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ABSTRACT

This paper demonstrates a new system identification approach of using Lanczos coordinates in place of modal coordinates. Identified experimental Lanczos vectors can be directly used in many structural dynamics analysis applications. A multi-input, multi-output frequency-domain technique was used to extract system matrices and an unsymmetric block Lanczos algorithm was used to reduce the order of the experimental model. A cantilever beam example showed promising results, indicating that a new system identification approach using Lanczos coordinates is worthy of further study.

List of Symbols

$[\hat{A}],[\hat{B}]$ first-order system matrices
$[C]$ damping matrix
C complex space
$[D]$ force distribution matrix
$[E]$ $\equiv[F]^{-T}\equiv[[Y_1]\ldots[Y_r]]$, left Lanczos matrix
$[F]$ $\equiv[[X_1]\ldots[X_r]]$, right Lanczos matrix
$\{\hat{F}\}$ force vector for the first-order system
$\mathcal{F}$ Fourier transform
$\{f\}$ force vector
$[I]$ identity matrix
$\mathfrak{I}$ imaginary space
$[K]$ stiffness matrix
$[M]$ mass matrix
$\{p\}$ excitation vector
$\Re$ real space
$(\Re)^{n\times m}$ $n\times m$ (real) matrix
$[\hat{T}]$ block tridiagonal matrix
$\{x\}$ displacement vector

Greek

$\{\gamma\}$ a set of reduced coordinates
$[\psi_C]$ contracting transformation matrix
$[\psi_R]$ reconstructing transformation matrix
ω circular frequency

$[\ ]^T$ matrix transpose
$[\ ]^{-1}$ matrix inverse
$[\ ]^{-T}$ $\equiv[[\ ]^T]^{-1}$

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1. Introduction

In conventional modal analyses, natural frequencies, damping factors, and mode shapes of structures are identified from the experimental data [1]. These identified modal parameters have been used to: construct the experimental model, verify the mathematical model, and predict structural responses. Widely used system identification algorithms include: the damped complex exponential response method [2], the forced normal mode excitation method [3], the frequency response function method [4], the auto-regressive-moving-average (ARMA) method [5], the Ibrahim time domain (ITD) method [6], the simultaneous frequency domain (SFD) method [7], the polyreference method [8], and the eigensystem realization algorithm (ERA) [9].

Lanczos methods have been developed for finding some or all of the eigenvalues and eigenvectors of a large symmetric sparse matrix [10, 11]. However, cancellation and roundoff errors can render the Lanczos algorithm unstable due to the loss of orthogonality among the Lanczos vectors. Since this loss of orthogonality can be improved [12, 13], in the past few years, there has been a renewed interest in developing eigensolvers and dynamic response solvers based on Lanczos vectors [14, 15]. For dynamic analyses, an unsymmetric block Lanczos algorithm may be employed to: handle matrices with repeated (or closely-spaced) eigenvalues, permit dynamic response analyses with multiple simultaneously applied loads, and/or accommodate unsymmetric damping matrices [16, 17]. Lanczos vectors may also be used to develop reduced-order structural models for component mode synthesis and control applications [18].

This paper describes how unsymmetric block Lanczos vectors can be employed for the system identification of large structures having arbitrary damping and closely-spaced frequencies [19]. System matrices were estimated using a multi-input, multi-output modal parameter estimation algorithm based on excitation and response spectra [20]. The order of the identified experimental model was reduced using unsymmetric block Lanczos vectors for further applications. Simulated modal testing of a uniform cantilever beam was employed to evaluate the new system identification approach. Responses from analytical and reduced experimental models due to external step forces showed little differences, which indicated that the reduced-order Lanczos model was a good experimental model. In addition, sensor location selection and efficiency of Lanczos eigensolution for large structural problems were briefly discussed.

2. Equation of Motion

For general linear, time-invariant dynamic systems, it is assumed that there exists a discrete analytical model

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \tag{1}$$

where $[M]$, $[C]$, and $[K] \in \Re^{N\times N}$ are, respectively, the mass, damping, and stiffness matrices, $\{x(t)\} \in \Re^{N\times 1}$ is the displacement vector, $\{f(t)\} \in \Re^{N\times 1}$ is the force vector, and N is the number of physical degrees of freedom.

When dealing with dynamic systems having arbitrary linear damping, it is convenient to introduce a state vector formulation

$$\left[\hat{A}\right]\left\{\dot{X}(t)\right\}+\left[\hat{B}\right]\left\{X(t)\right\}=\left\{\hat{F}(t)\right\} \tag{2}$$

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where

This algorithm results in a block tridiagonal matrix

$$[F]^{-1}[A][F]=\left[\hat{T}\right]=\begin{bmatrix} [\mathcal{M}_1] & [\mathcal{G}_1]^T & & & \\ [\mathcal{B}_1] & [\mathcal{M}_2] & [\mathcal{G}_2]^T & 0 & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & 0 & & \cdot & \cdot & [\mathcal{G}_{r-1}]^T \\ & & & [\mathcal{B}_{r-1}] & [\mathcal{M}_r] \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} [F] &\equiv \left[[X_1]\ldots[X_r]\right],\ [X_i]\in\Re^{n\times p} \\ [F]^{-T} &\equiv [E]\equiv\left[[Y_1]\ldots[Y_r]\right],\ [Y_i]\in\Re^{n\times p} \\ [\mathcal{M}_i] &\in \Re^{p\times p} \\ [\mathcal{B}_i],[\mathcal{G}_i] &\in \Re^{p\times p} \text{ and upper - triangular matrices} \\ r &: \text{number of blocks} \\ p &: \text{size of blocks} \end{aligned} \tag{8}$$

The vectors comprising the $[X_j]$'s will be referred to as *right Lanczos vectors*, while the $[Y_j]$'s will be referred to as *left Lanczos vectors*. $[E]$ and $[F]$ will be referred to as left and right Lanczos matrices, respectively. Lanczos vectors (and matrices) will be always real contrary to eigenvectors as long as the system matrix $[A]$ in Eq. (6) is real. It is noted that the block-tridiagonal matrix $[\hat{T}]$ is not unique, but depends on the starting-vector matrices, $[X_1]$ and $[Y_1]$. Further algorithm development, including computational enhancement, can be found in Ref. [17].

(b) Starting Vectors

The block tridiagonal matrix $[\hat{T}]$ is not unique, but depends on the starting vectors (actually the matrices), $[X_1]$ and $[Y_1]$. In general, the starting-vector matrices $[X_1]$ and $[Y_1]$ may be chosen arbitrarily with $[X_1]^T[Y_1] = [I]$. When the external force is of the form given by Eq. (4), the starting vectors can be chosen in the direction of the static displacements, that is, appropriate starting vectors for use with Eq. (2) would be

$$[X_1]=\left[\hat{B}\right]^{-1}\left[\hat{H}\right]=\begin{bmatrix}[0]\\ \left[K^{-1}\right][D]\end{bmatrix} \tag{9}$$

Then $[Y_1]$ can be found by choosing a $[Y_1]$ which satisfies $[X_1]^T[Y_1] = [I]$, with $[Y_1]$ being determined by using a generalized matrix inversion. This choice gives the Lanczos vectors the important advantage that they automatically include the static displacement and avoid any possible need for a static correction.

(c) Model Order Reduction

In analytical dynamics, the mode-superposition version of the Rayleigh-Ritz method is frequently employed for model order-reduction, i.e., $\{X(t)\}$ is approximated by

$$\{X(t)\} \cong [X_R]\{\eta(t)\} \tag{10}$$

where $\{X(t)\} \in \Re^{n\times 1}$, $[X_R] \in C^{n\times m}$ contains a portion of the right (complex) eigenvectors, $\{\eta(t)\} \in C^{m\times 1}$, and $m \leq n$. Then Eq. (2), when premultiplied by $[Y_R]^T \in C^{m\times n}$ (which consists of a corresponding set of left eigenvectors), becomes

$$\left([Y_R]^T[\hat{A}][X_R]\right)\{\dot{\eta}(t)\} + \left([Y_R]^T[\hat{B}][X_R]\right)\{\eta(t)\} = [Y_R]^T\{\hat{F}(t)\} \tag{11}$$

Hence, the mode-superposition reduced-order model may be written in the form

$$[A_R]\{\dot{\eta}(t)\} + [B_R]\{\eta(t)\} = \{F(t)\} \tag{12}$$

where $[A_R]$, $[B_R] \in C^{m\times m}$ and $\{\eta(t)\}$, $\{F(t)\} \in C^{m\times 1}$.

The right and left Lanczos vectors can also be employed in a Rayleigh-Ritz fashion to form reduced order models. For example, Eq. (2) may be transformed to a standard form in order to formally employ Eq. (6) for model order reduction, i.e.,

$$[A]\{\dot{X}(t)\} + \{X(t)\} = [H]\{p(t)\} \tag{13}$$

where $[A] = [\hat{B}]^{-1}[\hat{A}] \in \Re^{n\times n}$, $[H] = [\hat{B}]^{-1}[\hat{H}] \in \Re^{n\times N_p}$, and $\{X(t\} \in \Re^{n\times 1}$. If $[\hat{B}]$ is a symmetric positive-definite matrix, the Cholesky decomposition of $[\hat{B}]$ can be used to avoid a generalized matrix inverse [15].

Using the standard first-order form of Eq. (13), the model order may now be reduced by introducing the Lanczos vectors defined in Eq. (6). $\{X(t)\}$ can be approximated by

$$\{X(t)\} \cong [X_L]\{Z(t)\} \tag{14}$$

where $[X_L] \in \Re^{n\times m}$, $m \leq n$, contains a portion of the right Lanczos matrix $[F]$. When Eq. (14) is substituted into Eq. (13) and the resulting equation is premultiplied by a matrix $[Y_L]^T \in \Re^{m\times n}$, which contains a portion of the left Lanczos matrix $[E]$, the result is

$$\left([Y_L]^T[A][X_L]\right)\{\dot{Z}(t)\} + \left([Y_L]^T[X_L]\right)\{Z(t)\} = [Y_L]^T[H]\{p(t)\} \tag{15}$$

Because of the biorthogonality between the right and left Lanczos vectors, Eq. (15) can be written in the condensed form

$$[A_L]\{\dot{Z}(t)\} + \{Z(t)\} = [H_L]\{p(t)\} \tag{16}$$

where $[A_L] \in \Re^{m\times m}$, $[H_L]=[Y_L]^T[H] \in \Re^{m\times N_p}$, $\{Z(t)\} \in \Re^{m\times 1}$, and $m \leq n$.

It is noted that, in Eq. (16), $[A_L]$ is not actually calculated from $([Y_L]^T[A][X_L])$. It may be formed, as in Eq. (7), by using the algorithm of Eq. (6), since $[A_L]$ is just a reduced-order form of $[\hat{T}]$. A reduced-order model, formulated according to Eqs, (13)-(16), emphasizes the lower-frequency characteristics of the original system [16], while $\{\dot{X}(t)\} + [\hat{A}]^{-1}[\hat{B}]\{X(t)\} = [\hat{A}]^{-1}[\hat{H}]\{p(t)\}$ yields the higher-frequency characteristics.

(a) Preliminary Data Reduction

The number of force (input) and response (output) measurements will normally be less than the physical degrees of freedom N. Based on the N_m available measurement degrees of freedom, consider the following system model

$$[M_m]\{\ddot{x}_m(t)\}+[C_m]\{\dot{x}_m(t)\}+[K_m]\{x_m(t)\}=[D_m]\{p(t)\} \tag{17}$$

where $[M_m]$, $[C_m]$, and $[K_m] \in \Re^{N_m \times N_m}$ are, respectively, the *equivalent* mass, damping, and stiffness matrices. $[D_m] \in \Re^{N_m \times N_m}$ is the force distribution matrix, $\{x_m(t)\} \in \Re^{N_m \times 1}$ is the displacement vector, $\{f(t)\} \in \Re^{N_m \times 1}$ is the force vector, and N_m is the number of measurement degrees of freedom.

To seek a means of reducing the model order without seriously degrading the major characteristics of the structure, one of the signal reduction techniques based on regression analysis or principal component analysis [20] can be employed

$$\{\gamma(t)\}=[\psi_c]\{x_m(t)\} \tag{18}$$

where $\{\gamma(t)\} \in \Re^{N_r \times 1}$ is a reduced set of measured data and $[\psi_c] \in \Re^{N_r \times N_m}$ is called a contracting transformation matrix. The number of reduced measurement degrees of freedom, $N_r (\leq N_m)$, can be determined by Gaussian pivot values (regression analysis) or singular values (principal component analysis).

Based on the reduced measurement degrees of freedom, therefore, consider

$$[M_r]\{\ddot{\gamma}(t)\}+[C_r]\{\dot{\gamma}(t)\}+[K_r]\{\gamma(t)\}=[D_r]\{p(t)\} \tag{19}$$

where $[M_r]$, $[C_r]$, and $[K_r] \in \Re^{N_r \times N_r}$ are, respectively, the *equivalent reduced* mass, damping, and stiffness matrices. And $[D_r] \in \Re^{N_r \times N_p}$ is the equivalent reduced force distribution matrix.

Once the analysis is completed, all the measured degrees of freedom or $\{x_m(t)\}$, including the mode shapes, can be approximated via

$$\{x_m(t)\} \cong [\psi_R]\{\gamma(t)\} \tag{20}$$

where $[\Psi_R] \in \Re^{N_m \times N_r}$ is called a reconstructing transformation matrix.

In Ref. [19], the independent analysis was used to determine the optimal number and location of sensors for large space structures. For example, if Eq. (33) in Ref. [20] is employed, i.e.,

$$\begin{bmatrix} \Re[\ddot{x}_i(\omega)]^T \\ \Im[\ddot{x}_i(\omega)]^T \end{bmatrix} [\psi_{di}]^T = \begin{bmatrix} \Re[\ddot{x}_d(\omega)]^T \\ \Im[\ddot{x}_d(\omega)]^T \end{bmatrix} \tag{21}$$

consider the complex matrix

$$[\ddot{x}(\omega_i)] \equiv \begin{bmatrix} \ddot{x}_1(\omega_1) & \dots & \ddot{x}_1(\omega_{N_\omega}) \\ \ddot{x}_2(\omega_1) & \dots & \ddot{x}_2(\omega_{N_\omega}) \\ & \vdots & \\ \ddot{x}_N(\omega_1) & \dots & \ddot{x}_N(\omega_{N_\omega}) \end{bmatrix} \tag{22}$$

which will be driven to the following form by Gaussian elimination with row and column pivoting (full pivoting):

$$\begin{bmatrix} \times & \times & \times & \times & \times & \times & \\ & \times & \times & \times & \times & \times & \\ & & \times & \times & \times & \times & \ldots\text{etc.} \\ & & & \times & \times & \times & \\ & & & & \times & \times & \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}\times\\\times\\\times\end{matrix}}\right\} i \text{ (independent coordinate)} \\ \left.\vphantom{\begin{matrix}\times\\\times\end{matrix}}\right\} d \text{ (dependent coordinate)} \end{matrix} \tag{23}$$

As a result, nearly-linearly-dependent rows are driven to the bottom d positions shown in Eq. (23) and the corresponding coordinates are selected as the dependent coordinate group. To find the real transformation matrices, however, the sampled spectra matrix of Eq. (22) was separated into real and imaginary parts and processed together in the Gaussian elimination process. Gaussian pivot values indicate the importance of each degree of freedom and can be used to select the optimal sensor locations.

(b) Parameter Estimation

To estimate the system parameters using only the acceleration data, consider a frequency-domain reduced-order model based on Eq. (19)

$$[M_r]\{\ddot{\gamma}(\omega)\} + [C_r]\left(\frac{1}{j\omega}\right)\{\ddot{\gamma}(\omega)\} + [K_r]\left(\frac{1}{-\omega^2}\right)\{\ddot{\gamma}(\omega)\} = [D_r]\{p(\omega)\} \tag{24}$$

where

$$\{\gamma(\omega)\} = \mathcal{F}[\{\gamma(t)\}] \tag{25}$$

and $\mathcal{F}[\quad]$ denotes the Fourier transform. To find a unique set of system matrices, it is necessary to reduce the number of unknown parameters, e.g., premultiplying Eq. (24) by the inverse of $[M_r]$

$$\{\ddot{\gamma}(\omega)\} + [\hat{C}]\left(\frac{1}{j\omega}\right)\{\ddot{\gamma}(\omega)\} + [\hat{K}]\left(\frac{1}{-\omega^2}\right)\{\ddot{\gamma}(t)\} = [\hat{D}]\{p(\omega)\} \tag{26}$$

where $[\hat{C}] = [M_r]^{-1}[C_r] \in \Re^{N_r \times N_r}$, $[\hat{K}] = [M_r]^{-1}[K_r] \in \Re^{N_r \times N_r}$, and $[\hat{D}] = [M_r]^{-1}[D_r] \in \Re^{N_r \times N_p}$.

If a data set $[\ddot{x}_m(\omega_i)]$ represents a set of N_ω measured response spectra, i.e.,

$$[\ddot{x}_m(\omega_i)] \equiv \left[\{\ddot{x}_m(\omega_1)\}\{\ddot{x}_m(\omega_2)\}\cdots\{\ddot{x}_m(\omega_{N_\omega})\}\right] \tag{27}$$

then a reduced set of response spectra may be obtained by using the contracting transformation $[\psi_c]$

$$[\ddot{\gamma}(\omega_i)] \equiv \left[\{\ddot{\gamma}(\omega_1)\}\{\ddot{\gamma}(\omega_2)\}\cdots\{\ddot{\gamma}(\omega_{N_\omega})\}\right] = [\psi_c][\ddot{x}_m(\omega_i)] \tag{28}$$

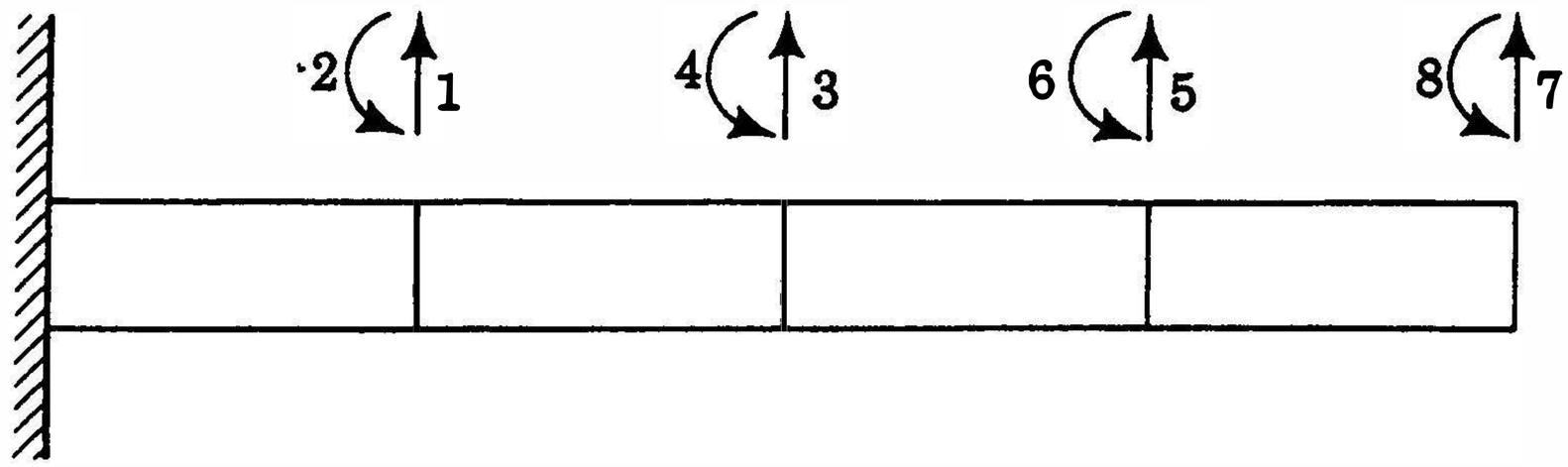

Fig. 1 8-DOF cantilever beam

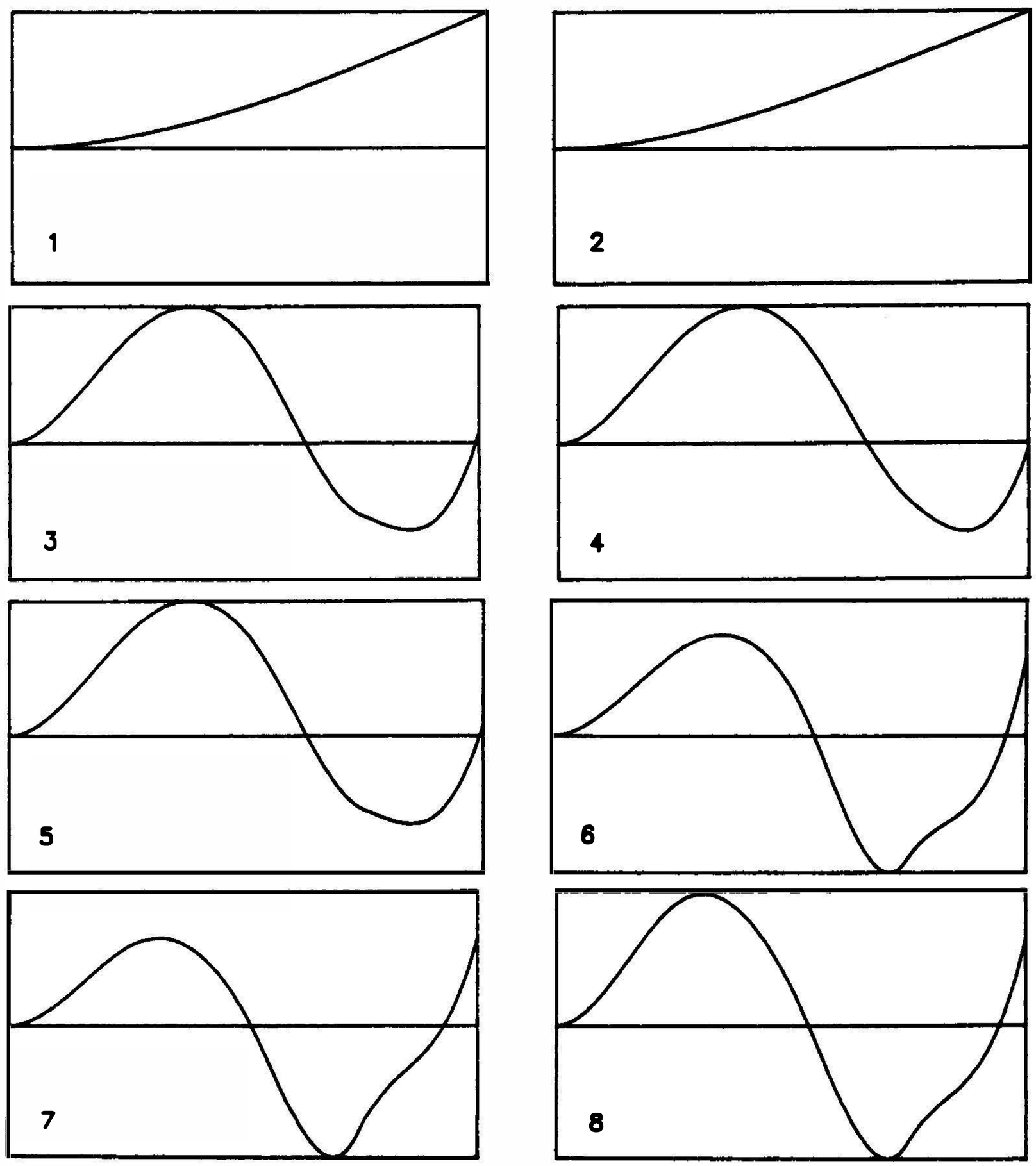

Fig. 2a Analytical Lanczos vectors (1st-8th)

TABLE 1 DAMPED FREQUENCIES AND DAMPING FACTORS OF A CANTILEVER BEAM

No.	Frequency	Damping
1	0.110217	0.08362
2	0.692830	0.05588
3	1.95307	0.05234
4	3.85316	0.05151
5	7.16710	0.05043
6	11.5104	0.05039
7	18.2480	0.05024
8	29.9407	0.05051

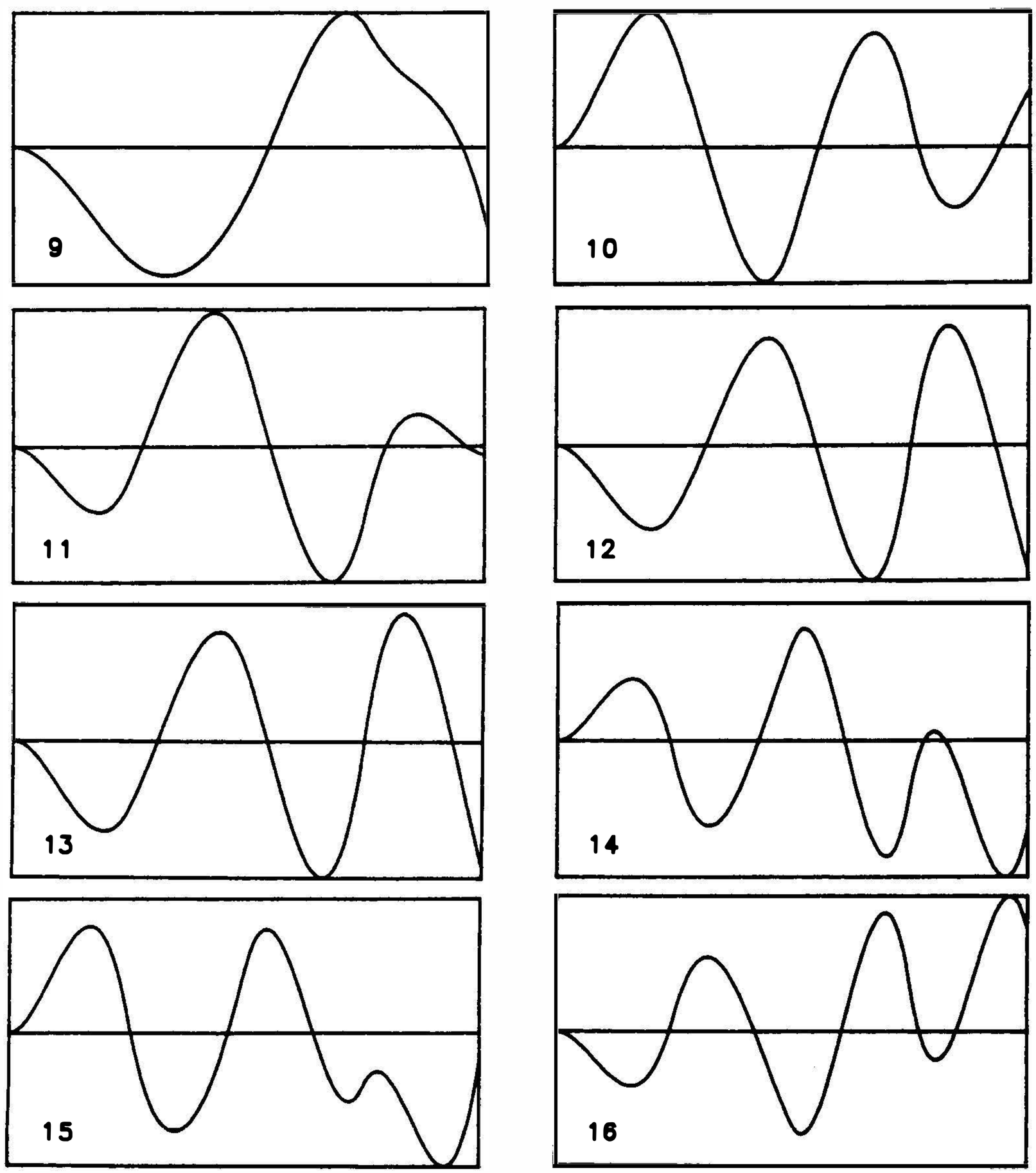

Fig. 2b Analytical Lanczos vectors (9th-16th)

To accommodate the condensed experimental acceleration spectra defined by Eq.(28), Eq. (26) may be expanded for N_ω discrete frequencies and rearranged to give

$$\left[\left[\hat{C}\right]\left[\hat{K}\right]\left[\hat{D}\right]\right]\begin{bmatrix}\left[\frac{1}{j\omega_1}\ddot{\gamma}(\omega_i)\right]\\ \left[\frac{1}{-\omega_i^2}\ddot{\gamma}(\omega_i)\right]\\ \left[-p(\omega_i)\right]\end{bmatrix}=\left[-\ddot{\gamma}(\omega_i)\right] \tag{29}$$

A standard least-squares problem for $[\hat{C}]$, $[\hat{K}]$, and $[\hat{D}]$ may be formulated by transposing and separating the real and imaginary parts of Eq. (29).

$$\begin{bmatrix}\left[\frac{1}{j\omega_1}\ddot{\gamma}(\omega_i)\right]^T & \left[\frac{1}{-\omega_i^2}\ddot{\gamma}(\omega_i)\right]^T & \left[-p(\omega_i)\right]^T\\ \left[\frac{1}{j\omega_1}\ddot{\gamma}(\omega_i)\right]^T & \left[\frac{1}{-\omega_i^2}\ddot{\gamma}(\omega_i)\right]^T & \left[-p(\omega_i)\right]^T\end{bmatrix}\begin{bmatrix}\left[\hat{C}\right]^T\\ \left[\hat{K}\right]^T\\ \left[\hat{D}\right]^T\end{bmatrix}=\begin{bmatrix}\left[-\ddot{\gamma}(\omega_i)\right]^T\\ \left[-\ddot{\gamma}(\omega_i)\right]^T\end{bmatrix} \tag{30}$$

or, in condensed form

$$\left[A_s\right]\left[Y_s\right]=\left[B_s\right] \tag{31}$$

where $[A_s]$ is the $2N_\omega \times (2N_r + N_p)$ known coefficient matrix, $[B_s]$ is the $2N_\omega \times N_r$ known matrix, and $[Y_s]$ is the $(2N_r + N_p) \times N_r$ unknown matrix.

Now the system matrix [A] in Eq. (13) can be estimated by: solving the least-squares problem of Eq. (31) using excitation and response spectra; transforming Eq. (26) into a time-domain model (equivalent to Eq. (1)); and changing it to a first order model (equivalent to Eq. (2)). Reference [19] discussed several least-squares algorithms in detail.

5. Numerical Simulation Results

(a) Proposed System Identification Approach

Figure 1 shows an 8-DOF finite element model of a uniform cantilever beam [19] which was employed to evaluate the new system identification procedure using numerical simulation. Table 1 shows the damped frequencies and damping factors of the model. This model has 5% modal damping for each mode, as well as some discrete damping at DOF = 1, 3, 5, and 7. As a result, it is necessary to use a state vector formulation. First, an analytical model of Eq. (1) was generated. By using special starting vectors, the analytical Lanczos vectors were generated (see Fig. 2), which will be compared with experimental Lanczos vectors. It is noted that Lanczos vectors always span the real space $\Re$, while eigenvectors may span the complex space C.

Response (acceleration) data were produced by numerical simulation and then transformed to response

spectra by the fast Fourier transform. For simulation, the type of input was random (at DOF = 5 and 7); the sampling rate was 10.53 Hz; the cut-off frequency was 4.112 Hz; the frequency resolution was 0.02056 Hz; and the number of samples was 512. Since the order of a model is small, the preliminary data reduction step was not employed for this example.

The experimental system model was identified by solving the least-squares problem of Eq. (31). The experimental Lanczos vectors were identified from Eq. (6). It can be seen that the experimental Lanczos vectors are different from the analytical ones. This difference exists because Lanczos vectors are not unique. However, both analytical and experimental vectors of higher-order exhibit increasingly more complex deflection shapes, i.e., vectors having a greater number of nodes [18].

The experimental reduced order model was constructed by using eight Lanczos vectors (see Fig. 3). To compare the analytical and reduced experimental models, responses (acceleration) due to external step forces at DOF = 5 and 7 (see Figs. 4 and 5) were generated. Figures 6 and 7 show the responses at DOF = 5, based on the analytical model of order 16 and the reduced experimental model of order 8, respectively,

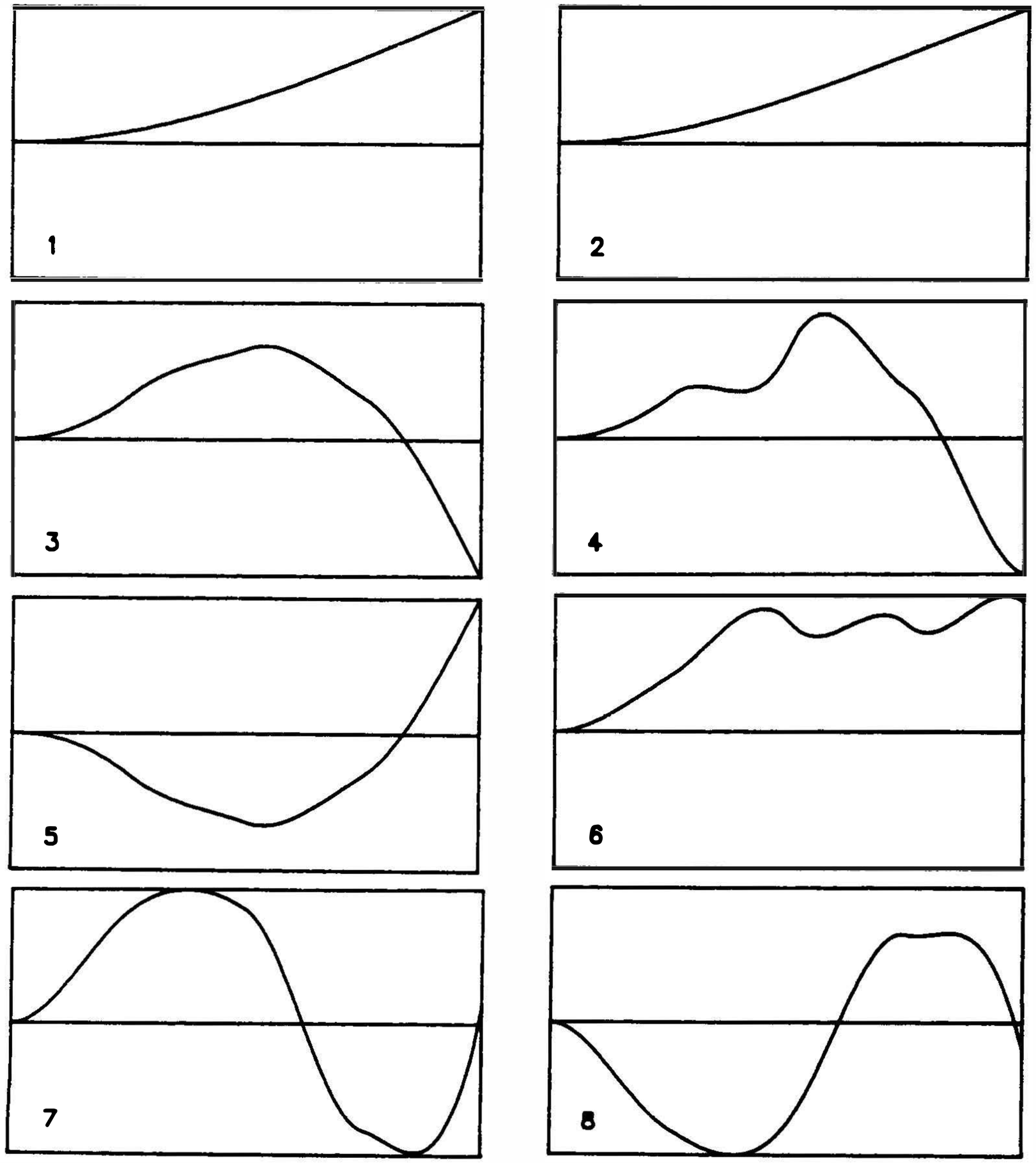

Fig. 3 Experimental Lanczos vectors (1st-8th)

and Figs. 8 and 9 show the responses at DOF = 7. The approximate responses are almost the same as the exact responses, which indicate that the reduced-order Lanczos model is a good experimental model. In this new system identification approach, a numerically expensive eigenvalue problem was not involved.

(b) Sensor Location Selection

Figure 10 shows a 60-DOF simplified finite element model of the Space Station Freedom. In this example, the preliminary data reduction based on regression analysis was used to reduce the number of measurement degrees of freedom, i.e., N_m of Eq. (17) to N_r of Eq. (19). Random input was applied at node number 4 and acceleration responses were generated by numerical simulation.

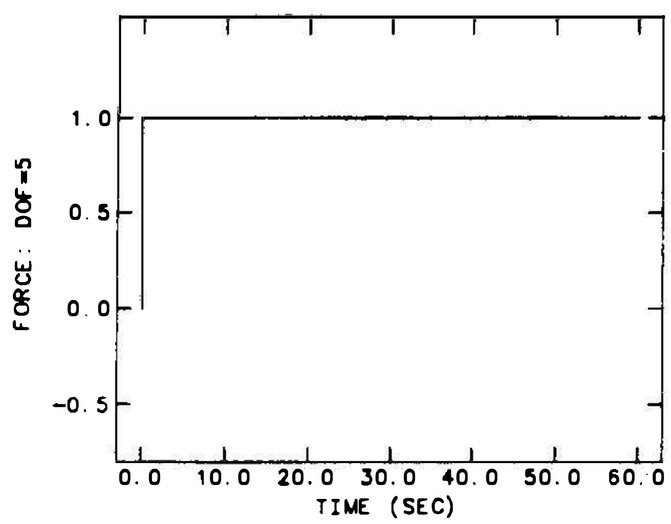

Fig. 4 External Step Force at DOF = 5

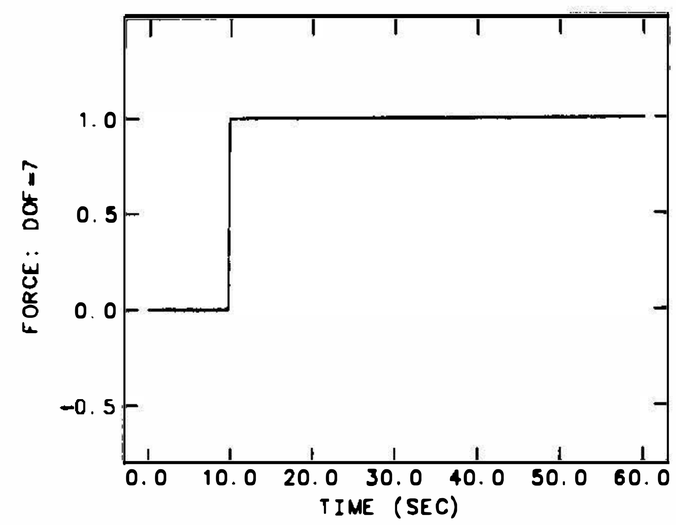

Fig. 5 External Step Force at DOF = 7

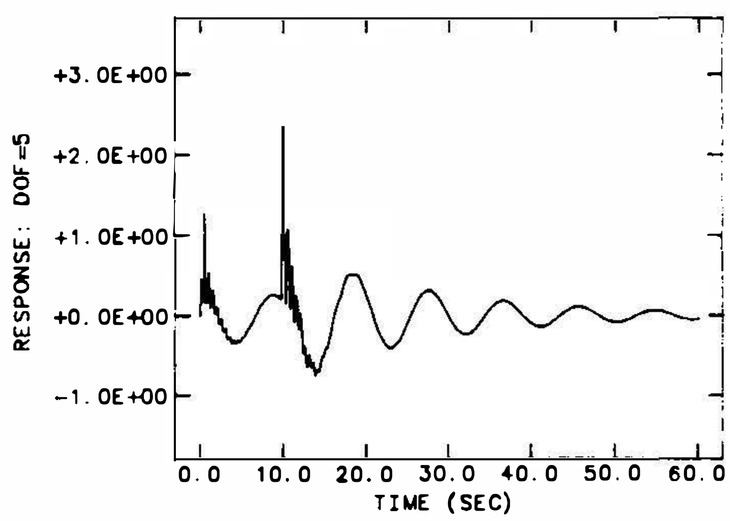

Fig. 6 Exact response at DOF = 5

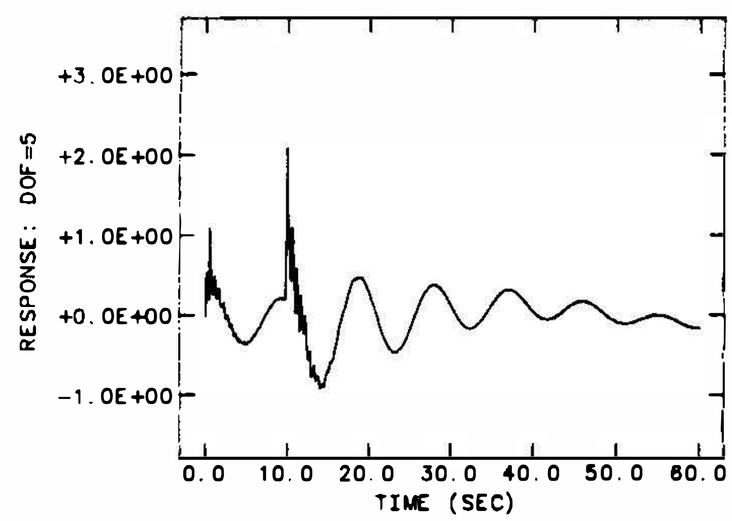

Fig. 7 Approximate response at DOF = 5

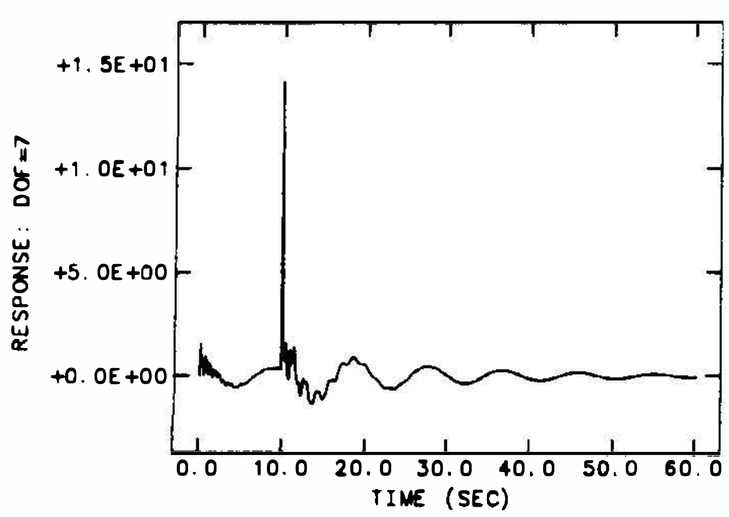

Fig. 8 Exact response at DOF = 7

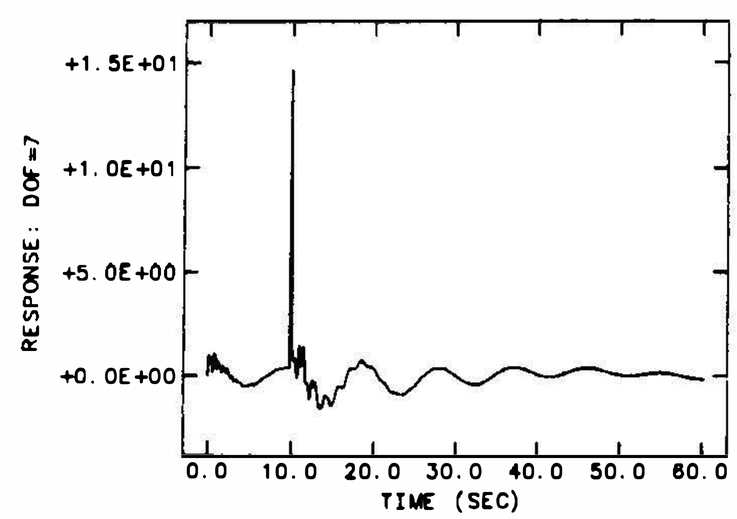

Fig. 9 Approximate response at DOF = 7

During the preliminary data reduction process of Eq. (23), a Gaussian pivot value for each degree of freedom (and corresponding coordinate) was determined. These pivot values in Fig. 11 indicated the importance of each sensor, which was used to determine the optimal number and location of sensors. Complete system identification results can be found in Ref. [19]

(c) Lanczos Eigensolution

To evaluate the efficiency of the Lanczos eigensolver for a large structural problem, the inverse power method and Givens method as well as the Lanczos method were tested by using MSC/NASTRAN on a Cyber 6400/6600 computer and a Cray X-MP/24 supercomputer. The test model is a 474-DOF finite element model of the Space Station Freedom first flight configuration (Fig. 12). The computed eigenvalues were exactly the same between the methods, but the computing cost for the Lanczos method was significantly reduced, as seen in Table 2.

TABLE 2 COMPUTING TIME FOR EIGENSOLUTIONS

Computer	Methods	CPU Time (sec)	I/O Time (sec)
Cyber	Inverse Power Method	1666.42	625.90
Cray	Inverse Power Method	74.98	35.20
Cray	Givens Method	12.62	0.70
Cray	Lancozos Method	2.30	1.65

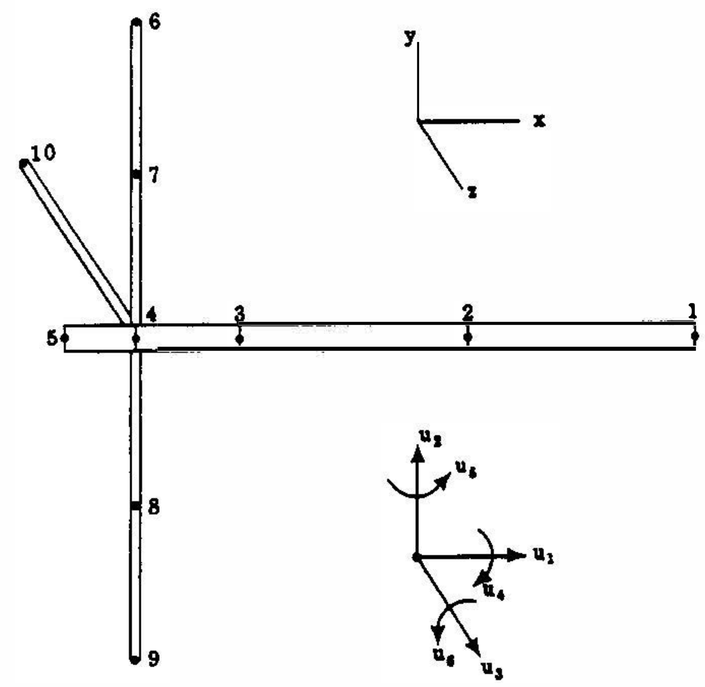

Fig. 10 60-DOF Simplified Space Station Freedom model

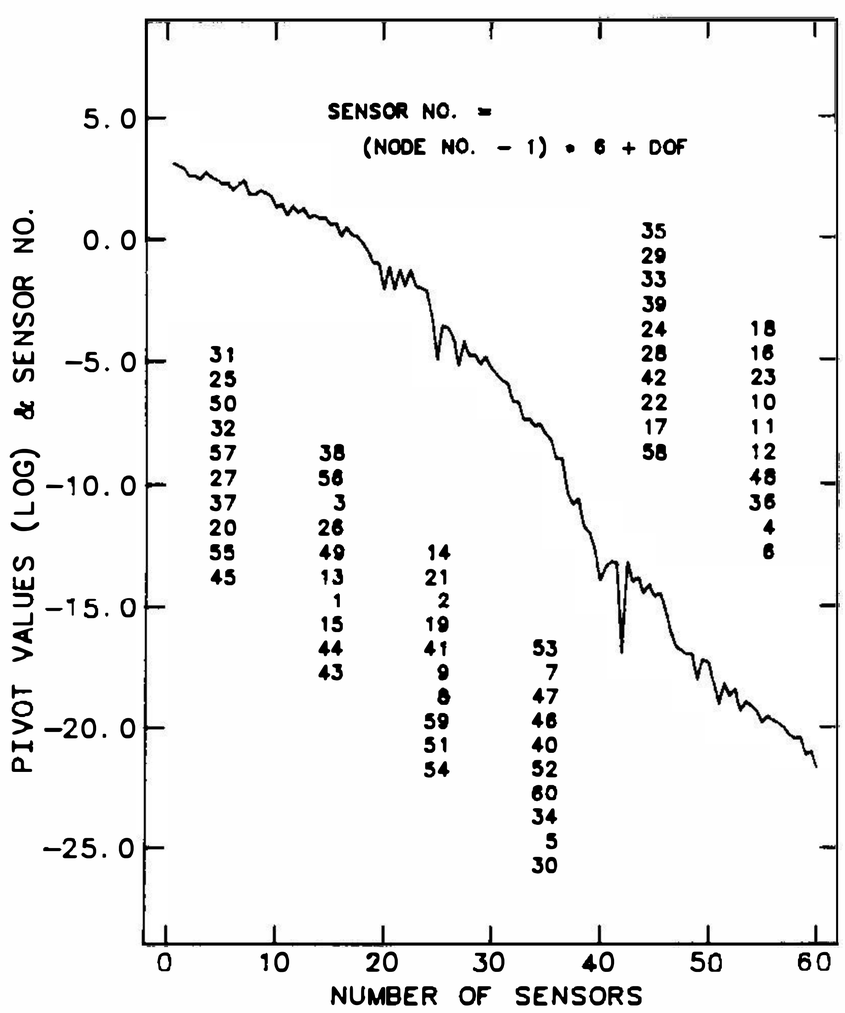

Fig. 11 Gaussian pivot values for sensor location selection

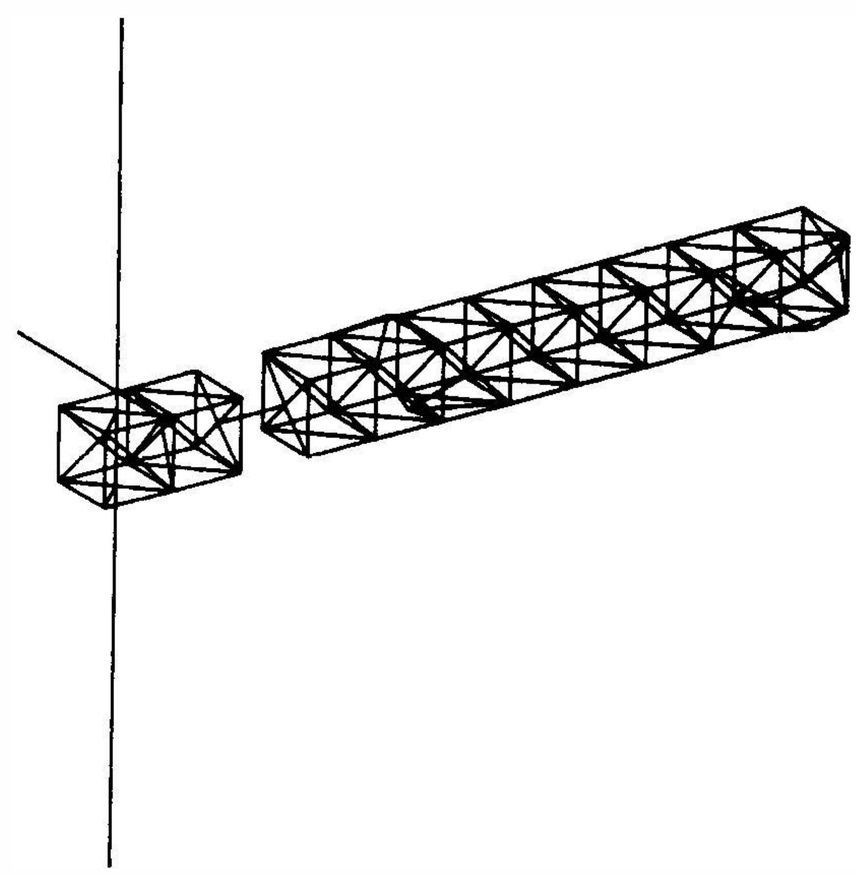

Fig. 12 474-DOF Space Station Freedom model

6. Summary and Conclusions

A new system identification approach using Lanczos coordinates has been presented. A multi-input, multi-output frequency-domain technique generated system matrices for a general linear, time-invariant dynamic system. The order of the identified dynamic system was reduced using Lanczos coordinates in place of conventional modal coordinates. Identified experimental Lanczos vectors can be used for other applications such as component mode synthesis and control analysis. A cantilever beam example showed promising results, indicating that a new system identification approach using Lanczos coordinates is worthy of further study. The sensor location selection process was demonstrated using the frequency-domain data reduction analysis. The computational efficiency of Lanczos method was compared to other conventional eigensolvers for a large structural problem.

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References

1. Ewins, D.J. *Modal Testing: Theory and Practice.* New York: John Wiley & Sons, Inc.; 1984.
2. Prony, R. "Essai Expérimental et Analytique sur les Lois de la Dilatabilité des Fluides Elastiques et sur Celles de la Force Expansive de la Vapeur de l'Eau et de la Vapeur de l'Alkool, a Differentes Témperatures." *J. de l'Ecole Polytechnique (Paris)* Vol. 1, Cahier 2, Floréal et Prairial, An. III, p 24-76, 1795.
3. Lewis, R.C.; Wrisley, D.L. "A system for the excitation of pure natural modes of complex structures." *J Aeronaut Sci* v 17 n 11 p 705-722 Nov 1950.
4. Stahle, C.V., Jr.; Forlifer, W.R. "Ground vibration testing of complex structures." *Flight Flutter Testing Symposium*, 1958. NASA-SP-385 p 83-90.
5. Gersch, W. "Estimation of the autoregressive parameters of a mixed autoregressive moving-average time series." *IEEE Trans Autom Control* v AC-15 p 583-588 Oct 1970.
6. Ibrahim, S.R.; Mikulcik, E.C. "A method for the direct identification of vibration parameters from the free response." *Shock Vib Bull*, v 47 pt 4 p 183-198 Sept 1977.
7. Coppolino, R.N. "A simultaneous frequency domain technique for estimation of modal parameters from measured data." *SAE Pap* 811046 Oct 1981.
8. Vold, H.; Rocklin, G.T. "The numerical implementation of a multi-input modal estimation method for mini-computers," *Proceedings of the 1st International Modal Analysis Conference,* Orlando, FL, Nov 8-10, 1982. p 542-548
9. Juang, J.N.; Pappa, R.S. "An eigensystem realization algorithm for modal parameter identification and model reduction." *J Guid Control Dyn,* v 8 n 5 p 620-627 Oct 1985.
10. Lanczos, C. "An iteration method for the solution of the eigenvalue problem of linear differential and integral operators." *J Res Natl Bur Stand (US)* v 45 n 4 p 255-282 Oct 1950.

11. Wilkinson, J.H. *The Algebraic Eigenvalue Problem.* Oxford: Claredon Press; 1965.
12. Paige, C.C. "Practical use of the symmetric Lanczos process with reorthogonalization." *BIT (Copenhagen)* v 10 n 2 p 183-195 1970.
13. Parlett, B.N.; Scott, D.S. "The Lanczos algorithm with selective orthogonalization." *Math Comp* v 33 n 145 p 217-238 1979.
14. Parlett, B.N.; Reid, J.K. "Tracking the progress of the Lanczos algorithm for large symmetric eigenproblems." *IMA J Numer Anal* v 1 p 135-155 1981.
15. Smith, I.M.; Heshmati, E.E. "Short communications: use of a Lanczos algorithm in dynamic analysis of structures." *Earthquake Eng Struct Dyn* v 11 n 4 p 585-588 Jul/Aug 1983.
16. Kim, H.M.; Craig, R.R., Jr. "Structural dynamics analysis using an unsymmetric block Lanczos algorithm." *Int J Numer Methods Eng* v 26 p 2305-2318 1988.
17. Kim, H.M.; Craig, R.R., Jr. "Technical note: computational enhancement of an unsymmetric block Lanczos algorithm." *Int J Numer Methods Eng* v 30 p 1083-1089 1990.
18. Craig, R.R., Jr.; Kim, H.M.; Su, T.J. "Some applications of Lanczos vectors in structural dynamics." *Proceedings of the 6th International Modal Analysis Conference,* Kissimmee, FL, Feb 1-4, 1988. v 1 p 501-506
19. Kim, H.M. *System Identification for Large Space Structures.* Ph.D. Dissertation, The University of Texas at Austin, 1988.
20. Craig, R.R., Jr.; Kurdila, A.J.; Kim, H.M. "State-space formulation of multi-shaker modal analysis." *Int J Anal Exp Modal Anal* v 5 n 3 p 169-183 July 1990.

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