Studies in Nonlinear Dynamics and Econometrics

Quarterly Journal
April 1996, Volume 1, Number 1
The MIT Press

Studies in Nonlinear Dynamics and Econometrics (ISSN 1081-1826) is a quarterly journal published electronically on the Internet by The MIT Press, Cambridge, Massachusetts, 02142. Subscriptions and address changes should be addressed to MIT Press Journals, 55 Hayward Street, Cambridge, MA 02142; (617)253-2889; e-mail: journals-orders@mit.edu. Subscription rates are: Individuals $40.00, Institutions $130.00. Canadians add additional 7% GST. Prices subject to change without notice.

Permission to photocopy articles for internal or personal use, or the internal or personal use of specific clients, is granted by the copyright owner for users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the per-copy fee of $10.00 per article is paid directly to the CCC, 222 Rosewood Drive, Danvers, MA 01923. The fee code for users of the Transactional Reporting Service is 0747-9360/96 $10.00. For those organizations that have been granted a photocopy license with CCC, a separate system of payment has been arranged. Address all other inquiries to the Subsidiary Rights Manager, MIT Press Journals, 55 Hayward Street, Cambridge, MA 02142; (617)253-2864; e-mail: journals-rights@mit.edu.

© 1996 by the Massachusetts Institute of Technology

On Cycles and Chaos in Economics

Jess Benhabib
Department of Economics
New York University
benhabib@fasecon.econ.nyu.edu

The possibility of cycles and chaos arising from nonlinear dynamics in economics emerged in the literature in the 1980s, and it came as a surprise.[1] The possibility of deterministic cycles in economic models had been noted before, for example in the well-known multiplier-accelerator models, but not in equilibrium models with complete markets, no frictions, and full intertemporal arbitrage.[2] The reason for the surprise was understandable: deterministic fluctuations in equilibrium models involve predictable changes in relative prices which should be ruled out by intertemporal arbitrage. In models of overlapping generations, however, finite lives can restrict complete arbitrage over time. As a result, some people thought, and still think, that cycles that are shorter than the agents' postulated life-spans would not be possible in equilibrium models, and therefore are irrelevant for business-cycle analysis. This view is clearly wrong, and of course ignores the extensive literature on cycles and chaos in optimal growth models with infinitely lived agents. In such models deterministic cycles in relative prices occur easily, but the amplitudes of the cycles remain within bounds of the discount rate.[3] It is not difficult to show in the context of multisector growth models, say with Cobb-Douglas production functions, that for any positive discount rate there is a large class of technologies for which cycles occur. (See Benhabib and Rustichini [1990].) Getting chaos, however, is harder. Recent works by Sorger (1992), by Mitra (1995), and by Nishimura and Yano (1995) give lower bounds for the discount rate, below which chaos is ruled out for one-sector models of optimal growth. Yet even in that context, going to a multisector framework may considerably lower the bounds on the discount rate thus far established.

A second reason for the attention that chaotic dynamics received in the economics literature regards prediction. The common wisdom has been that economic fluctuations are driven by exogenous shocks. Chaotic dynamics not only supplied an alternative explanation for at least some part of economic fluctuations, but also provided an excuse for economists' difficulties with forecasting. Sometimes, however, an important feature of chaotic dynamics that makes forecasting difficult, namely, sensitive dependence on initial conditions, is used in a cavalier way to explain short-run dynamics, forgetting that the effect of sensitive dependence becomes significant only after some periods, but not in the very short run.

When it became obvious that very-standard equilibrium models could easily generate cycles and chaos, the attention in the literature naturally turned to the empirical plausibility of such dynamics. The most interesting approach, inspired by developments in natural sciences and mathematics, was also atheoretical, and reminiscent of VAR methods of time series.[4] The idea was to try to infer whether a particular economic time series was generated by a deterministic, low (at most four- or five-) dimensional system that was chaotic, or whether it came from a simple (linear) stochastic system. It is not difficult to see that such inferences are hard to make when the time-series data is short, as is the case with most economic series, with the exception of financial data. It is not surprising, then, that many applications of this approach are in the area of finance, but even there, where we have very high-frequency data, it is hard to pick up fluctuations that may occur at lower

[1] Among the many authors contributing to this early literature were, Benhabib, Boldrin, Day, Deneckere, Grandmont, and Montrucchio. See the survey in this volume by Nishimura and Sorger (1996).

[2] Some earlier examples include the well-known cyclical equilibria in overlapping generations model by David Gale (1973), and some not very robust cyclical counterexamples to the turnpike theorems by Sutherland (1970).

[3] For example if the discount rate is 4%, relative prices can change by up to 12% over a three-year upswing; how quantities respond would depend on production possibilities and labor-supply elasticities.

[4] Among the many authors who contributed to this literature are Brock, Dechert, LeBaron, Scheinkman, and others. A good starting point is the book by Brock, Hsieh, and LeBaron (1991).

© 1996 by the Massachusetts Institute of Technology

business-cycle frequencies. Another serious limitation of the atheoretical approach is that its conclusions must be limited to determining whether or not the time series come from a low-dimensional (aggregative) chaotic system. When resources in the economy are close to fully utilized, much of the fluctuations and disruptions may occur at the micro level and between sectors. In this context the underlying system may not be "low" dimensional, and it may also be contaminated by noise. Not surprisingly, therefore, many of the results that emerge from this empirical approach are either inconclusive or simply reject low-dimensional chaos.

Another (and maybe more fruitful) approach would be to bring back economic theory for purposes of empirically evaluating the plausibility of chaos and cycles. Maybe less-aggregative models of economic dynamics could be estimated or appropriately calibrated using microeconomic data. We could then directly check whether the estimated or calibrated parameters generate significant nonlinearities and fall within the theoretical range that would generate cycles and chaos. Of course, the theoretical characterization of high-dimensional systems generating chaotic dynamics is still not fully developed, and is difficult to apply. In the meantime, we may complement theoretical methods by simulating sensibly parametrized models to obtain some insight into their dynamics.

References

Benhabib, Jess and Aldo Rustichini (1990), "Equilibrium Cycling with Small Discounting," *Journal of Economic Theory* 5, 423–432.

Brock, William A., David Hsieh, and Blake LeBaron (1991), *Nonlinear Dynamics, Chaos and Instability.* Cambridge, Massachusetts: MIT Press.

Gale, David (1973), "Pure Exchange Equilibrium of Dynamic Economics Models," *Journal of Economic Theory* 6, 12–36.

Mitra, T. (1996), "An Exact Discount Factor Restriction for Period Three Cycles in Dynamic Optimization Models," *Journal of Economic Theory* 69, 281–305.

Nishimura, Kazuo and Gerhard Sorger (1996), "Optimal Cycles and Chaos: A Survey," *Studies in Nonlinear Dynamics and Econometrics* 1, 11–28.

Nishimura, Kazuo and M. Yano (1996), "On the Least Upper Bound of Discount Factors that are Compatible with Optimal Period-Three Cycles, *Journal of Economic Theory* 69, 306–333.

Sorger, Gerhard (1992), "On the Minimum Rate of Impatience for Complicated Optimal Growth Paths," *Journal of Economic Theory* 56, 160–179.

Sutherland, William A. (1970), "On Optimal Development in a Multi-Sector Economy: The Discounted Case," *Review of Economic Studies* 37, 585–589.

Advisory Panel

Jess Benhabib, New York University

William A. Brock, University of Wisconsin-Madison

Jean-Michel Grandmont, CEPREMAP-France

Jose Scheinkman, University of Chicago

Halbert White, University of California-San Diego

Editorial Board

Bruce Mizrach (editor), Rutgers University

Michele Boldrin, University of Carlos III

Tim Bollerslev, University of Virginia

Carl Chiarella, University of Technology-Sydney

W. Davis Dechert, University of Houston

Paul De Grauwe, KU Leuven

David A. Hsieh, Duke University

Kenneth F. Kroner, BZW Barclays Global Investors

Blake LeBaron, University of Wisconsin-Madison

Stefan Mittnik, University of Kiel

Luigi Montrucchio, University of Turin

Kazuo Nishimura, Kyoto University

James Ramsey, New York University

Pietro Reichlin, Rome University

Timo Terasvirta, Stockholm School of Economics

Ruey Tsay, University of Chicago

Stanley E. Zin, Carnegie-Mellon University

Editorial Policy

The *SNDE* is formed in recognition that advances in statistics and dynamical systems theory may increase our understanding of economic and financial markets. The journal will seek both theoretical and applied papers that characterize and motivate nonlinear phenomena. Researchers will be encouraged to assist replication of empirical results by providing copies of data and programs online. Algorithms and rapid communications will also be published.

ISSN 1081-1826