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by T. W. Lim, Lockheed Engineering and Sciences Company

ABSTRACT

This paper addresses a systematic procedure of placing actuators and sensors to identify modal parameters of flexible structures. In this procedure, using a pre-test analysis model, actuator locations are selected to minimize the condition number of the controllability matrix and sensors are placed to obtain a full-rank Hankel matrix with the minimum condition number. Since the rank of the Hankel matrix determines the number of modes identified by a modal parameter identification algorithm such as the Eigensystem Realization Algorithm, modal identification of the target modes for a modal testing can be ensured. A suboptimal method, called the Effective Independence approach, is employed to select the best locations among the candidate actuator/sensor locations. In this approach, the candidate actuator and sensor locations contributing little to the linear independence of the controllability matrix and the Hankel matrix, respectively, are removed iteratively until a desired number of actuator/sensor locations remains. Unlike computationally intensive search techniques, this approach is computationally efficient. An example using a ten-bay truss structure is presented to illustrate the design procedure.

List of Symbols

- $[A]$ see Eq. (13)
- $[\hat{A}]$ see Eq. (7)
- $[B]$ see Eq. (13)
- $[\hat{B}]$ see Eq. (7)
- $[C]$ see Eq. (7)
- $[C_p]$ proportional damping matrix
- $[D]$ excitation measurement matrix
- $[E_W]$ see Eq. (21)
- ea_i collective EI contribution of the i^{th} actuator
- ef_i EI contribution of the i^{th} candidate actuator DOF
- eh_j EI contribution of the j^{th} row of $[H_{vw}]$
- es_i collective EI contribution of the i^{th} sensor
- ew_j EI contribution of the j^{th} column of $[W]$
- ey_i EI contribution of the i^{th} candidate sensor DOF
- $[F]$ actuator location matrix
- $\{f\}$ excitation vector
- $[H]$ Hankel matrix
- $[H_{vw}]$ see Eq. (27)
- h discretization time step
- $[I]$ identity matrix
- $[K]$ stiffness matrix
- $[M]$ mass matrix
- n number of system DOFs
- n_a desired number of actuator DOFs
- n_b total number of candidate actuator DOFs

T. W. Lim, currently Assistant Professor, Aerospace Engineering Department, 2004 Learned Hall, The University of Kansas, Lawrence, KS 66045.

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n_c number of candidate sensor DOFs
n_f number of excitation sources
n_m number of modes incorporated in transform ion, see Eq. (2)
n_t number of target modes
n_y number of sensors
$\{q\}$ displacement in the modal coordinates
T see Eq. (13)
$\{u\}$ displacement vector
$[V]$ see Eq. (25)
$[V_\alpha]$ observability matrix
$[W]$ see Eq. (19)
$[W_\beta]$ controllability matrix
$\{x\}$ state vector
$[Y]$ sensor location matrix
$[Y_d]$ displacement measurement matrix
$[Y_k]$ Markov parameter
$[Y_v]$ velocity measurement matrix
$\{y\}$ sensor measurement vector
$[Z]$ diagonal matrix of modal damping ratios

Greek Symbols

α see Eq. (16)
β see Eq. (17)
Δ_{ea}, Δ_{es} tolerance values
ζ_i modal damping ratio of the i^{th} mode
$\rho(\ldots)$ rank of a matrix
$[\Phi]$ normalized mode shape matrix
$[\Phi_f]$ see Eq. (2)
$[\Phi_y]$ see Eq. (8)
$[\Omega]$ diagonal matrix of natural frequencies
ω_i natural frequency of the i^{th} mode

1. Introduction

Time domain modal identification algorithms such as the Eigensystem Realization Algorithm (ERA) [1], the Polyreference technique [2], or the Ibrahim Time Domain method [3] have been increasingly employed in identifying modal parameters of flexible structural systems. The Hankel matrix, which is a product of the observability and controllability matrices, is a building block of the ERA and the Polyreference technique and the number of identified modes depends on the rank of the Hankel matrix. The recovery of target modes are ensured when sensors and actuators are placed on a test structure to provide a full rank Hankel matrix. The target modes are defined as the selected modes to be identified through a modal survey. The target modes are typically selected from a pre-test analysis of a test structure. In a typical modal test, the number of available sensors and actuators is limited and is often required to be minimized. Therefore, a systematic method of placing actuators and sensors in an optimum way is needed.

In conjunction with the control of flexible structures, a large number of investigations has been performed to find the optimal sensor and actuator locations. Reference [4] contains a list of methods purposed in this respect. Pertaining to the actuator/sensor placement methods for modal parameter identification, a relatively small number of studies is available. A sensor placement method utilizing the Effective Independence (EI) information was developed [4] and applied to place sensors for on-orbit modal identification [5]. In this method, the EI contribution of individual sensor locations to the total rank of a mode shape matrix is computed and used to remove less effective candidate sensor locations. The number of sensors cannot be reduced below the number of target modes because the mode shape matrix becomes rank deficient. In a modal test, the number of actuators is frequently smaller than the number of target modes. Therefore, a direct extension of this EI sensor placement method to an actuator placement is inappropriate.

This paper presents a systematic approach to selecting locations of actuators and sensors to recover given target modes. Initially, actuator locations are selected to minimize the condition number of the controllability matrix. The sensors are subsequently placed to obtain a full-rank Hankel matrix using the previously selected actuator locations. The use of the controllability matrix in the proposed method allows the reduction of the number of candidate actuator degrees-of-freedom (DOFs) below the number of target modes. Hence, it is possible to satisfy the constraint imposed on the number of actuator DOFs for a modal test. Also, instead of computing the EI contribution of a mode shape matrix for the sensor placement [4], the proposed approach computes the EI contribution of the Hankel matrix, which contains additional information on the frequencies and modal damping ratios of target modes. Since the resulting actuator/sensor locations will have a full rank Hankel matrix, the recovery of target modes is ensured. As an illustrative example, a sensor/actuator placement study for a ten-bay truss structure is presented.

2. Theo[illegible] for Actuator/Sensor [illegible]

The equation of motion governing the response of lightly-damped flexible structures is frequently represented by

$$[M]\{\ddot{u}\}+[C_p]\{\dot{u}\}+[K]\{u\}=[F]\{f\} \tag{1}$$

where the matrices $[M]$, $[C_p]$ and $[K]$ are $n \times n$ system mass, proportional damping, and stiffness matrices respectively; $\{u\}$ and $\{f\}$ are $n \times 1$ physical diplacement vector and $n_f \times 1$ excitation vector, respectively; the matrix $[F]$ defines the actuator locations in the equation. Employing the transformation $\{u\} = [\Phi]\{q\}$, Eq. (1) can be written in the modal coordinates as

$$\{\ddot{q}\}+2[Z][\Omega]\{\dot{q}\}+[\Omega]^2\{q\}=[\Phi]^T[F]\{f\}=[\Phi_f]^T\{f\} \tag{2}$$

where $[Z] = \text{diag}\ \{\zeta_i\}$ and $[\Omega] = \text{diag}\ \{\omega_i\}$ in which ζ_i and ω_i are the modal damping ratio and natural frequency of the i^{th} mode, respectively; $[\Phi]$ is an $n \times n_m$ mode shape matrix normalized to yield a unity generalized mass matrix; and $\{q\}$ represents the displacement in the modal coordinates. The quantity n_m is the number of the modes incorporated in the transformation.

In general, an n_y-dimensional sensor measurement vector is defined as

$$\{y\}=[Y_d]\{q\}+[Y_v]\{\dot{q}\}+[D]\{f\} \tag{3}$$

where $[Y_d]$, $[Y_v]$, and $[D]$ are the matrices with appropriate dimensions and they depend on the types and locations of the measurements. Define a state vector

$$\{x\}=\left\{\{q\}^T\{\dot{q}\}^T\right\}^T \tag{4}$$

Then, an equivalent state-space representation of Eqs. (2) and (3) becomes

$$\{\dot{x}\}=[\hat{A}]\{x\}+[\hat{B}]\{f\} \tag{5}$$

$$\{y\}=[C]\{x\}+[D]\{f\} \tag{6}$$

where

$$[\hat{A}]=\begin{bmatrix}0 & [I]\\ -[\Omega^2] & -2[Z][\Omega]\end{bmatrix} \qquad [\hat{B}]=\begin{bmatrix}0\\ [\Phi_f]^T\end{bmatrix} \qquad [C]=[[Y_d]\ \ [Y_v]] \tag{7}$$

When accelerometers are employed to measure the response, the matrices in Eqs. (6) and (7) become

$$[Y_d] = -[Y][\Phi][\Omega]^2 = -[\Phi_y][\Omega]^2 \tag{8}$$

$$[Y_v] = -2[Y][\Phi][Z][\Omega] = -2[\Phi_y][Z][\Omega] \tag{9}$$

$$[D] = [Y][\Phi][\Phi_f]^T = [\Phi_y][\Phi_f]^T \tag{10}$$

where the matrix $[Y]$ contains information on the sensor locations. Discretize Eqs. (5) and (6) to obtain for $k = 0,1,2,\ldots$

$$\{x(k+1)\} = [A]\{x(k)\} + [B]\{f(k)\} \tag{11}$$

$$\{y(k)\} = [C]\{x(k)\} + [D]\{f(k)\} \tag{12}$$

where

$$[A] = e^{[\hat{A}]h} \quad ; \quad [B] = \left(\int_0^h e^{[\hat{A}]T}dT\right)[\hat{B}] \tag{13}$$

where h is the discretization time step.

When the system given in Eqs. (11) and (12) is subject to an initial pulse input, the time domain response description is given as

$$\{y(0)\} = [D]\{f(0)\} \text{ and } \{y(k)\} = [C][A]^{k-1}[B]\{f(0)\} \text{ for } k = 1,2,3,\ldots \tag{14}$$

where the matrix product $[Y_k] = [C][A]^{k-1}[B]$ is known as the Markov parameter. The Eigensystem Realization Algorithm (ERA) begins by forming the block Hankel matrix

$$[H(\alpha,\beta)] = \begin{bmatrix} [Y_1] & [Y_2] & [Y_3] & \cdots & [Y_\beta] \\ [Y_2] & [Y_3] & [Y_4] & \cdots & [Y_{\beta+1}] \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ [Y_\alpha] & [Y_{\alpha+1}] & [Y_{\alpha+2}] & \cdots & [Y_{\alpha+\beta-1}] \end{bmatrix} \tag{15}$$

The Hankel matrix is a product of the observability matrix, $[V_\alpha]$, and the controllability matrix, $[W_\beta]$. They are defined as

$$[V_\alpha] = \begin{bmatrix} [C] \\ [C][A] \\ \cdot \\ \cdot \\ \cdot \\ [C][A]^{\alpha-1} \end{bmatrix} \tag{16}$$

$$\left[W_{\beta}\right]=\left[[B]\;\;[A][B]\ldots[A]^{\beta-1}[B]\right] \tag{17}$$

The ERA performs the singular value decomposition on the Hankel matrix and the number of identified modes depends on the rank of the Hankel matrix. The rank of $[H(\alpha,\beta)]$ is bounded by

$$\rho\left(\left[H(\alpha,\beta)\right]\right)\le \min\left(\rho\left(\left[V_{\alpha}\right]\right),\rho\left(\left[W_{\beta}\right]\right)\right) \tag{18}$$

where $\rho(\ldots)$ represents the rank of a matrix. Therefore, in order to identify all the modes incorporated in the matrix $[A]$, the observability and controllability matrices should have full rank, i.e., $\rho([V_{\alpha}]) = \rho([W_{\beta}]) = 2n_m$. To select the best actuator/sensor locations, target modes are incorporated in the matrix $[A]$. Target modes are the modes to be identified in a modal test. They are selected based on the the usage of the identified modes, e.g., structural load analysis model validation or flexible plant model verification for control system design. In this paper, it is assumed that a set of target modes is given.

3. Actuator Placement

The first step for actuator placement is to select locations on the structure where the excitation can be applied. The selected locations are referred to as candidate actuator locations and they may be influenced by the support configuration of a test structure, types of actuators available, and target modes. The number of the candidate actuator locations may be much larger than the number of the target modes. However, in a typical modal test, the number of actuators used is smaller than the number of the target modes. Therefore, the candidate actuator locations need to be reduced. The actuator placement problem is to find the "best" locations among the candidate actuator locations in order to excite the target modes properly. There may be many sets of actuator locations containing the desired number of actuator locations that provide full-rank controllability matrices. The best set of actuator locations is the one that minimizes the condition number of the controllability matrix.

Once the candidate actuator locations are selected, the matrix $[F]$ in Eq. (2) is determined. All n_t number of target modes are incorporated to define the matrices $[\hat{A}]$ and $[\hat{B}]$ in Eq. (7). The rank of the matrix $[\hat{A}]$ now becomes $2n_t$. The controllability matrix can be obtained using Eqs. (7), (13), and (17). The rank of the controllability matrix is determined by examining the matrix [6]

$$[W]=\left[[B]\;\;[A][B]\ldots[A]^{\rho[A]-\rho[B]}[B]\right] \tag{19}$$

The rank of the matrix $[A]$ is $2n_t$ and fixed. The maximum rank of the matrix $[B]$ is bounded by

$$\rho([B])\le\rho\left(\left[\hat{B}\right]\right)=\rho\left(\left[\Phi_f\right]^T\right)=\min\left(n_t,n_b\right) \tag{20}$$

where n_b is the total number of the candidate actuator DOFs. As the actuator selection process progresses, n_b is reduced and $\rho([B])$ may be changed. Assume that the candidate actuator DOFs are to be reduced to the desired number of actuator DOFs (n_a). If n_a is assumed to be smaller than n_t, the lower bound of $\rho([B])$ becomes n_a. Thus, the matrix $[A]$ in Eq. (19) is raised to the $(2n_t-n_a)^{th}$ power during the selection process. Each column of the matrices, $[A]^{i-1}[B]$ $(i=1,2,\ldots,2n_t-n_a+1)$, corresponds to each actuator DOF.

The contribution of each candidate actuator DOF to the rank of the controllability matrix $[W]$ is evaluated. Consider a matrix product

$$[E_W] = [W]^T\left([W]^T\right)^+ \tag{21}$$

where $[E_W]$ is identified as an orthogonal projector [7] onto the column space of $[W]^T$ and the symbol + represents a pseudo-inverse. The matrix $[E_W]$ is an idempotent matrix [7], i.e., $[E_w]^2 = [E_w]$. One important characteristic of the idempotent matrix is that its trace is equal to its rank. Also, as was shown by Kammer [4], the diagonal elements of $[E_W]$ represent the contributions of the corresponding actuator locations to the rank of $[W]$. The contribution of the j^{th} column of the matrix $[W]$ to the total rank of the matix, defined as the effective independence (EI) contribution [4] of the j^{th} actuator location, thus becomes

$$ew_j = [E_W]_{j,j} = \left[[W]^T\left([W]^T\right)^+\right]_{j,j} \tag{22}$$

where ew_j is the EI contribution of the j^{th} column of the matrix $[W]$. This approach provides a systematic procedure for reducing the number of candidate actuator DOFs without significantly deteriorating the condition number of the matrix $[W]$ by removing less effective candidate actuator DOFs. The collective EI of i^{th} actuator DOF to the controllability matrix $[W]$ thus becomes

$$ea_i = \sum_{k=1}^{2n_t - n_a + 1} ew_{i+(k-1)n_b} \tag{23}$$

where ea_i is the collective EI contribution of the i^{th} actuator DOF. For each iteration, the candidate actuator DOFs are removed corresponding to the ea_i values satisfying

$$ea_i \le \min\left(ea_1\ ea_2\ \ldots\ ea_{n_b}\right) + \Delta_{ea} \tag{24}$$

where Δ_{ea} is a tolerance value. The tolerance value is used to delete simultaneously the actuator DOFs which essentially have the same minimum EI contributions and to control the number of actuator DOFs removed in each iteration. Since the EI method is a suboptimal approach, the condition number of the controllability matrix is computed and monitored at each iteration. Once a number of candidate actuator DOFs is removed, a new $[B]$ matrix is defined accordingly and the EI evaluation is continued iteratively using Eqs. (19), (22)-(24) until the desired number of actuator DOFs remains or a further reduction of candidate actuator DOFs is not feasible due to a signifcant degradation of the condition number of the controllability matrix.

4. Sensor Placement

The best actuator locations which minimize the condition number of the controllability matrix have been selected. Therefore, the controllability matrix is now fixed. The task is to select the sensor locations to ensure the recovery of the target modes, i.e., to obtain a full-rank Hankel matrix. The first step is to select the locations where the sensor placement is feasible. These feasible sensor locations are referred to as candidate sensor locations and they may be influenced by the types of sensors available, the adequacy of mounting locations, and the dynamic characteristics of target modes. The number of the candidate sensor DOFs (n_c) is typically much larger than the number of the target modes (n_t). As suggested in Refs. [4] and [5], the minimum number of sensors required to recover all the target modes in n_t. Therefore, it is necessary to reduce n_c to the vicinity of n_t. The sensor placement problem is to find the "best" locations among the candidate sensor locations in order to recover the target modes properly. There may be many sets of sensor locations containing the desired number of sensors that provide full-rank Hankel matrices. Since the ERA identifies modes using the singular value decomposition, the best identification results will be obtained when the magnitudes of singular values corresponding to the target modes are relatively the same. Thus, the best sensor locations are those which minimize the condition number of the Hankel matrix.

Once the candidate sensor locations are selected, the matrix $[Y]$ in Eqs. (8)-(10) is detennined. The observability matrix is then obtained using Eqs. (7), (13), and (16). The rank of the observability matrix is detennined by examining the matrix [6]

$$[V]=\begin{bmatrix} [C] \\ [C][A] \\ \cdot \\ \cdot \\ \cdot \\ [C][A]^{\rho[A]-\rho[C]} \end{bmatrix} \tag{25}$$

The rank of the matrix $[C]$ is bounded by

$$\rho([C]) \leq \rho([\Phi_y]) = \min(n_c, n_t) \tag{26}$$

Since the minimum number of sensor DOFs is n_t and the matrix $[C]$ is selected to be non-singular, $\rho([C])$ is always n_t. Thus, the matrix $[A]$ in Eq. (25) is raised the n_t^{th} power. Each row of $[C][A]^{i-1}$ ($i=1,2,\ldots,n_t+1$) corresponds to each sensor DOF. Using Eqs. (19) and (25), the Hankel matrix becomes

$$[H_{VW}] = [H(n_t+1, 2n_t-n_a+1)] = [V][W] \tag{27}$$

where the best actuator DOFs, previously selected, are used to define the matrix $[W]$.

The contribution of each candidate sensor DOF to the rank of the hankel matrix $[H_{VW}]$ is evaluated by computing the EI contribution of each candidate sensor DOF in a way analogous to computing the EI contribution for actuator placement. The contribution of the j^{th} row of the matrix $[H_{VW}]$ to the total rank of the matrix is computed by

$$eh_j = \left([H_{VW}][H_{VW}]^+\right)_{j,j} \tag{28}$$

where eh_j is the EI contribution of the j^{th} row of the matrix $[H_{VW}]$. The collective EI of the i^{th} sensor DOF to the Hankel matrix thus becomes

$$es_i = \sum_{k=1}^{n_t+1} eh_{i+(k-1)n_c} \tag{29}$$

where es_i is the collective EI contribution of the i^{th} sensor DOF. For each iteration, the candidate sensor DOFs are removed corresponding to the es_i values satisfying

$$es_i \leq \min\left(es_1 \ es_2 \ \ldots \ es_{n_c}\right) + \Delta_{es} \tag{30}$$

where Δ_{es} is a tolerance value and is used to delete simultaneously the sensor DOFs which essentially have the same minimum EI contributions and to control the number of sensor DOFs removed in each iteration. Since the EI method is a suboptimal approach, the singular values of the Hankel matrix are computed and monitored at each iteration. Once a number of candidate sensor DOFs is removed, a new $[C]$ matrix is defined and the EI evaluation continues iteratively using Eqs. (25), (27)-(30) until the desired number of sensor DOFs remains or a further reduction of the number of candidate sensor DOFs is not feasible due to a significant degradation of the singular values of the Hankel matrix.

5. Pre-Selection Procedure for the Reduction of Candidate Actuator/Sensor Locations

The EI evaluation in Eq. (22) is computationally efficient since the calculation of the pseudo-inverse usually requires the inversion of the matrix $[W][W]^T$, which is a $2n_t \times 2n_t$ matrix, and the number of the target modes (n_t) is typically much smaller compared to the size of the finite element DOFs even for large, complicated space structures. However, the number of the candidate actuator DOFs (n_b) can be much larger than n_t. In this case, it will be costly computationally to set up the controllability matrix and to perform a large number of iterations to reduce the candidate actuator locations. In order to enhance the computational efficiency without significantly degrading actuator placement results, a pre-selection procedure is proposed to reduce n_b to a smaller number.

The maximum and minimum ranks of the controllability matrix are bounded by $\rho([A]) = 2n_t$ and $\rho([B])$, respectively. In turn, $\rho([B])$ is bounded by Eq. (20). Therefore, it is possible to reduce n_b to the vicinity of n_t without deteriorating $\rho([B])$ using the EI method. The EI values are computed for the mode shape matrix $[\Phi_f]$ using

$$ef_i = \left(\left[\Phi_f\right]\left[\Phi_f\right]^+\right)_{i,i} \tag{31}$$

where ef_i is the EI contribution of the i^{th} candidate actuator DOF. The DOFs contributing the least to the rank of the matrix $[\Phi_f]$ are removed iteratively until the remaining number of candidate actuator DOFs is the same as n_t or further reduction is infeasible due to a significant change in the condition number of the $[\Phi_f]$ matrix. This reduced set of candidate actuator DOFs is then used to perform the further reduction to the number of desired actuator DOFs using Eqs. (19), (21)-(24).

Similarly, when the number of the candidate sensor DOFs (n_c) is much larger than n_t, it will be costly computationally to set up the Hankel matrix and to perform a large number of iterations to reduce the candidate sensor locations. In order to enhance the computational efficiency without degrading sensor placement results significantly, a pre-selection procedure is proposed to reduce n_c to a smaller number. The approach presented in Ref. [4] to select the best sensor locations is used as the sensor pre-selection procedure in this paper. The minimum number of the sensor DOFs required is n_t. Therefore, it is proposed than n_c be reduced to the vicinity of $2n_t$ without deteriorating $\rho([C])$ using the EI method. The EI values are computed for the mode shape matrix $[\Phi_y]$ using

$$ey_i = \left([\Phi_y][\Phi_y]^+ \right)_{i,i} \tag{32}$$

where ey_i is the EI contribution of the i^{th} candidate sensor DOF. In a similar way as described previously, the DOFs contributing the least to the rank of the matrix $[\Phi_y]$ are removed iterartively until the remaining number of candidate sensor DOFs is at the vicinity of $2n_t$ or further reduction is infeasible due to a significant deterioration in the condition number of the $[\Phi_y]$ matrix. This reduced set of candidate sensor DOFs is then used to perform the further reduction to the minimum number of sensor DOFs using Eqs. (25), (27)-(30).

6. Numerical Example

The cantilevered, ten-bay truss structure shown in Fig. 1 is employed to demonstrate the proposed procedure. The truss contains 135 truss members and 120 finite element DOFs (3 DOFs per node). Each bay of the truss is a cube with the side dimension of 0.5 m. The modulus of elasticity and the cross-sectional area ofthe truss members are 6.89×10^{10} N/m^2 and 7.348×10^{-5}m^2, respectively. The mass density of the battens and longerons is 2.024×10^3 kg/m^3 and the mass density of the diagonals is 2.173×10^3 kg/m^3. Using a general purpose finite element program the natural frequencies and the mode shapes are calculated. The first nine modes below 200 Hz are considered as the target modes for modal testing. The natural frequencies and descriptions of the target modes are shown in Table 1. The target modes include the first, second and third bending modes, the first and second torsional modes, and the first axial mode. The bending modes are paired due to the symmetry of the truss.

Considering the symmetry of the truss, two nodes per bay are selected as candidate actuator/sensor locations. The resulting 20 candidate locations (60 in DOFs) are indicated in Fig. 1. In this example, the candidate actuator and sensor locations are selected identically. For a modal test of the truss, it is assumed that a single actuator is available and is capable of exciting up to three DOFs simultaneously at a given node location. The number of available accelerometers is approximately the number of the target modes and is required to be minimized.

Since the number of the target modes is nine, i.e., $n_t = 9$ and the number of the candidate actuator/sensor DOFs is 60, i.e., $n_b = n_c = 60$, the dimensions of the matrices $[A]$, $[B]$, and $[C]$ are 18×18, 18×60 and 60×18, respectively. Due to the identical candidate actuator/sensor locations, the matrices $[\Phi_f]$ and $[\Phi_y]$ are the same and their dimension is 60×9. Considering that the desired number of the actuator DOFs is

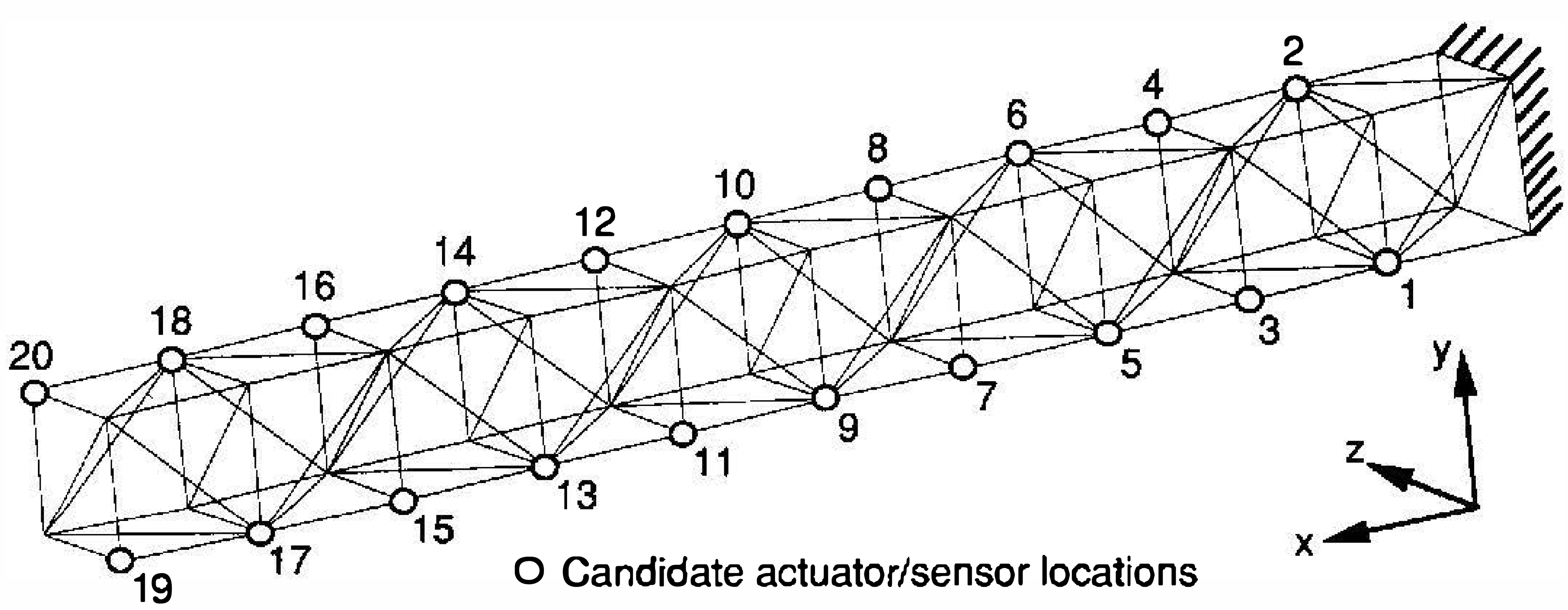

Fig. 1 A ten-bay truss structure

three, i.e., $n_a = 3$ and the rank of matrix $[A]$ is 18, the controllability matrix in Eq. (19) is raised to the 15^{th} power of matrix $[A]$ and its dimension becomes 18×960. A pre-selection procedure reduces n_b to the vicinity of n_t by applying the EI method to the matrix $[\Phi_f]$ using Eq. (31). Once the pre-selection procedure is performed, the column dimension of the controllability matrix is significantly reduced and will be about $16 \times 9 = 144$ compared to 960. Similarly, the observability matrix in Eq. (25) has a dimension of 600×18 since the matrix $[A]$ is raised to the 9^{th} power. The pre-selection procedure reduces n_c to the vicinity of $2n_t$ by applying the EI method to the matrix $[\Phi_y]$ using Eq. (32). Once the pre-selection procedure is performed, the row dimension of the Hankel matrix is significantly reduced and will be at the vicinity of $10 \times 18 = 180$ compared to 600. The actuator/sensor placement method proposed in this paper can be applied to select the best actuator/sensor DOFs without performing the pre-selection procedure. However, the number of iterations taken to reduce the size of the controllability and Hankel matrices would be larger and it will cost more computationally.

The results of the pre-selection procedure using the EI method and the matrix $[\Phi_f]$ (=$[\Phi_y]$) are shown in Fig. 2 and the condition number of the matrix $[\Phi_f]$ corresponding to each iteration is shown in Fig. 3. The tolerance value used for the removal of the candidate DOFs is 10^{-6}. The candidate y-DOFs and z-DOFs exhibit higher EI values than the x-DOFs since eight modes out of the nine target modes show bending and torsional motions and there is one axial mode. The EI evaluation stops at iteration 18 since the condition number of the matrix $[\Phi_f]$ for the remaining 12 DOFs shows an order of magnitude change. The loss of the two x-DOFs contributes to the result. Therefore, the 14 DOFs obtained at iteration 17 are used as a reduced set of candidate actuator DOFs. The eighteen DOFs at iteration 16 are used as a reduced set of candidate sensor DOFs since $2n_t = 18$.

Using the reduced set of candidate actuator/sensor DOFs, the best actuator/sensor DOFs are investigated. The controllability matrix $[W]$ corresponding to the reduced candidate actuator DOFs are assembled and the EI contribution is computed using Eqs. (21)-(23) with $n_t = 9$, $n_a = 3$, and $n_b = 14$. For the tolerance value (Δ_{ea}) in Eq. (24), 10^{-6} is used. Figure 4 shows the results of the EI evaluation. Unlike the results of the EI method using the mode shape matrix shown in Fig. 2, the x-DOFs at the tip of the truss are maintained as the essential DOFs to excite the target modes. Condition numbers during the iteration are also shown in Fig. 4 and their values are maintained approximately at the same level. The locations 19 and 20 exhibit the same effectiveness in exciting the target modes. Since it is assumed that there is one exciter, the three DOFs at the node 19 are selected as the best actuator DOFs.

Now the best actuator location is determined, the controllability matrix is fixed. The observability matrix $[V]$ corresponding to the reduced candidate sensor DOFs are assembled with $n_t = 9$ and $n_c = 18$. The EI contribution of the Hankel matrix in Eq. (27) is then computed using Eqs. (28) and (29). For the tolerance

TABLE 1 TARGET MODES OF THE TEN-BAY TRUSS

Mode number	Frequency (Hz)	Description
1	15.1	First bending mode
2	15.5	First bending mode
3	60.9	First torsional mode
4	76.9	Second bending mode
5	81.2	Second bending mode
6	154.2	First axial mode
7	176.1	Third bending mode
8	180.9	Second torsional mode
9	185.8	Third bending mode

value (Δ_{es}) in Eq. (30), 10^{-6} is used. Once the sensor DOFs to be removed are determined, the observability matrix corresponding to the remaining sensor DOFs are reassembled using Eq. (25) and the EI evaluation of the Hankel matrix continues until the locations for the desired number of sensors ($n_s = n_t = 9$) is obtained. Figure 5 shows the results of the EI evaluation. Iteration stops since a further reduction would result in the number of sensors smaller than n_s. The y and z sensor DOFs at the locations 5 and 6 and the x, y, and z DOFs at the locations 9 and 10 are selected as the best sensor DOFs. The singular values of the resulting Hankel matrix corresponding to ten best sensor DOFs, three best actuator DOFs and nine target modes are computed and shown in Fig. 6. The largest 18 singular values correspond to the nine target modes and are relatively equal in magnitude indicating that the target modes are well identified.

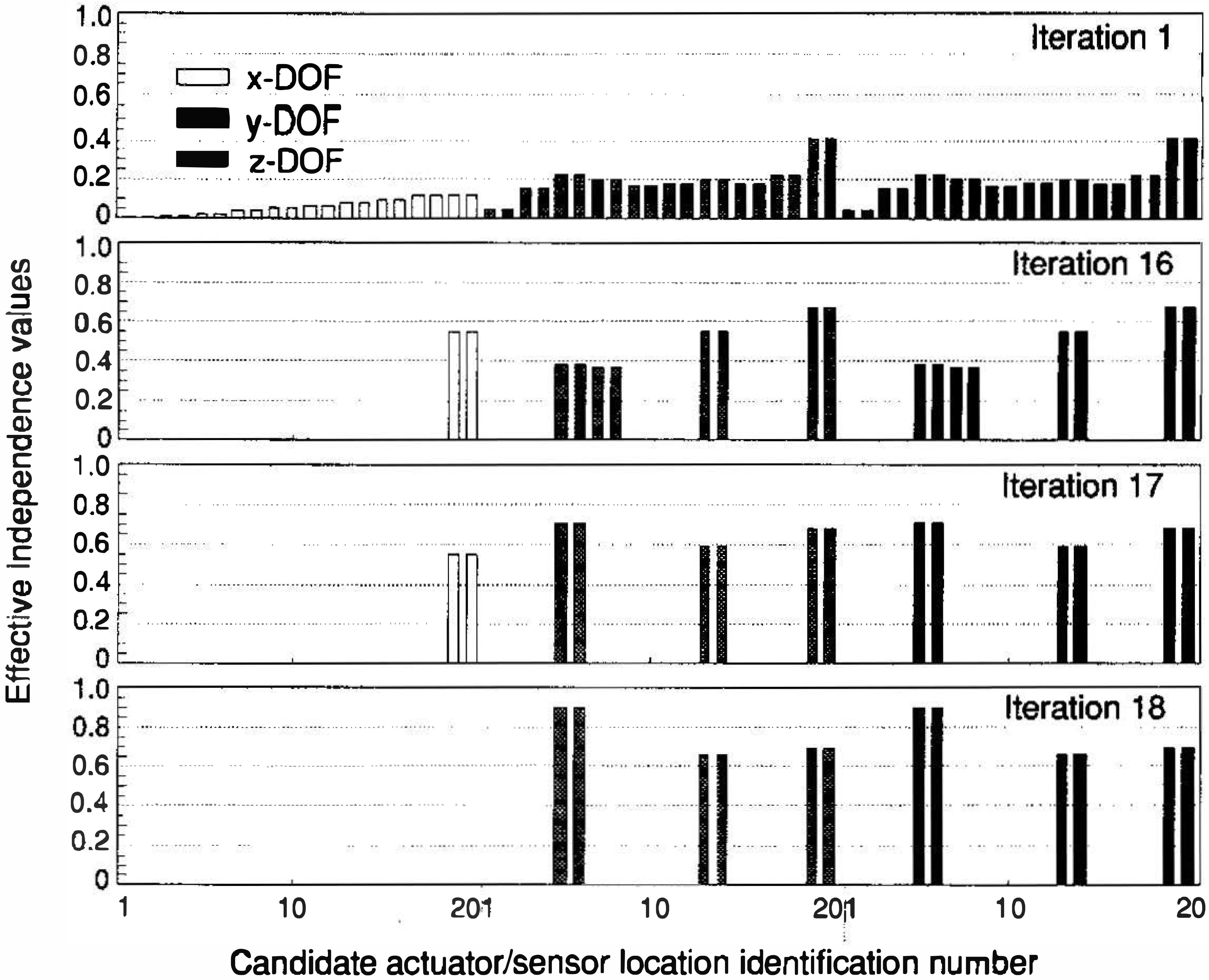

Fig. 2 Selection of reduced candidate actuator/sensor DOFs using EI method

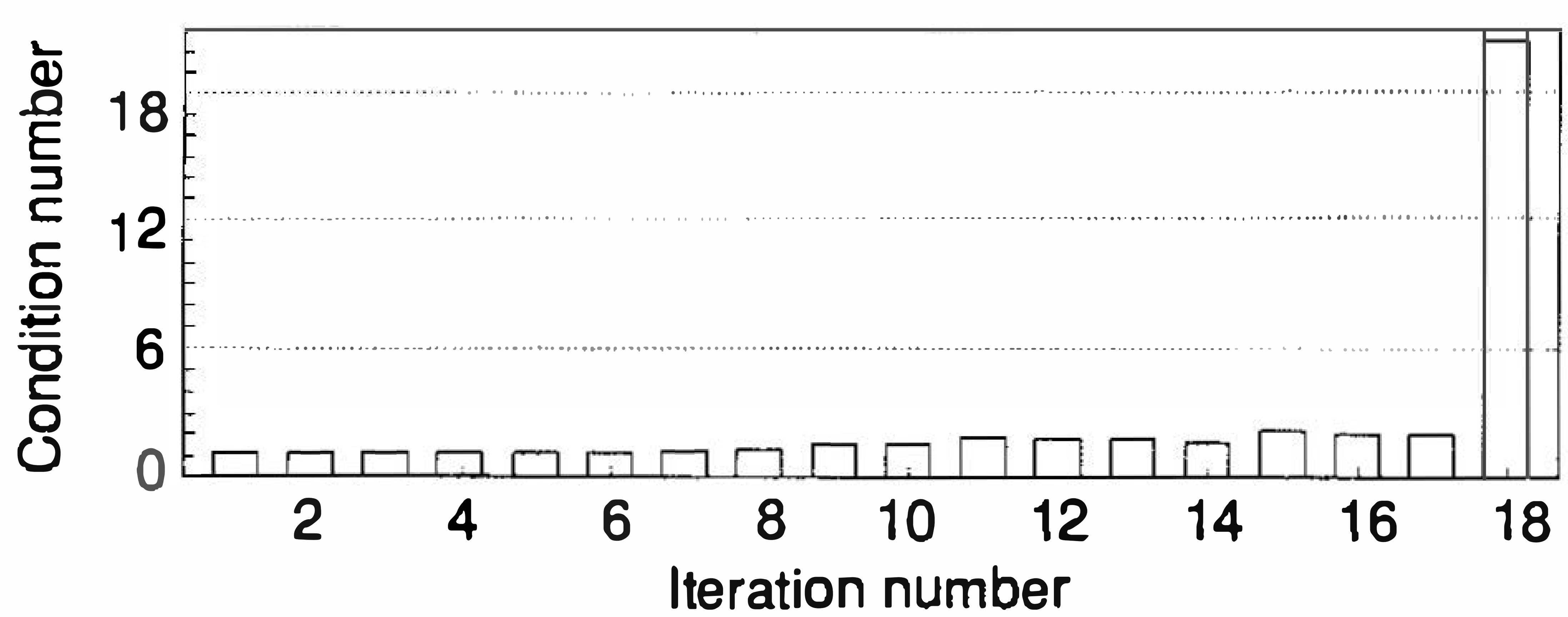

Fig. 3 Condition number of the mode shape matrix

7. Concluding Remarks

A systematic procedure of placing actuators and sensors for the modal parameter identification of flexible structures was presented. For a given set of target modes, this procedure selects the locations of actuators and sensors so that the rank of the Hankel matrix, which is a building block of a time-domain modal parameter identification algorithm such as Eigensystem Realization Algorithm, may always be twice the number of the target modes. Therefore, the recovery of the target modes is ensured. The use of the controllability matrix in the proposed method allows the reduction of the candidate actuator DOFs below the number of the target modes. Hence, it is possible to satisfy the constraint imposed on the number of actuator DOFs which is frequently smaller than the number of the target modes for a modal test. Unlike the Effective Independence (EI) sensor placement technique based on the mode shape matrix alone, the proposed method operates on the Hankel matrix which contains the Markov parameters. Therefore, the combined information on natural frequencies, mode shapes, and modal damping ratios are considered for the sensor placement.

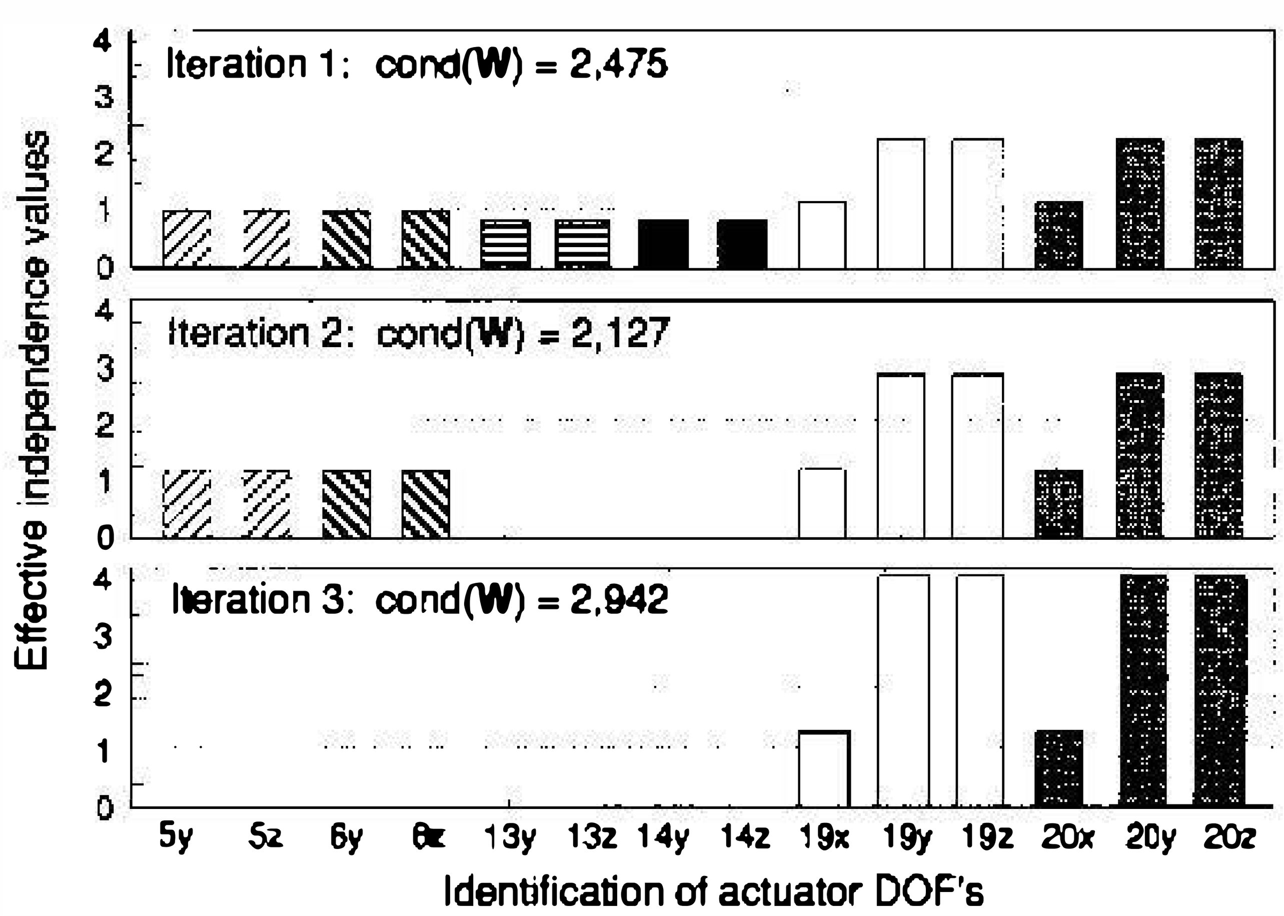

Fig. 4 Selection of the best actuator locations

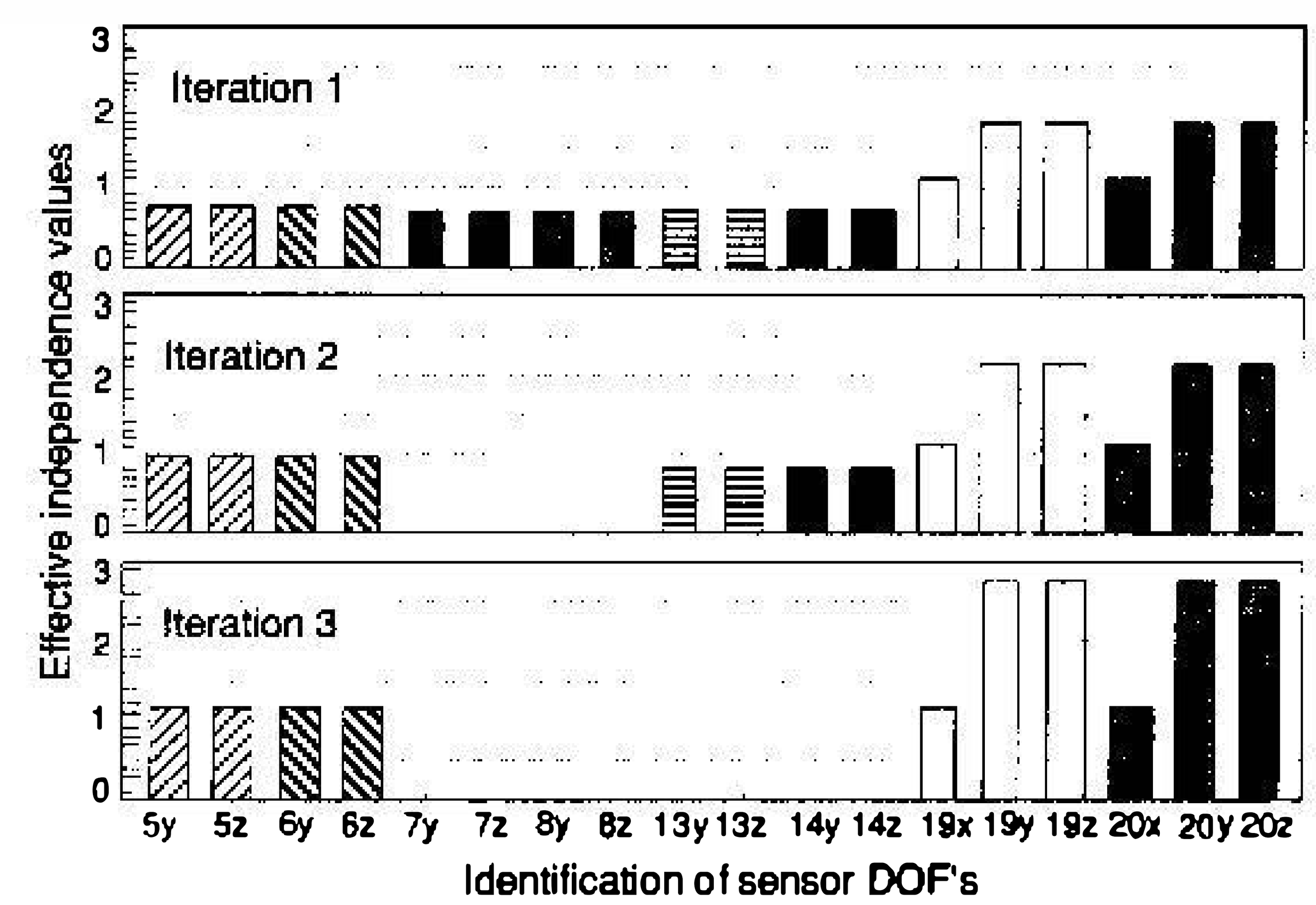

Fig. 5 Selection of the best sensor locations

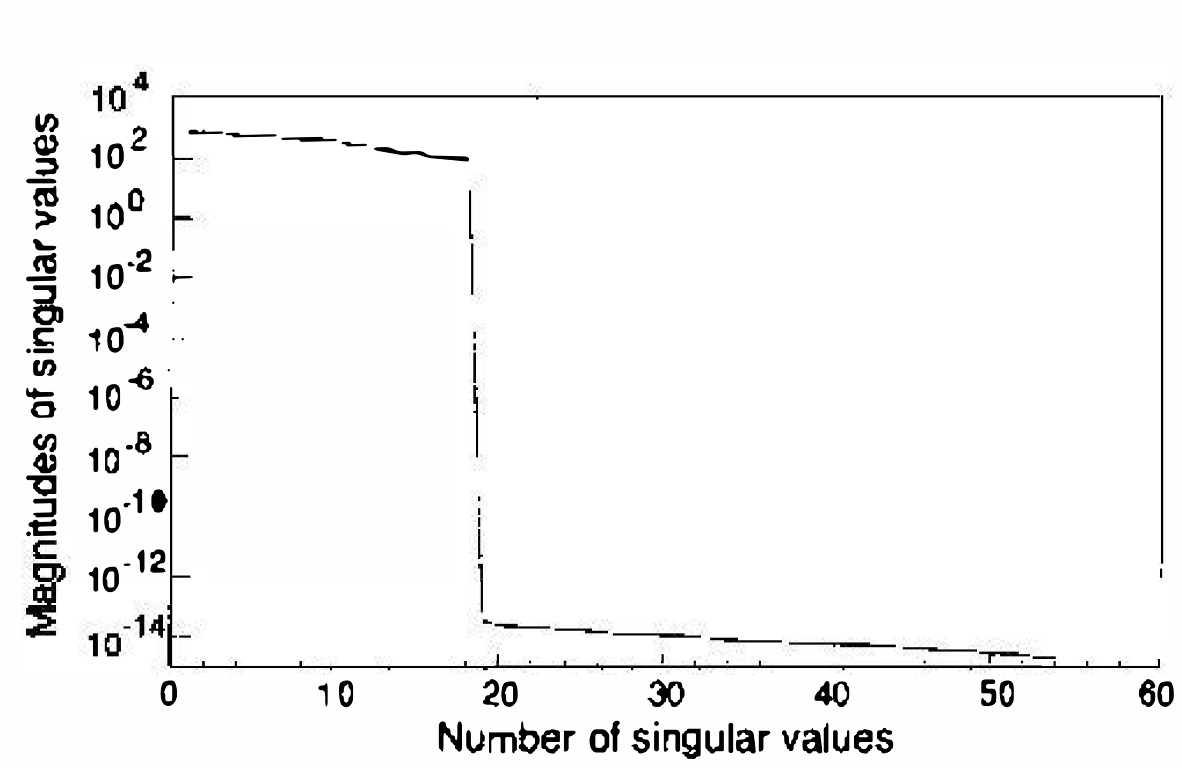

Fig. 6 Singular values of the Hankel matrix corresponding to the final actuator/sensor locations

The proposed actuator/sensor placement procedure employs the EI approach to reduce the candidate actuator/sensor locations. Using the EI method, the rank contribution of each candidate actuator location to the controllability matrix is evaluated for actuator placement and the rank contribution of each sensor location to the Hankel matrix is computed for sensor placement. Even though the EI calculation is not intensive computationally, the use of the pre-selection procedure for reducing the candidate actuator/sensor locations based on the mode shape matrix is recommended to enhance the computational efficiency.

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