both translation (translational residues) and slope (rotational residues) at each test location to define the mode shapes. The results show that combined use of translational and rotational degrees of freedom reduces the effects of experimental measurement uncertainty and provides a significantly improved spatial description of mode shapes.

1. Introduction

The use of experimental rotational degrees of freedom in modeling structures has recently become an area of significant interest [1]. The need for rotational data originally stems from the desire to develop accurate finite element models of structural systems. Typically, finite element models contain information representing three translations (x, y, z) and three rotations (θ_x, θ_y, θ_z) at each node in the model. Unfortunately, current experimental modal analysis methods infer all characteristics solely from translation data. This mismatch in the data further exacerbates the already difficult fine tuning process of computational models [2]. The inability to easily measure rotational degrees of freedom has precluded the incorporation of more sophisticated elements (e.g. beam elements) in structural dynamics modification efforts [3]. Furthermore, description of mode shapes extracted from experimental data can be enhanced by combining both translational and rotational modal residue data.

It is apparent that the use of rotational degrees of freedom in experimental modal analysis has many potential uses [4-7]. However, their implementation has not matured because of limited experimental methods available for measurement of rotational degrees of freedom. One objective of this paper is to discuss the utilization of a novel transducer system, with rotational capabilities, for experimental modal analysis applications. The system is capable of simultaneously measuring one translational and two rotational deflections of a vibrating surface in a noncontacting fashion. The second objective is to demonstrate that rotational capabilities of this measurement technique provide a means to integrate

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translational and rotational modal residues and better define the spatial descriptions of mode shapes.

The following section will briefly discuss the operation of the novel transducer system. The transducer is subsequently applied to two structures to examine the measurement and utilization of the rotational data in modal testing. A simple one-dimensional cantilever beam is first analyzed to produce estimates of both translational and rotational modal residues. Several polynomial curve fitting methods are then compared to examine how to refine the mode shapes by combining the translational and rotational residues. The concept is extended to two dimensions and explored in a more practical sense by analyzing a printed circuit board.

2. Translational/Rotational Measurement Technique

The measurement system is schematically illustrated in Fig. 1 [8]. The system is based on positional measurement of two collimated light beams reflected from a planar reflective target. The light beams are created by lasers and the locations of their respective reflections are measured with two separate two-dimensional photodetectors. Using the geometric orientation between the origins of the light beams and the corresponding reflections onto the photodetectors, vertical translation (y) as well as roll (θ_x) and pitch (θ_z) angular deflections of the planar target can be determined. The time varying x-z coordinate signals from the photodetectors are digitized and then processed to estimate the three target pose (position and attitude) variables. The algorithm used to extract the pose variables from the geometric layout and photodetector signals is based on kinematic closure principles developed in Refs. [8,9].

The basic operation of the transducer is depicted by the block diagram shown in Fig. 1. The vibration time based signals (x-z positional outputs from each photodetector) are filtered and then acquired using a simultaneous sample and hold module attached to a Metrabyte A/D board installed in a Compaq 386 computer. The kinematic closure algorithm is then applied to the photodetector signals to calculate the corresponding time variant vertical translation, pitch angle, and roll angle functions. Data processing is performed offline and not in real time.

The system has been evaluated and was shown to possess the performance parameters shown in Table 1 [8]. The theoretical upper frequency limit of the transducer is 30 kHz as dictated by the analog response of the photodetector transimpedance amplifiers. However, the maximum sampling rate of the analog to digital hardware used was 2.4 kHz. Therefore, based on the Nyquist sampling criteria, the present prototype was frequency limited to a 1.0 kHz usable frequency range. Extensive evaluations have demonstrated the measurement technique's ability to simultaneously measure high quality translation and angular deflection vibration data [10].

3. Modal Analysis of a Cantilever Beam

Experimental modal analysis of a simple cantilever beam was performed to demonstrate use of the transducer and utilization of rotational degrees of freedom. A brass beam was used for the experiment with approximate dimensions of 225 x 25 x 3 mm. Nine flat circular front surface mirrors (18.0 mm diameter and 0.2 mm thickness) were mounted equidistantly along the beam with double-sided tape to provide reflective targets as shown in Fig. 1.

An instrumented modal hammer was used as the excitation source. The centerline of the beam at mirror site 3 was chosen as the driving point. The transducer was moved to each mirror site along the beam and five ensemble sets of time input and response data were acquired at each site. Examples of typical time

TABLE 1 PERFORMANCE SPECIFICATIONS OF THE THREE DEGREE OF FREEDOM LASER VIBROMETER

Measurement Variable	Workspace	Absolute Error (RMS)	Precision
Vertical Translation	+2.5 mm to -2.5 mm	0.04 mm	0.01 mm
Roll Angle	+5 deg to -5 deg	0.03 deg	0.01 deg
Pitch Angle	+5 deg to -5 deg	0.02 deg	0.01 deg

Frequency Range: Electrical = 30 kHz, A/D Board = 1 KHz

Target Footprint (minimum target size): 2 mm by 3 mm rectangle

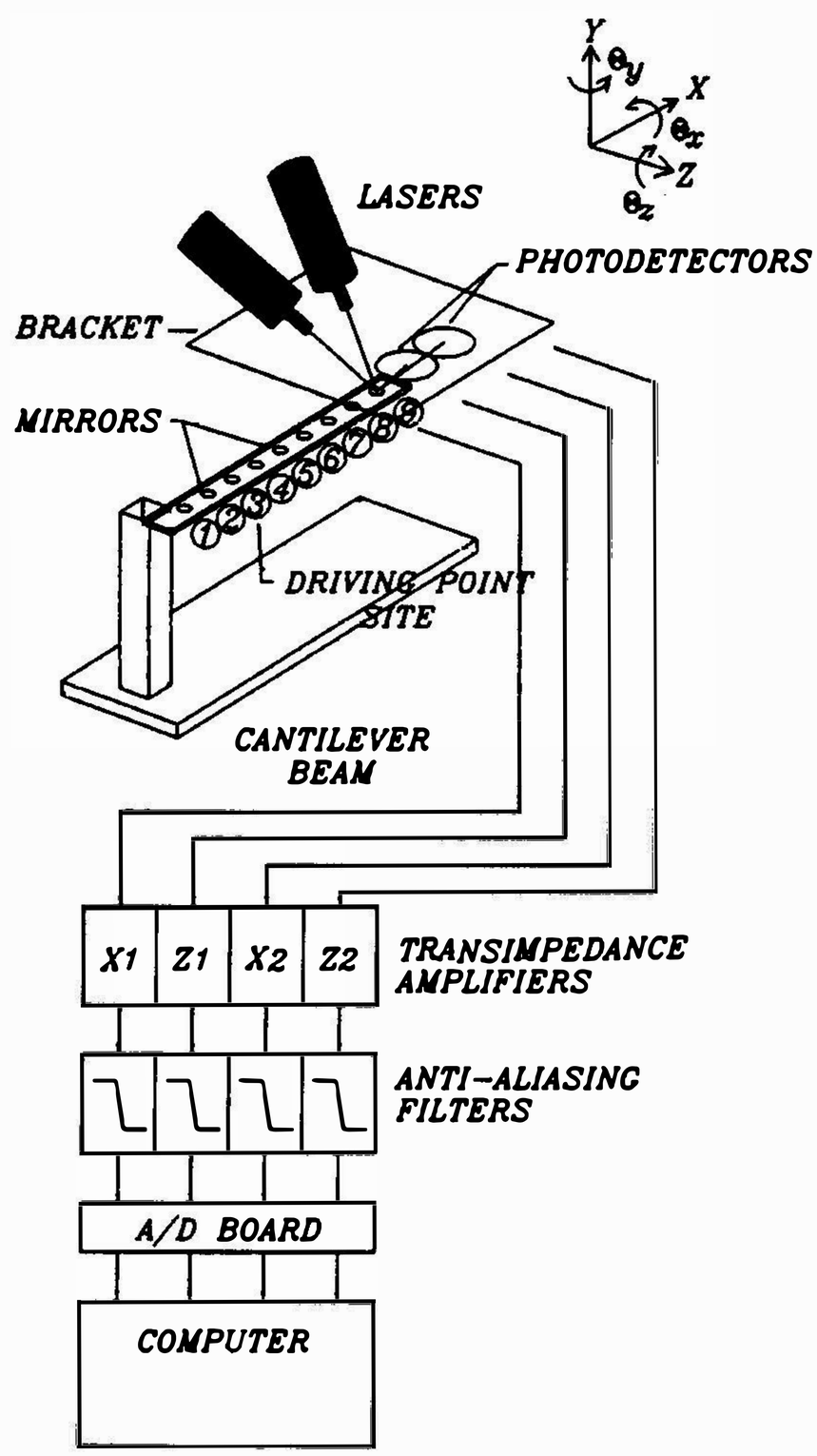

Fig. 1 Schematic of three degree of freedom laser vibrometer

response signals at test site 9 from a hammer impact at test site 3 are illustrated in Fig. 2. All the signals possess the anticipated characteristics. The hammer impulse is shown in Fig. 2(a). Since the transducer prototype digitizing hardware does not have pretrigger capability slight initial signal truncation is apparent in the waveforms. The translation (y) response in Fig. 2(b) and pitch (θ_z) response in Fig. 2(c) have the greatest magnitudes while the roll (θ_x) response (Fig. 2(d)) is small. Next, time histories were processed with a FFT for frequency domain analysis. Because of the slowly decaying nature of the waveforms apparent in Fig. 2, an exponential window was applied to the response time signals. FFT results at each test site were averaged over the five ensemble data sets at each site to estimate the respective auto and cross spectral quantities. Finally, the frequency response functions (FRF) and ordinary coherence functions for each of the nine test sites along the beam were formed.

The frequency response function and coherence plots for test site 9 are shown in Fig. 3. Solid lines are for vertical translation (y), dashed lines represent pitch angle (θ_z), and dotted lines are for roll angle (θ_x). Coherence for all three pose variables was excellent across the frequency range of interest. The magnitude of all three FRF's show resonant peaks at 31 and 197 Hz. As expected the magnitudes of the translation and pitch FRF's are significant, while the roll response is approximately 20 dB lower. Theoretically, the first two bending modes of the beam should occur near 34 and 217 Hz [11] which represents good agreement with the measured results.

It is expected that beam bending modes should be readily detectable from either the translation or pitch measurements. Further, the roll measurement should theoretically be zero. However, the roll FRF in Fig. 3 shows two distinguishable peaks at the beam bending frequencies. This is attributed to two possible causes: 1) a slight rotational misalignment of the transducer axis relative to the beam centerline axis or 2) a departure from theoretical test conditions (e.g., an imperfect clamped boundary condition or misplacement of the hammer excitation off the exact beam centerline). It should be noted that a rotational misalignment of approximately two degrees between the beam centerline and the transducer is sufficient to cause the artifactual roll measurement. Inspection of the phase plot in Fig. 3, shows that at the resonant peaks, the expected 180 degree phase shift occurs. The translation phase starts at zero degrees as expected since the

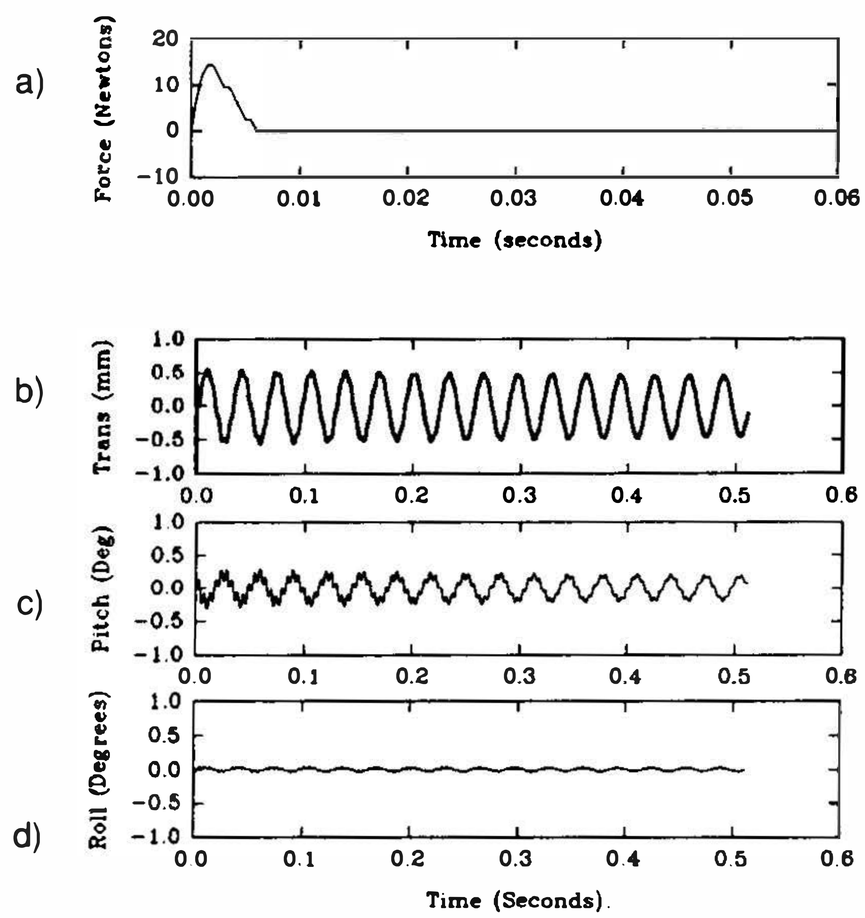

Fig. 2 Typical time response signals from the cantilever beam impact test at site 9: a) hammer impulse; b) translation (y); c) pitch (θ_z); d) roll (θ_x)

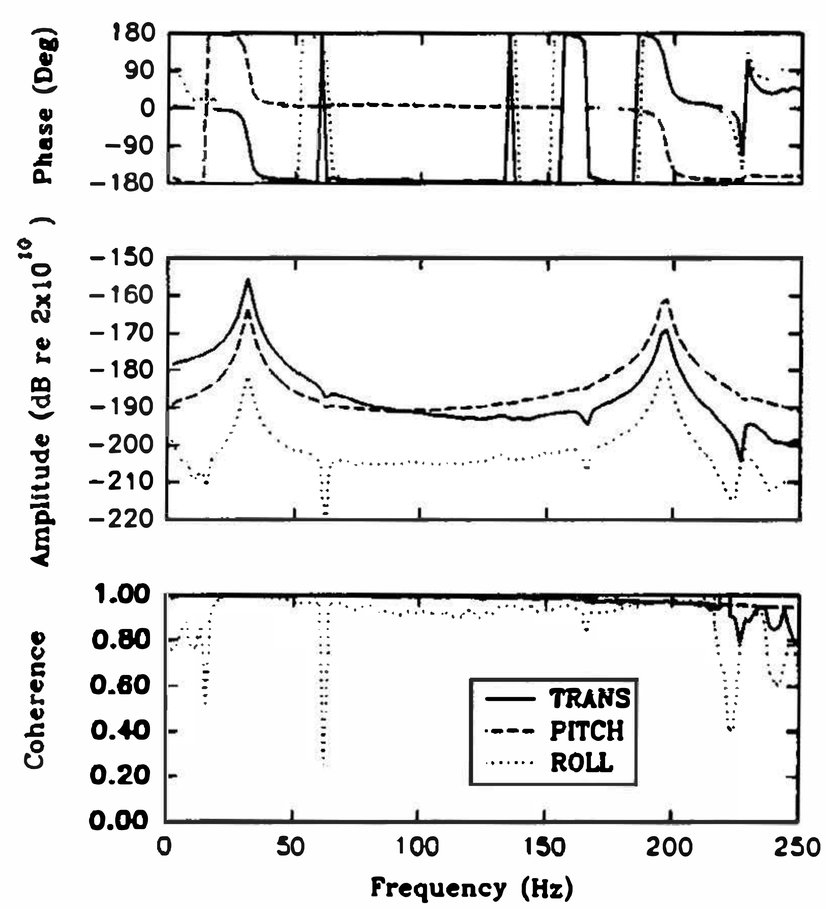

Fig. 3 Typical frequency response and coherence functions from the cantilever beam impact test at site 9 (dB reference: 2×10^{10} mm/N for translation; 2×10^{10} degrees/N for rotation)

initial impact was in the upward (positive) vertical direction. The pitch starts 180 degrees out of phase as expected since positive pitch measured by the transducer is measured down relative to the horizontal.

(a) Modal Analysis Results

Modal parameters were extracted from this family of 27 frequency response functions (9 translation, 9 pitch, and 9 roll) by a complex exponential algorithm [12]. Figure 4(a) presents the mode shapes for the first two bending modes of the test beam using only the translational (y) data. The theoretical mode shapes are represented by the solid lines while the measured data are represented by the triangles and circles. In general, there was very good agreement between the theoretical and the measured values. The first mode has one node at the clamped end while the second mode has two nodes, one at the clamped end of the beam and one that was located at approximately 165 mm along the beam. Based on Bernoulli-Euler beam theory, the distal node of the second mode should occur 176 mm from the clamped end [11].

A less traditional way to view the bending modes is shown in Fig. 4(b) where pitch angle (θ_z) residues are plotted. The pitch (θ_z) modes are actually the spatial derivatives of the translation modes with respect to position along the length of the beam. The measured values are overlaid on the theoretical values and good agreement is again illustrated. One of the practical uses of the pitch mode shape is to better define the location of an anti-node. The location of the anti-node is apparent where the pitch mode shape crosses the zero line. For mode two, the anti-node is located 105 mm from the clamped end by interpolating the pitch measurements between sites 4 and 5. This is excellent agreement with the theoretical value of 106 mm. If only a translational transducer were used, an additional measurement site would be required at the anti-node position to verify its location. Another use of the pitch mode shape would be to check that adequate spatial resolution was used when setting up the test grid. The appearance of inflection points in the pitch mode shape would infer an inadequate grid resolution and possible spatial aliasing.

(b) Integration of Translational and Rotational Mode Shape Data

The translation and angular modal information contained in Fig. 4 may be combined to refine the description of the mode shapes. Since angular deflection represents the spatial derivative at each respective measurement site, the two can be superimposed. However, precautions must be taken to accomplish this merger correctly. The most critical aspect is scaling of the respective modal deflection coefficients with respect to one another. In applications when only an animated form of the mode shape is desired (e.g., computer graphic presentation of Fig. 4), the transducer sensitivities and modal algorithm scaling are immaterial, and often ignored. This is possible since only relative values are important. However, the combination of translational and rotational modal data requires close scrutiny because of the relationship between the spatial translation (represented by translation modal coefficient) and its spatial derivative (represented by the rotational modal coefficient).

To maintain the proper relationship between the two variables, it is necessary to base the mode shape description on the modal residues. This can be explained by realizing that the modal residue represents the amplitude of the corresponding impulse response function. For the cantilever beam impulse test described above, the translation residue units are mm/N-s. Similarly, the rotational modal residue units are deg/N-s, because a force and not a moment induced the response. The resulting vibration response amplitudes would be in units of mm (translation) or degrees (rotation) and would be numerically correct with respect to the magnitude of the applied impulse. Furthermore, the proper dependent relationship between the modal translation and its spatial derivative, expressed by the rotational residue is maintained.

The restriction of using modal residues to merge the rotation and translation data has several ramifications with respect to how the modal data is acquired and processed.

1. The vibration data must be acquired in correct engineering units through use of the appropriate calibration factors and instrumentation gains.
2. The modal extraction algorithms used must be capable of estimating the modal residues and not simply a "modal deflection". This precludes the use of many algorithms such as the Quadrature Response Method and many simple circle fitting methods. Also implied, is the ability of the algorithm to estimate the residues from either accelerance, mobility or compliance data.
3. Once the residues are estimated, any scaling factor applied for graphic display must be multiplied identically to both the translation and rotational values. This ensures that the relationship between the residues is maintained. For a one-dimensional case such as the cantilever beam, this would imply that if the beam length were scaled by a factor c_1 and the translation residue was scaled by a factor c_2, then the rotational residue must be scaled by a factor c_2/c_1.

Since the complex exponential algorithm utilized in this effort produced residue estimates, the two modal data sets are numerically consistent and can be combined. By plotting short straight line segment centered at the translation residues and with their slopes determined from the rotational residues, a more descriptive illustration of the mode shapes can be depicted as in Fig. 5. It is apparent that even this simple combination presents a more informative view of the mode shapes than in their individual forms.

(c) Integration of Translational and Rotational Mode Shape Data Via Polynomial Curve Fitting

The next logical step in combining translational and rotational residues after superposition is to fit a function to the respective translation and slope data. By comparing different fitting methods that include

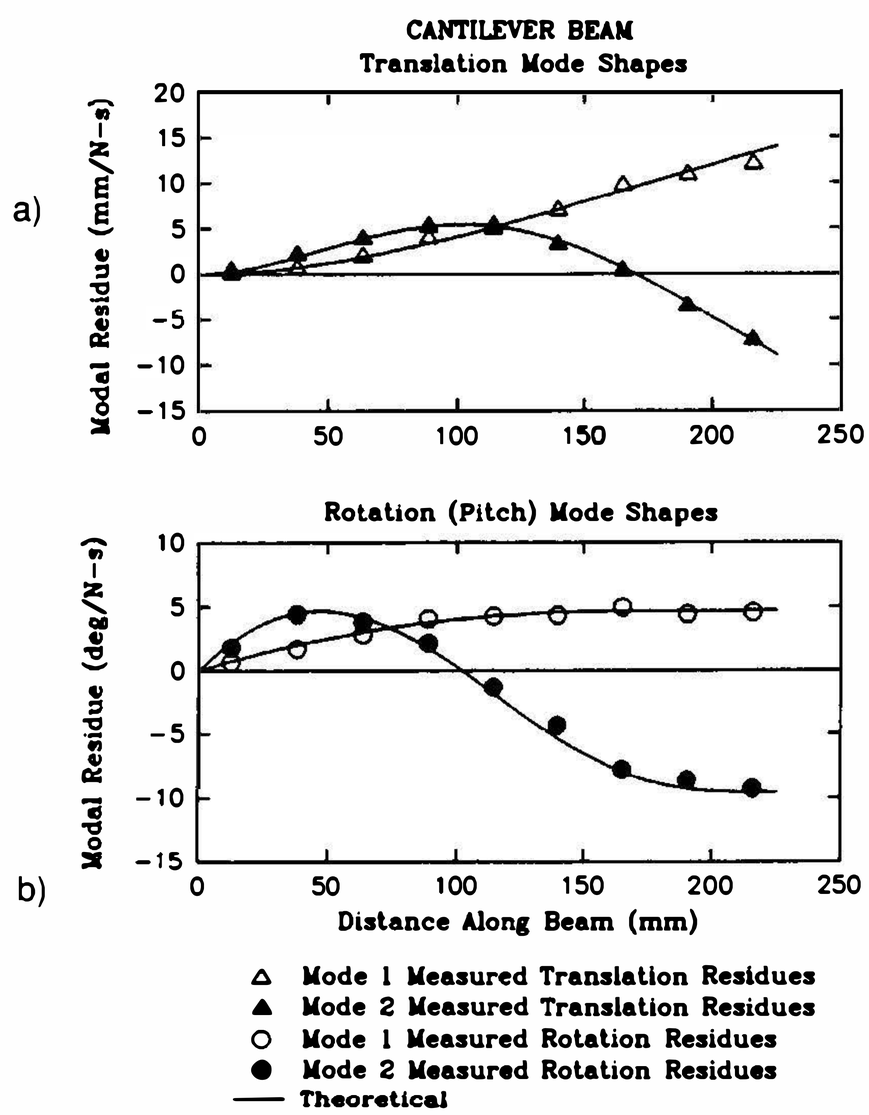

Fig. 4 Experimental mode shapes from the cantilever beam test: a) translation residues (y); b) rotation residues (θ_z)

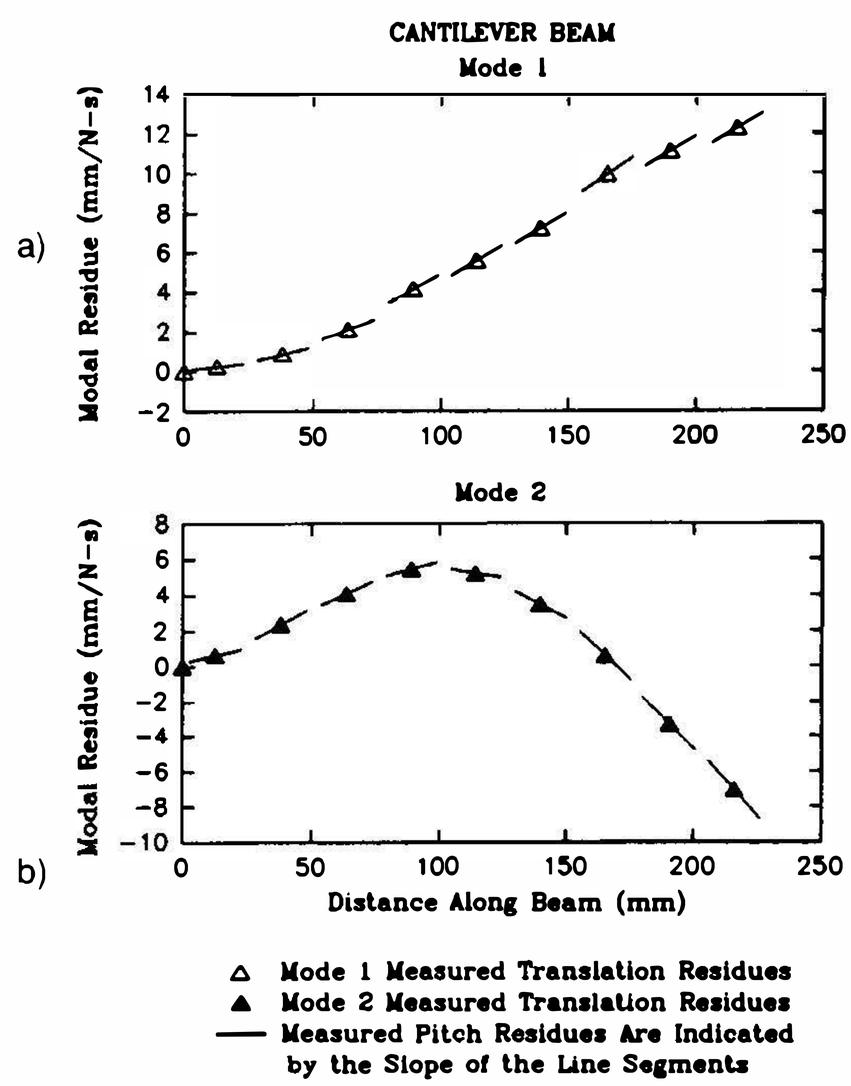

Fig. 5 Experimental mode shapes from cantilever beam by superimposing the translation (y) and rotation residues (θ_z): a) mode 1; b) mode 2

translational and rotational data against those using only translation data, it is possible to examine potential benefits. Furthermore, several analytical methods have been proposed which estimate rotational data solely from measured translational data [2,5,13]. Through this comparison, it will be possible to examine the magnitude of potential errors in the rotational information from an actual test.

There are two general categories to curve fitting data, fitting a single function to all of the available data (i.e., global fit) and fitting separate functions to separate data subsets and forcing continuity between subsets (i.e., local fit). While many types of functions can be used for curve fitting, the work in this paper is restricted to the use of polynomials.

Typically, when curve fitting in a global sense, there may be more data available than absolutely necessary for the chosen polynomial. Therefore, the fitting is usually done in a least squares sense. It should be noted that while local fitting generates many polynomials, continuity between these polynomials must be maintained. The continuity is accomplished by ensuring that the derivatives at the boundaries of each respective polynomial are equal. The cubic spline local fit approach used herein forced curvature continuity (second derivative of position) between adjacent polynomials. Previously when only translational modal translations were available, slopes at the polynomial boundaries were extracted from such curve fitting results. However, now that slope values may be measured experimentally, they may be used as additional data to improve the fitting procedures.

The order of the polynomial chosen dictates the characteristics of the mode which can be approximated. Too low an order will not be able to follow the basic trends in the data set. If the order is too high, the fitted equation may also artifactually track noise in addition to thc basic trends. Also, the order dictates the minimum amount of data that is needed (e.g., a third order cubic equation requires four coefficients to be calculated which implies at least four pieces of information are needed).

The translational and pitch rotational residues for the first two mode shapes of a cantilever beam provided a basis to examine the application of polynomial functions to describe mode shapes. Several different fitting approaches were applied to the respective modal residue data sets. The first category delineated between a spline or global curve fitting approach. The second category varied the polynomial order (3rd, 4th, or 5th). The last category investigated which data set (translation (y) only, or translation (y) and pitch rotation (θ_z)) was used to calculate polynomial coefficients.

In order to provide a quantitative comparison, an overall error term for both the translation (y) and rotation (dy/dx) were calculated. A root mean square (RMS) error was calculated for the translation and pitch as shown in Eq. 1. These errors were combined to form an aggregate error for each case as shown in Eq. 2. The aggregate errors were then used for comparisons between the various fits studied.

$$RAM\ Error = \sqrt{\frac{\sum_{i=1}^{N} (measured\ data_i - fit\ value_i)^2}{N}} \tag{1}$$

where:

$$N = number\ of\ data\ points\ measured$$

$$Aggregate\ Error = \sqrt{(Translation\ RMS\ Error)^2 + (Pitch\ RMS\ Error)^2} \tag{2}$$

The results from the various fits for each of the two modes are tabulated in Table 2. The best fitting method for both modes based on the aggregate error proved to be that used in Case 7 and Case 14 respectively, a global fifth order curve fit using both the measured translational and rotational data. The large errors produced in Cases 9 and 10 resulted from trying to fit a cubic equation to the second mode of

the beam. By increasing the order of the polynomial from three to four, this problem was alleviated. Because the reduction in error between fourth and fifth order polynomials for both modes (Mode 1: Cases 5 and 7; Mode 2: Cases 12 and 14) was small, fourth order fits will be used in subsequent discussions.

Two significant conclusions can be drawn from analysis of Table 2:

1. Including rotational data (spatial derivatives) with the translational data in the fits yields significantly better results than simply using translations. The aggregate error for translation and pitch is much smaller than the aggregate error for the same fit that does not utilize the rotational data.
2. While more accurate representation of translation data alone is possible in those fits that do not include rotational data (e.g., Case 1) calculated slopes then compare very poorly to those measured experimentally. Including rotational data in the fitting procedure degrades translational representation marginally while significantly improving slope modelling.

These two concepts are further illustrated in Figs. 6, 7, and 8. Figure 6(a) and 6(b) shows the translation and rotational mode shapes, respectively, for the cantilever beam mode 1 using the results from Case 1. The polynomial fit is obtained by a cubic spline using only the translation data and is compared to the theoretical and experimental residues values. Excellent agreement between the measured translation data and the cubic spline fits are seen for the translation mode shape in Fig. 6(a). By comparing the measured data points with the theoretical curve, it can be seen that the data has some noise associated with it. Typically, when a numerical derivative of noisy data is calculated, the effect of the noise is magnified. This can be readily

TABLE 2 SUMMARY OF POLYNOMIAL CURVE FIT ERRORS FOR CANTILEVER BEAM TEST

Case	Mode	Type of Fit	Poly. Order	Data Set	Translation (y) RMS Error	Pitch (dy/dx) RMS Error	Aggregate Error
1	1	Spline	3	t	0.0	0.434	0.434
2	1	Global	3	t	0.080	0.327	0.337
3	1	Global	3	t-p	0.163	0.730	0.179
4	1	Global	4	t	0.078	0.390	0.397
5	1	Global	4	t-p	0.157	0.073	0.173
6	1	Global	5	t	0.075	0.479	0.485
7	1	Global	5	t-p	0.156	0.067	0.169
8	2	Spline	3	t	0.0	0.418	0.418
9	2	Global	3	t	0.158	0.929	0.942
10	2	Global	3	t-p	0.349	0.617	0.709
11	2	Global	4	t	0.024	0.319	0.320
12	2	Global	4	t-p	0.141	0.104	0.175
13	2	Global	5	t	0.023	0.369	0.369
14	2	Global	5	t-p	0.138	0.103	0.173

t = translation modal residues; p = pitch (rotation) modal residues

seen in Fig. 6(b), where the derivative of the cubic spline oscillates about the theoretical and measured rotational values. This implies that a small amount of noise in the translation data can produce a large error in its derivative. This has important practical implications since previous methods using cubic splines have been proposed as an approach to estimate rotational degrees of freedom from translational data [2]. Therefore, estimating rotational degrees of freedom by this method may be suspect unless the residue values are highly accurate and virtually noise free.

Although, not quite as dramatic, a similar trend is illustrated in Fig. 7 where a fourth order global curve fit of the translation data, Case 4, is compared to the theoretical curves and the measured data. Figure 7(a) shows that this polynomial fit does not reproduce the noise as much as the cubic splines did. This is expected due to its global least squares fitting attributes. Once again, excellent agreement is seen in Fig. 7(a) between theory, measured, and polynomial translation mode shapes. However, again its derivative expressed as the rotational mode shape produces significant errors particularly near the free end of the beam. It is obvious in Fig. 7(a) that the measured translation data diverges from the theoretical curve near the free end and the noise effect is magnified in the derivative, Fig. 7(b).

The numerical results in Table 2 indicate a marked decrease in curve fit error if both translational and the rotational residues are included in the fitted data set. To this end, Fig. 8 compares a fourth order global curve fit determined using both translation and rotation data from Case 5 with theoretical and measured values. The fitted translation mode shape in Fig. 8(a), again compares well to both the theoretical and measured values. Note that by comparing this fit with the previous, Fig. 7(a), it is seen that the polynomial fit with both translation and rotation data in Fig. 8(a), is closer to the theoretical value. While this improved polynomial fit may not match the experimental translation values perfectly, in reality it better represents the expected theoretical function. The utility of using both translation and rotation data in mode shape fitting is best exemplified when comparing pitch mode shapes in Figs. 7(a) and 8(b). Including rotational

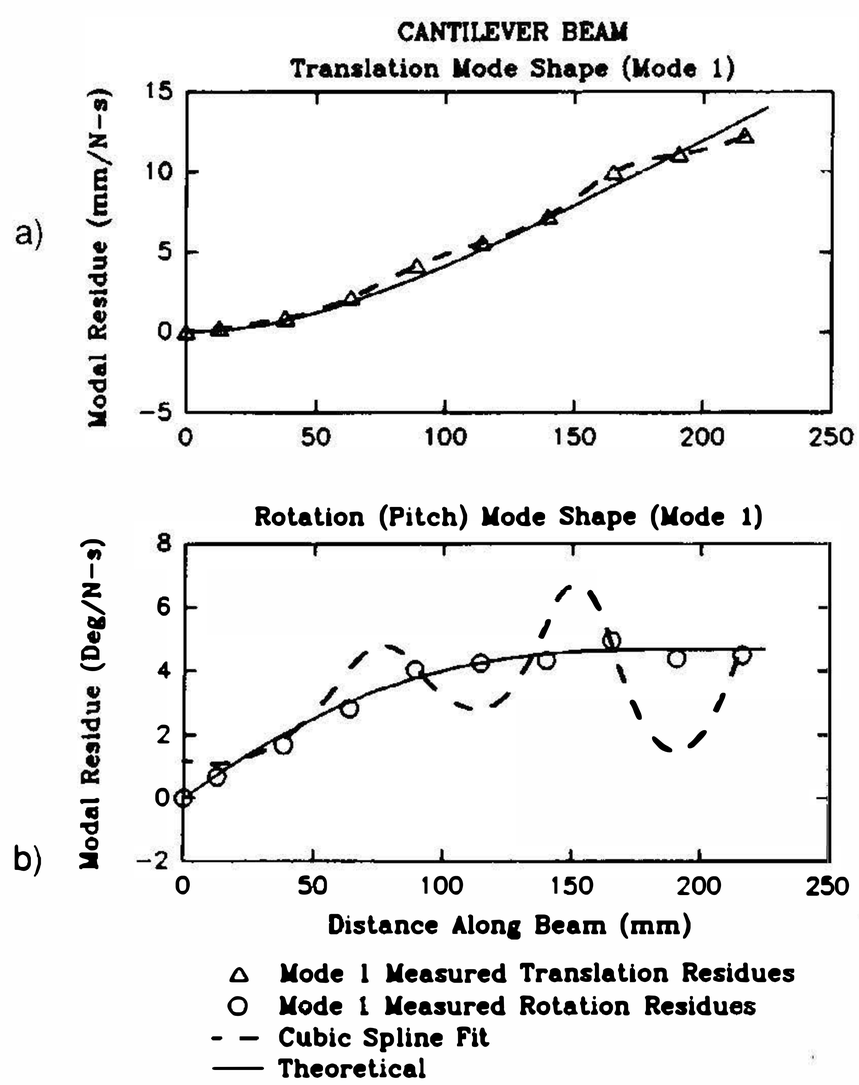

Fig. 6 **Comparison of cantilever beam mode 1 from theory, experiment and a local cubic spline polynomial fit using only translation residues (*y*) (Table 2 - Case 1): a) translation mode; b) rotation mode**

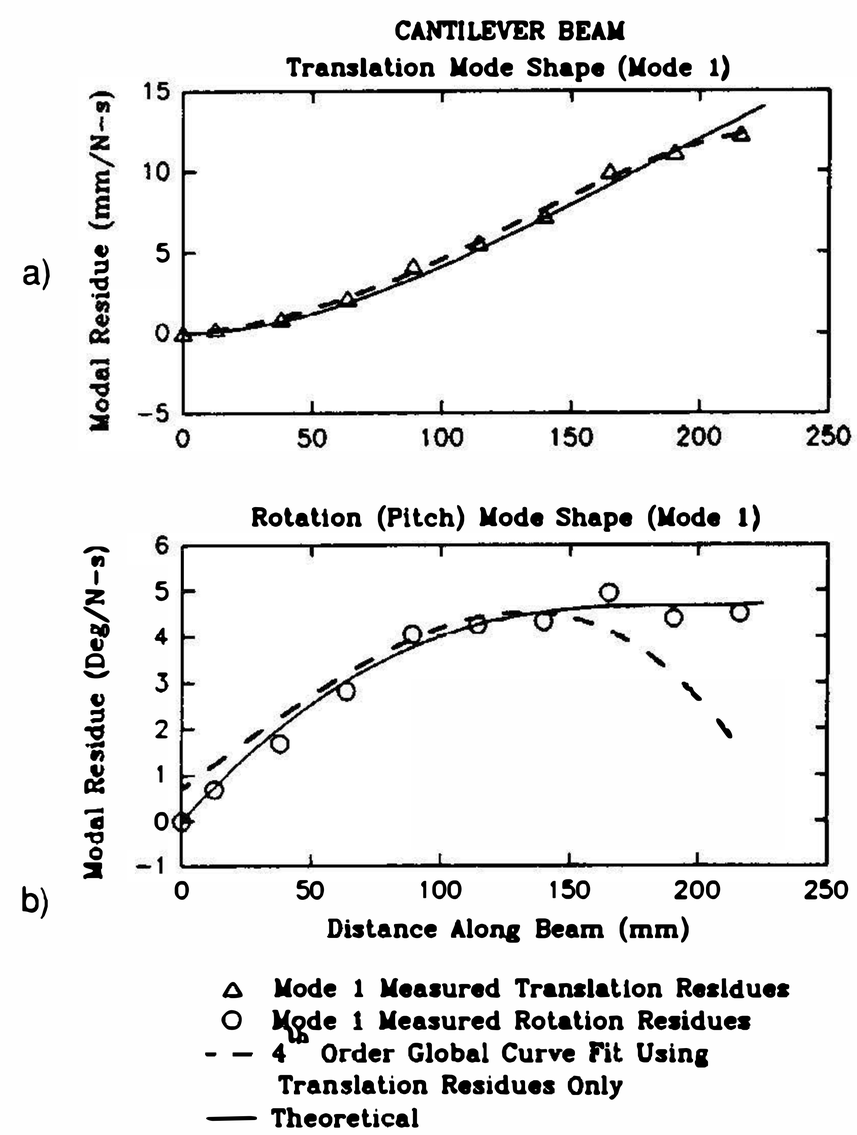

Fig. 7 Comparison of cantilever beam mode 1 from theory, experiment and a global 4th order polynomial fit using only translation residues (*y*) (Table 2 - Case 4): a) translation mode; b) rotation mode

data can further help remove fitted mode shape artifact caused by experimental noise as seen in the significantly improved pitch mode fit.

Similar results to those discussed for mode 1 were seen in the analysis of mode 2. Mode 2 had less measurement noise associated with it than mode 1, so the results are not as dramatic. However, the conclusions remain the same. Figure 9 compares a fourth order global curve fit of both the translation and rotation data from Case 12 with the theoretical and measured values. Both the fitted translation and rotation values compare well with theory and measurement.

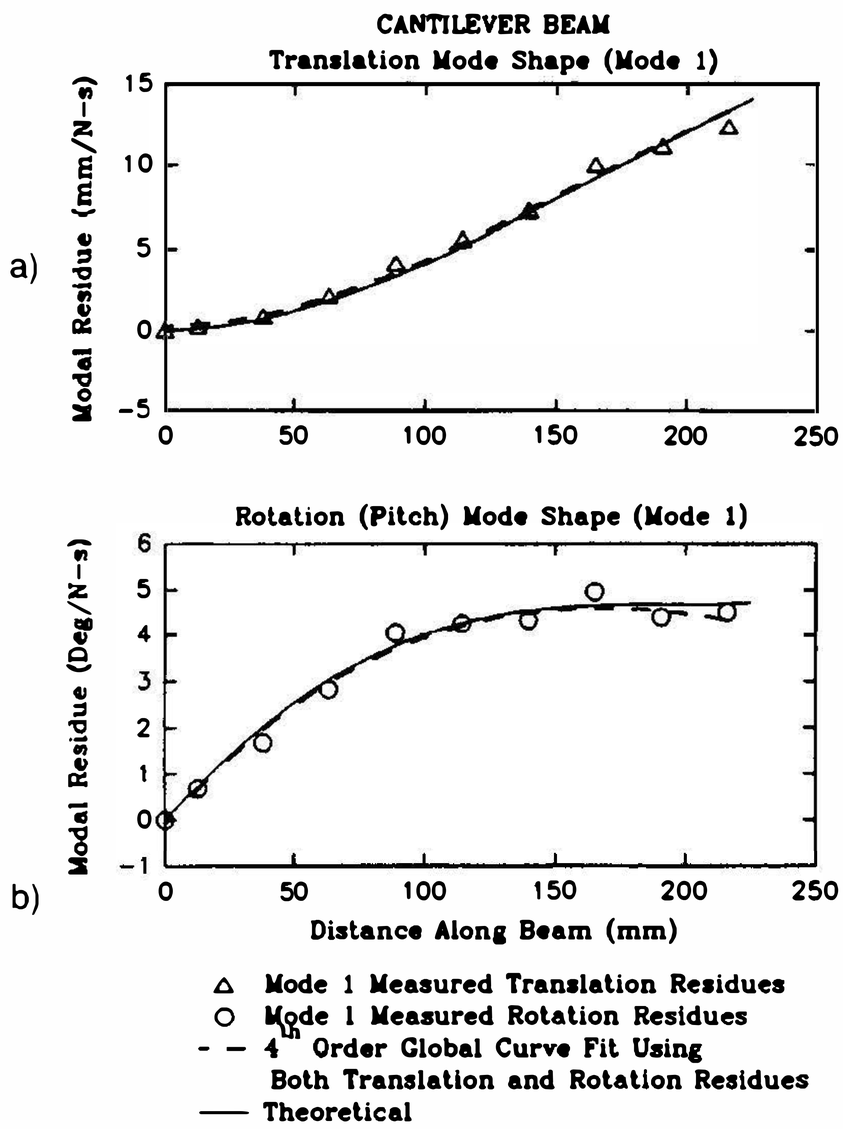

Fig. 8 Comparison of cantilever beam mode 1 from theory, experiment and a global 4th order polynomial fit using both translation (y) and rotation residues (θ_z) (Table 2 - Case 5): a) translation mode; b) rotation mode

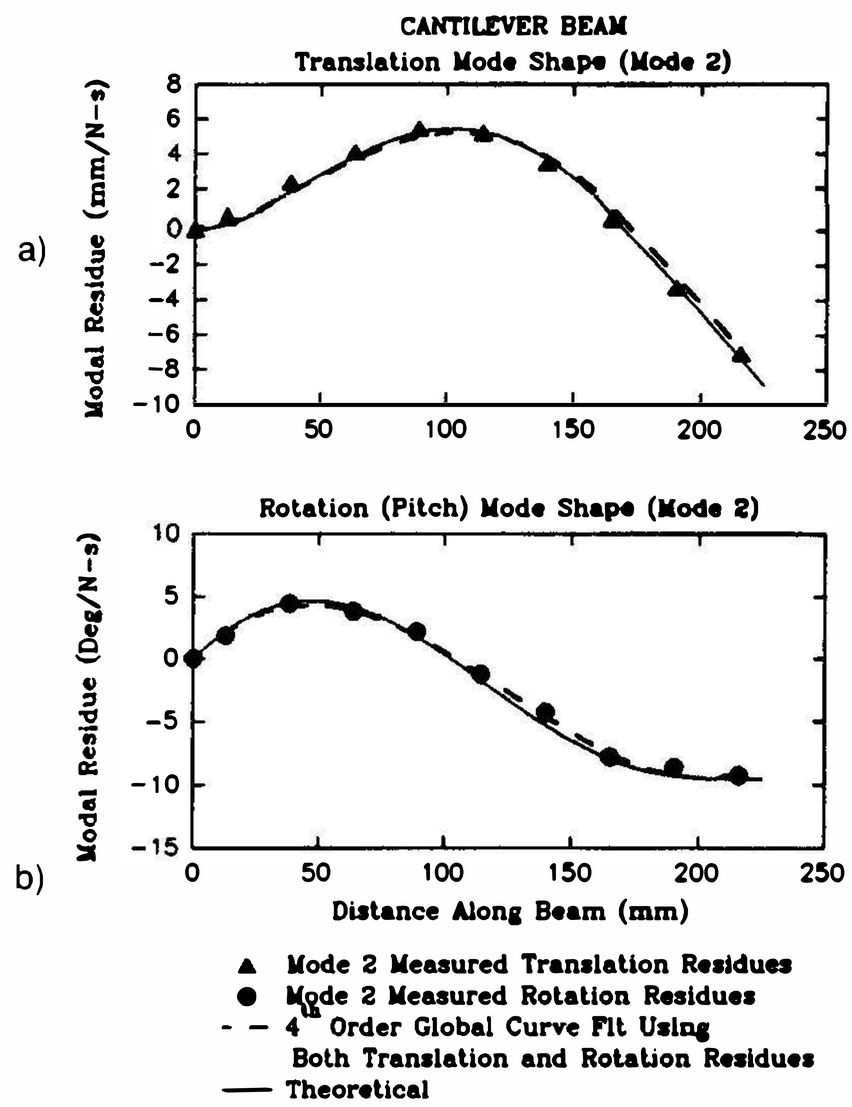

Fig. 9 Comparison of cantilever beam mode 2 from theory, experiment and a global 4th order polynomial fit using both translation (y) and rotation residues (θ_z) (Table 2 - Case 12): a) translation mode; b) rotation mode

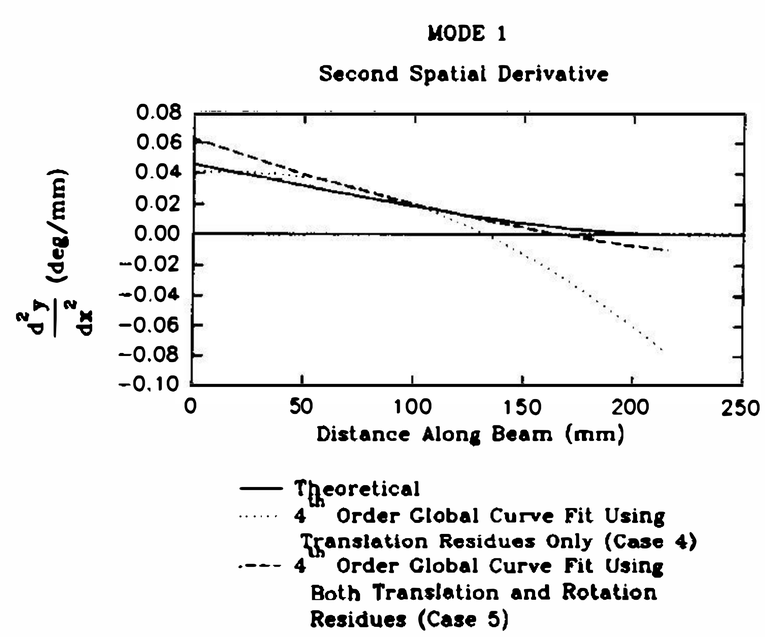

Fig. 10 Comparison of second spatial derivative for cantilever beam mode 1

Some applications, such as near field structural intensity, use second derivative information. The ability to measure both translation and first derivative information improves estimation of second derivative quantities as illustrated in Fig. 10. Theoretical second derivative for mode 1 is compared to estimated quantities utilizing the fourth order polynomial equations developed for Cases 4 and 5. It is evident that the polynomial equation that was developed using both translation and rotation data from Case 5 is a better estimator than the corresponding model developed using only translations from Case 4.

4. Modal Analysis of a Circuit Board

Experimental modal analysis of a printed circuit board shown in Fig. 11 was performed to further illustrate the integration of translational (y) and rotational (θ_x, θ_z) data for mode shape refinement. Whereas the cantilever beam was basically a one-dimensional structure, the circuit board is two-dimensional. This provides a better opportunity to explore the transducer's capabilities for simultaneous measurement of roll (θ_x) and pitch (θ_z) and the integration of the respective residue data. Furthermore Wong, et. al. [14] showed that rotational information is important for analyzing electrical components on printed circuit boards located near nodal lines.

The experimental setup was similar to that used for the cantilever beam. The board was rigidly clamped at the larger of its two mounting interfaces while all other edges remained free. The circuit board is

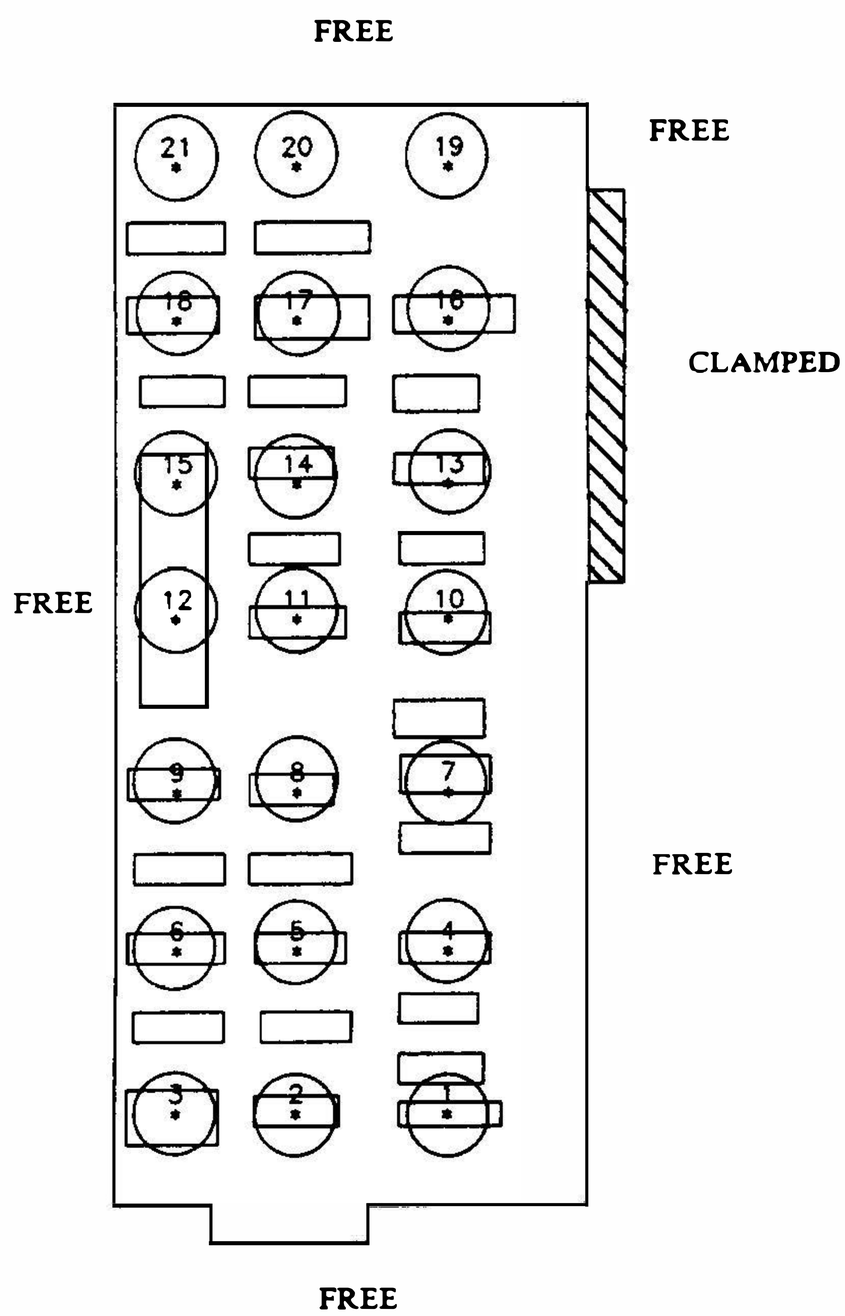

Fig. 11 Circuit board component layout, circles marked by asterisks denote mirror target placement

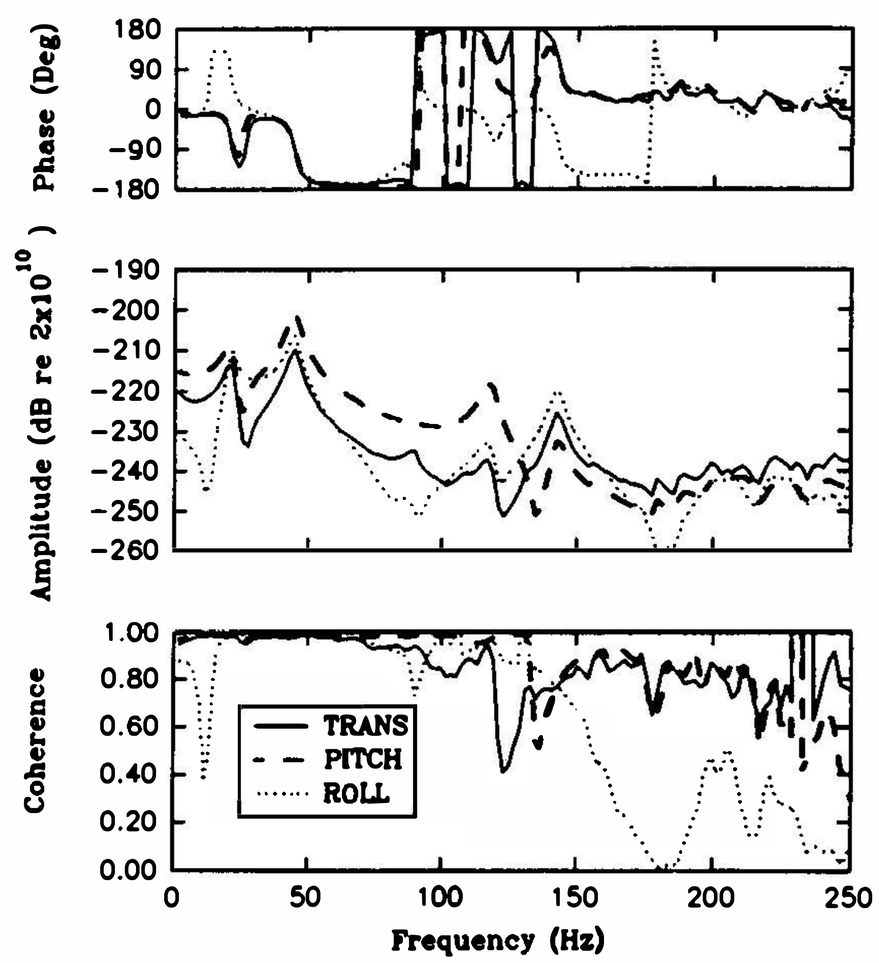

Fig. 12 Typical frequency response and coherence functions from the printed circuit board impact test at site 10 (dB reference: 2×10^{10} mm/N for translation; 2×10^{10} degrees/N for rotation)

approximately 230 mm by 90 mm with 32 electronic chips of various sizes mounted directly onto the board. Twenty one flat circular front surface mirrors were mounted on the circuit board in a three by seven matrix to provide reflective target sites as shown in Fig. 11. The numbers in Fig. 11 indicate the measurement sites with site 16 used for the instrumented hammer impact driving point. A similar testing procedure to that described for the cantilever beam was used.

A typical frequency response and corresponding coherence function for mirror site 10 is shown in Fig. 12. The solid line represents vertical translation (y), the dashed line the pitch angle (θ_z), and the dotted line the roll angle (θ_x). The magnitude of all three FRF's shows four resonant peaks that occur at 20, 44, 117 and 142 Hz with corresponding 180 degree phase shifts. The coherence indicates that generally below 150 Hz the measurements are of good quality. The impact excitation was sufficiently flat across the frequency range indicating that the coherence drops above 150 Hz are due to the decreased signal amplitudes in this frequency range.

Two-Dimensional Mode Shape Refinement by Polynomial Curve Fitting

Again, modal parameters were extracted using the complex exponential algorithm. Due to the two-dimensional nature of the circuit board, the integration of the translation and rotational residues requires additional development.

Polynomial fitting methods were restricted to global schemes. Local methods were not investigated due to the results obtained in the one-dimensional cantilever beam case. The discussion on polynomial curve fitting order from the previous section is still valid with respect to the two-dimensional case. The parameters varied in this study were polynomial order and data set type (e.g., translation residues only or both translation and rotation residues).

Two different fitting methods were examined for expressing the two-dimensional modal surface. The first was a bicubic surface which is cubic in both the x and z coordinate directions including all of the cross terms for a total of 16 coefficients. The second surface was a quadratic cubic which is quadratic in the x coordinate and cubic in the z direction with all the associated cross terms producing 12 polynomial coefficients. Once again the two different data sets examined were translation data only, and translation

TABLE 3 SUMMARY OF POLYNOMIAL CURVE FIT ERRORS FOR CIRCUIT BOARD EXPERIMENT

Case	Mode	Surface	Data Set	Translation *y* RMS Error	Pitch *dy/dx* RMS Error	Roll *dy/dz* RMS Error	Aggregate Error
1	1	c-c	t-s	0.69	0.80	0.36	1.11
2	1	q-c	t-s	0.71	0.88	0.35	1.19
3	1	q-c	t	0.36	2.91	1.40	3.25
4	2	c-c	t-s	2.05	1.65	1.05	2.83
5	2	q-c	t-s	2.08	1.68	1.05	2.87
6	2	q-c	t	1.16	7.70	3.97	8.74
7	3	c-c	t-s	0.78	0.72	0.70	1.27
8	3	q-c	t-s	0.78	0.76	0.70	1.29
9	3	q-c	t	0.41	3.06	1.68	3.51
10	4	c-c	t-s	0.28	0.40	0.37	0.62
11	4	q-c	t-s	0.28	0.49	0.38	0.68
12	4	q-c	t	0.20	1.02	0.47	1.14

c-c = bicubic surface; q-c = quadratic-cubic surface; t-s = translations and slopes in x and y directions; t = translations only

and rotation data. Rotational data for this example is comprised of both a pitch (θ_z) and a roll value (θ_x), or in other words, a slope in the x direction (dy/dx) and a slope in the z direction (dy/dz).

RMS errors were calculated using Eq. (1) for translation, roll, and pitch values. An aggregate error similar to that calculated in Eq. (2) was produced for each case studied. Results from the various fits are tabulated in Table 3. Table 3 does not include results from a bicubic surface fit using just translation data in that experimental data were measured on a 3 by 7 grid. For purely translation data, the three pieces of information available in the x coordinate direction were insufficient to calculate the four lateral cubic coefficients. It is, however, possible to fit a bicubic surface using both translations and rotations since in the x direction there are essentially six available pieces of information (3 translations and 3 slopes).

Examination of Table 3 shows essentially the same conclusions that were found in the one-dimensional case of the cantilever beam. The highest order surface, bicubic in this case, estimated using both translation and rotation data yields the most accurate fit. As with the cantilever beam data, translation error was only marginally larger when both translation and rotation data were used as compared to the fit from translation data only. However, this is more than offset by the improvement made in rotation estimates. Furthermore, if slope data is available, only two data points in either the x or z directions are needed to fit a cubic equation. Consequently, for typical modal tests fewer data sites are needed when measuring both translations and rotations to obtain comparable fits if only translations were measured.

Raw translation residues for mode 3 are shown in Fig. 13(a) and the bicubic polynomial fit for mode 3 is shown in Fig. 13(b). All of the mode shapes and their corresponding fits produced similar results. Overall, the bicubic fits have a smoothing effect on the data.

5. Concluding Remarks

This paper has discussed the utilization of a novel transducer system for noncontacting measurement of structural translational and rotational vibration. Modal analysis results using both translation and rotation data yielded interpretations beyond those which are available from purely translational transducers.

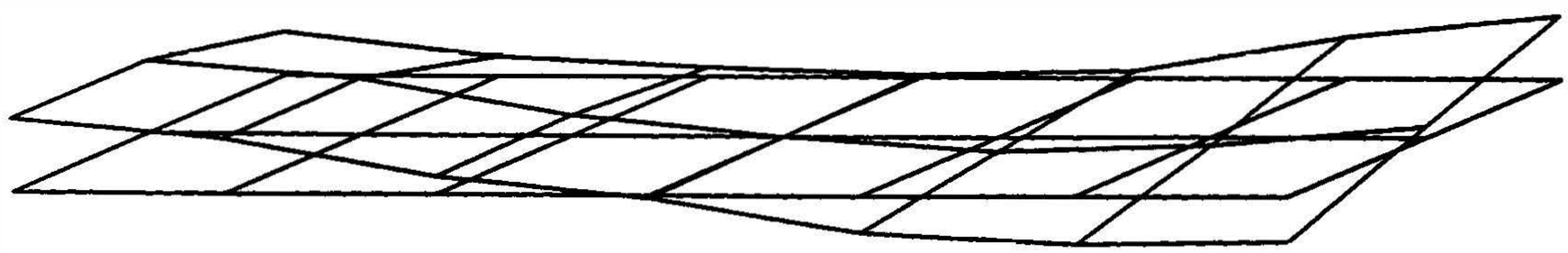

Fig. 13a Experimentally measured mode 3 for the circuit board

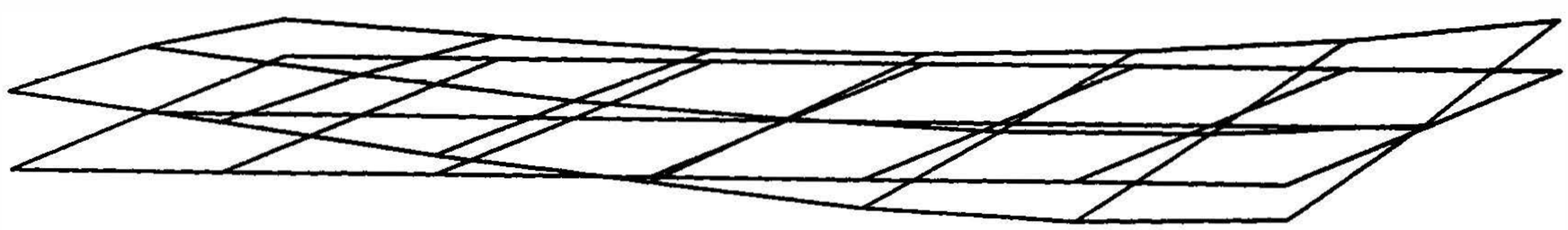

Fig. 13b Circuit board mode 3 determined from a bicubic global polynomial fit using both the translation (y) and rotation (θ_x, θ_z) residues (Table 3 - Case 7)

Significantly improved definition of mode shape spatial characteristics were observed by integrating the translational and rotational modal deflections at each response measurement site. The study has revealed several points with respect to the use of translation and rotational residues in modal analysis:

1. Locations of maximum translation are more apparent when incorporating rotational data. Since the rotational mode represents the first spatial derivative of the translational modes, zero crossings in the rotational data represent maxima in the translational data. Therefore, it is possible to more accurately locate the position of maximum dynamic excursion in a mode shape, regardless of where the experimental measurement sites were actually placed.
2. The polynomial mode shapes derived from both translational and rotational modal residues tended to more closely correspond to the theoretical solutions. The process tended to compensate for uncertainty of the individual residue values and produced an overall more accurate mode shape.
3. The study revealed that significant errors may arise when trying to estimate rotational degrees of freedom from polynomial mode shapes developed from translational residues acquired in an actual experimental test. Small discrepancies in the residues can produce large errors when the spatial derivatives are needed.
4. The implications of these observations go beyond better definition of mode shapes and into assessment of the measurement grid used to test the structure. Since spatial derivatives can be more readily observed in rotational data, a test grid which is too coarse to fully resolve the mode shape, is more apparent.
5. Measurement of rotational vibrations provides direct experimental values for first spatial derivatives in modal analysis, and consequently can produce significantly better estimates of structural intensity.
6. Combined use of translational and rotational vibration measurements allows data collection at fewer test sites to produce mode shapes with an accuracy similar to that obtained with purely translational measurements.

The concept of using polynomial functions to represent the mode shapes requires some judgement to achieve useful results. The polynomial order must be selected appropriately with respect to the structure under consideration. Most importantly the polynomial must be able to replicate the basic shape of the mode. Evaluation of the modal characteristics developed from translational residues provides a good basis to select the proper polynomial order. In general global polynomial fitting produced better results than local fitting approaches.

The results from this work indicate that rotational degrees of freedom possess great utility and value for the experimental modal analysis community. The ability to develop more refined experimental mode shapes, in addition to their documented need in finite element and structural dynamic modification work, will drive further investigation into effective measurement and use of structural rotational degrees of freedom.

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