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FORTRAN Programs for Running the TR Test: A Guide and Examples

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Abstract. *This paper provides a guide to two FORTRAN programs, called TRT1.FOR and TRT2.FOR, written to run the two stages of the Ramsey and Rothman (1996) time-reversibility (TR) test. Original FORTRAN source code and executable (on an IBM-compatible PC) versions of the programs are available. The major step in successfully running the programs is setting up two ASCII control files, called TRT1.CNL and TRT2.CNL. Two examples are provided.*

1 Introduction

Ramsey and Rothman (1996) introduced and applied a time-domain test of time reversibility, the TR test, to a wide set of long-run macroeconomic data. They argued that the framework of time reversibility is a useful context in which to address the problem of business-cycle symmetry, a topic that has received increasing attention in the empirical macroeconomics literature since the publication of Neftci's (1984) seminal article.

The purpose of this paper is to provide a guide to a pair of FORTRAN programs that were written by the author in order to implement the TR test. These programs enable the user to carry out the two stages of the TR test as presented in Ramsey and Rothman (1996), i.e., calculating the test statistics on the raw unfiltered data and on ARMA residuals. Both programs also compute the portmanteau variant of the TR test used by Rothman (1994).

The paper proceeds as follows. The TR test is briefly discussed in Section 2, and instructions for running the two FORTRAN programs, along with two examples, are given in Section 3. Section 4 concludes.

2 The TR Test

A formal statistical definition of time reversibility is as follows:

Definition 1. *A time series $\{X_t\}$ is time reversible if for every positive integer n, every $t_1, t_2, \ldots, t_n \in \mathbb{R}$, and all $m \in \mathbb{N}$, the vectors $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ and $(X_{-t_1+m}, X_{-t_2+m}, \ldots, X_{-t_n+m})$ have the same joint probability distributions.*

Under Definition 1, it can be shown that time reversibility implies stationarity. It is easily seen that: (1) independently and identically distributed (iid) processes are time reversible; and (2) stationary Gaussian processes are time reversible.

The TR test is based on the equality between certain pairs of moments from the joint probability distributions for a time-reversible time series $\{X_t\}$. Theorem 1 from Ramsey and Rothman (1996) establishes that $\{X_t\}$ is time reversible only if

$$E[X_t^i \cdot X_{t-k}^j] = E[X_t^j \cdot X_{t-k}^i] \tag{2.1}$$

for $i, j, k \in \mathbb{N}$, where the expectation is taken with respect to each respective joint distribution, and it is assumed that the mean of $\{X_t\}$ is zero.

For $i = j = 1$, Equation (1) reduces to the tautology that the autocovariance at lag k is equal to itself. When at least one of i, j is greater than one, $i, j \in \mathbb{N}$, the two terms in Equation (1) are called generalized autocovariances, following Welsh and Jernigan (1983). From Equation (1) it follows that if there exists a lag k for which these two moments are not equal, then $\{X_t\}$ is time irreversible.

Ramsey and Rothman (1996) focus on the $i = 2$, $j = 1$ case, i.e., they examine the difference between two bicovariances and define the symmetric bicovariance functions as

$$\gamma_{2,1}(k) = E[X_t^2 \cdot X_{t-k}] - E[X_t \cdot X_{t-k}^2] \tag{2.2}$$

for all integer values of k. If $\{X_t\}$ is time reversible, then $\gamma_{2,1}(k) = 0 \ \forall k \in \mathbb{N}$.

Within the context of stationary processes, time irreversibility can stem from two sources: (1) the underlying model may be nonlinear even though the innovations are symmetrically (perhaps normally) distributed; or (2) the underlying innovations may be drawn from a non-Gaussian probability distribution while the model is linear. Ramsey and Rothman (1996) refer to the first case as "Type I" time irreversibility and to the latter as "Type II" time irreversibility. Type II time irreversibility is consistent with a non-Gaussian linear process.

The TR test statistics consist of a simple method of moments estimate of the symmetric bicovariance function given by Equation (2), i.e., they are constructed as follows:

$$\hat{\gamma}_{2,1}(k) = \hat{B}_{2,1}(k) - \hat{B}_{1,2}(k) \tag{2.3}$$

for various integer values of k, where:

$$\begin{gathered} \hat{B}_{2,1}(k) = (T-k)^{-1} \cdot \sum_{t=k+1}^{T} X_t^2 \cdot X_{t-k} \\ \text{and} \\ \hat{B}_{1,2}(k) = (T-k)^{-1} \cdot \sum_{t=k+1}^{T} X_t \cdot X_{t-k}^2, \end{gathered} \tag{2.4}$$

where T is the sample size of a realization of a mean-zero stationary time series $\{X_t\}$.

$\hat{\gamma}_{2,1}(k)$ is an unbiased estimator of $\gamma_{2,1}(k)$. Under some additional restrictions, $\hat{\gamma}_{2,1}(k)$ is also consistent and has an asymptotic normal distribution. The exact small-sample expression for $Var(\hat{\gamma}_{2,1}(k))$ for $\{X_t\}$ iid follows from some straightforward calculations.

If a given series exhibits no serial correlation, $\hat{\gamma}_{2,1}(k)$ can be calculated on the raw data for a range of values of k. Since $\hat{\gamma}_{2,1}(k)$ is asymptotically normally distributed, rejection regions can be calculated using $Var(\hat{\gamma}_{2,1}(k))$ for the iid case.

The following procedure is appropriate in using the TR test for the case in which the given series exhibits serial correlation. In the first stage, $\hat{\gamma}_{2,1}(k)$ is calculated on the unfiltered but presumed stationary series.[1] An estimate of the variance of $\hat{\gamma}_{2,1}(k)$ is obtained by fitting an ARMA model to the data, and then simulating a series using the estimated ARMA coefficient values and generating a sequence of standard normal innovations. $\hat{\gamma}_{2,1}(k)$ is calculated for each such replication for N iterations. The estimated variance of $\hat{\gamma}_{2,1}(k)$ is then obtained from the replicated values of $\hat{\gamma}_{2,1}(k)$ in the usual manner. The FORTRAN program TRT1.FOR computes this first stage of the TR test. Rejections of time reversibility obtained with TRT1.FOR are consistent with both Type I and Type II time irreversibility.

Type I and Type II time irreversibility can be distinguished in the second stage of the TR test. If the process is Type II, i.e., the model is ARMA with non-Gaussian innovations, then the innovations themselves, which would be obtained by detransforming the series, are iid and therefore time reversible. So, by estimating the ARMA model and obtaining the residuals from it, the residuals by definition will be approximately time reversible (the degree of approximation being due to the low level of correlation in the residuals). Alternatively, if the process is Type I, then subject to the effects of the approximation mitigating the power of the test, the TR test applied to the residuals will reject the null hypothesis with probability greater than the size of the test. The second stage of the TR test can be carried out with the FORTRAN program TRT2.FOR.

[1]Before carrying out any calculations, the given series $\{X_t\}$ is standardized by subtracting off the sample mean and dividing through by the estimated standard deviation.

In both stages of the TR test, a sequence of $\gamma_{2,1}(k)$ values are computed across different lag lengths k. Recent work, however, has shown that the focus on individual time-horizon statistics, by ignoring the possible interdependences among the statistics at different horizons, can be misleading. To account for such potential interdependence, Ramsey and Rothman (1996) argued that it is useful to estimate the p-value of the largest (in absolute value) standardized TR test statistics. The FORTRAN programs TRT1.FOR and TRT2.FOR both make this calculation for the set of individual TR test statistics.

Rothman (1990, 1994) also considered a portmanteau version of the TR test. For a sequence of individual TR test statistics, define the portmanteau statistic as:

$$P(k_m) = \sum_{k=1}^{k=k_m} [\hat{\gamma}_{2,1}(k)/[Var(\hat{\gamma}_{2,1}(k)]^{1/2}]^2. \tag{2.5}$$

That is, the TR test portmanteau statistic $P(k_m)$ is computed by summing the squared standardized individual TR test statistics. The FORTRAN programs TRT1.FOR and TRT2.FOR both calculate $P(k_m)$ and estimate its p-value for a given range of individual TR test statistics.

Note that two Monte Carlo simulations are run in the TRT1.FOR program: the first to estimate the standard deviations of the TR test statistics, and the second to estimate the p-value of the maximum (in absolute value) TR test statistic across all lag values and the p-value of the portmanteau statistic (described below). In TRT2.FOR, however, only one Monte Carlo simulation is run: to estimate the p-values for the joint tests.

3 The FORTRAN Programs: TRT1.FOR and TRT2.FOR

The FORTRAN programs TRT1.FOR and TRT2.FOR run the first and second stages, respectively, of the Ramsey and Rothman (1996) TR test. Upon execution, the programs first read the contents of two ASCII control files, called TRT1.CNL and TRT2.CNL, to obtain information about file names, program parameters, etc. Thus, to run these programs, it is necessary to correctly set up these control files.

3.1 TRT1.CNL

TRT1.CNL is the control file for program TRT1.FOR. It should have 10 lines, specified as follows.

Line 1: An integer-valued variable indicating the number of iterations for Monte Carlo simulations.

Line 2: An integer-valued variable indicating the order p of the ARMA(p, q) model used to estimate the standard deviation of the series.

Line 3: An integer-valued variable indicating the order q of the ARMA(p, q) model used to estimate the standard deviation of the series.

Line 4: An integer-valued variable, called “ITRAN” by the program, used to request that a certain transformation be carried out on the data. ITRAN is set as follows:

$$\begin{aligned}
\text{ITRAN} &= 0,\ \text{use raw data;}\\
\text{ITRAN} &= 1,\ \text{use logarithm of data;}\\
\text{ITRAN} &= 2,\ \text{use log-first differences of data;}\\
\text{ITRAN} &= 3,\ \text{use raw (no log) first differences of data;}\\
\text{ITRAN} &= 4,\ \text{use log-linear detrended data; and}\\
\text{ITRAN} &= 5,\ \text{use raw linear detrended data.}
\end{aligned}$$

Line 5: An integer-valued variable indicating the number of observations in the series. Note: the executable version of the program is set to handle a maximum of 1,500 observations.

Line 6: An integer-valued variable indicating the maximum lag at which to calculate TR test statistics. Note: the executable version of the program is set to handle a maximum lag of 50.

Line 7: An integer seed for the random number generator in the range (0, 2147483646).

Table 1
Sample TRT1.CNL control file.

```
100
1
0
3
80
5
25443332
nomgnp.asc
Log N&P Nominal GNP, 1909–1988
t1nomgnp.out
```

Line 8: An alpha-numeric character variable, length up to 12 characters, giving the name of the file containing data.

Line 9: An alpha-numeric character variable, with length up to 50 characters, giving a description of the data.

Line 10: An alpha-numeric character variable, with length up to 12 characters, giving the name of the output file to which the program's results will be written.

Consider the sample TRT1.CNL file presented in Table 1. It sets up the TRT1.FOR program to run on the growth rates of the extended Nelson and Plosser nominal-GNP series analyzed by Ramsey and Rothman (1996). An AR(1) model is fitted to the series to estimate, in a Monte Carlo simulation with 100 iterations, the standard deviation of the $\hat{\gamma}_{2,1}(k)$ values. Note that the "raw data" in this example, contained in the file named "nomgnp.dat," is in logarithms; by setting ITRAN = 3, the program tests the log-first differenced series.

The output file produced by this run is presented in Table 2. While the fifth line of TRT1.CNL indicates that this series has 80 observations, in the T1NOMGNP.OUT output file, the number of observations in series is listed as 79, reflecting the loss of one observation by taking (log) first differences. A comparison of the standardized $\hat{\gamma}_{2,1}(k)$ values reported in Table 2 with those from the second row of Table 1 of Ramsey and Rothman (1996) show some slight differences, primarily reflecting the use of different random number generators in the Monte Carlo simulations.[2]

3.2 TRT2.CNL

TRT2.CNL is the control file for the TRT2.FOR program. Its specification is quite similar to that for TRT1.CNL, except that: (1) line 1 indicates the number of iterations for the single Monte Carlo run done by the program (recall that two Monte Carlo simulations are carried out in TRT1.FOR); and (2) lines 2 and 3 reflect that TRT2.FOR is run on either ARMA residuals or the presumed white-noise raw data. That is:

Line 1: An integer-valued variable indicating the number of iterations for the Monte Carlo simulation;

Line 2: An integer-valued variable indicating the order p of the ARMA(p, q) model used to get residuals. Note: set to 0 if raw series appears to be white noise; and

Line 3: An integer-valued variable indicating the order q of the ARMA(p, q) model used to get residuals. Note: set to 0 if raw series appears to be white noise.

Lines 4–10 for TRT2.CNL are specified exactly as for TRT1.CNL. Consider the sample TRT2.CNL file in Table 3. It sets up the TRT2.FOR program to calculate the TR test on the AR(5) residuals for the extended Nelson and Plosser GNP-deflator inflation rate. The output file produced by this run is given in Table 4. The strong evidence in favor of Type I time irreversibility for this series matches well the results reported in the seventh line of Table 2 in Ramsey and Rothman (1996).

[2]Both the TRT1.FOR and TRT2.FOR programs use IMSL random number generators. The simulations run for Ramsey and Rothman (1996) used Numerical Recipes random number generators from Press et al. (1986).

Table 2
Output file produced by a sample TRT1.CNL file, T1NOMGNP.OUT.

```
TR Test Program: Tested Data Not Passed Through ARMA Filter

-Written by Philip Rothman, East Carolina University

-Date Program Run (month,date,year): 05/19/96

-Data used from file: nomgnp.asc

-Log N&P Nominal GNP, 1909–1988

-Raw 1st differences in file used

-ARMA(p,q) Model Fitted to Series to Estimate Standard Deviation of TR Test Statistics. Order of ARMA(p,q) Model: p = 1, q = 0

-Number of observations in series:   79

-Number of iterations in MC simulations:    100

-Initial integer seed [in range (0,2147483646)]:    25443332

-Standardized TR Statistics

   k = 1, TR(k)/SDTR(k) =  −.110

   k = 2, TR(k)/SDTR(k) = −3.068

   k = 3, TR(k)/SDTR(k) =  1.197

   k = 4, TR(k)/SDTR(k) =  −.591

   k = 5, TR(k)/SDTR(k) =  −.113

-Joint Test Results

   Absolute Value of Maximum Standardized TR Statistic:        3.068
   P-Value of Maximum Standardized TR Statistic:                .020

   Portmanteau statistic calculated across lags 1 to:          5
   Value of portmanteau statistic:                            11.222
   P-Value of portmanteau statistic:                            .120

-Note: Rejections with this program are consistent with both Type I and Type II Time Irreversibility as defined in
Ramsey & Rothman (1996).
```

Table 3
A sample TRT2.CNL file.

```
100
1
0
3
99
5
25443332
gnpdefl.asc
Log of N&P GNP Price Deflator, 1889–1988
t2gnpdef.out
```

4 Conclusions

Both FORTRAN source code and executable (to run on IBM-compatible personal computers) versions of the programs are available. To run the programs in a non-IBM PC environment, it is clearly necessary to recompile and relink the original source code. However, it is important to note that these programs call some IMSL routines. Accordingly, it is necessary to either have access to IMSL or substitute the IMSL routines with alternatives to run the programs on a non-IBM PC machine. This is also true if the user wishes to make any

Table 4
Output file produced by a sample TRT2.CNL file, T2GNPDEF.OUT.

```
TR Test Program: Test Run on ARMA Residuals

-Written by Philip Rothman, East Carolina University

-Date Program Run (month,date,year): 05/17/96

-Data used from file: gnpdefl.asc

-Log of N&P GNP Price Deflator, 1889–1988

-Raw 1st differences in file used

-Order of ARMA(p,q) Model Fitted to Series: p= 1, q = 0

-Number of observations in series:    99

-Number of iterations in MC simulations:    100

-Initial integer seed [in range (0,2147483646)]:    25443332

-Standardized TR Statistics

   k = 1, TR(k)/SDTR(k) =   2.150

   k = 2, TR(k)/SDTR(k) = −1.163

   k = 3, TR(k)/SDTR(k) =    .918

   k = 4, TR(k)/SDTR(k) =   3.410

   k = 5, TR(k)/SDTR(k) =   2.118

-Joint Test Results

   Absolute Value of Maximum Standardized TR Statistic:       3.410
   P-Value of Maximum Standardized TR Statistic:               .000

   Portmanteau statistic calculated across lags 1 to:         5
   Value of portmanteau statistic:                           22.930
   P-Value of portmanteau statistic:                           .000
```

Note: Rejections with this program are consistent with Type I Time Irreversibility as defined in Ramsey & Rothman (1996).

changes to the source code (such as, e.g., increasing the maximum series length that the programs can analyze), even if the programs will be run on an IBM-compatible PC. The author stands ready, however, to help the user implement any changes in the original source code.

References

Neftci, Salih (1984). “Are economic time series asymmetric over the business cycle?” *Journal of Political Economy*, 92: 307–328.

Press, William H., B. P. Flannery, Saul A. Teukolsky, and William T. Vetterling (1986). *Numerical Recipes: The Art of Scientific Computing*. Cambridge: Cambridge University Press.

Ramsey, James B., and Philip Rothman (1996). “Time irreversibility and business cycle asymmetry.” *Journal of Money, Credit, and Banking*, 28: 1–21.

Rothman, Philip (1990). “Characterization of the time irreversibility of economic time series.” Ph.D. Dissertation, Department of Economics, New York University.

Rothman, Philip (1994). “The time reversibility test with application to financial data.” In W. Semmler, (ed.), *Business Cycles: Theory and Methods*. Boston: Kluwer Academic, pp. 389–403.

Welsh, Anne K., and Robert W. Jernigan (1983). “A statistic to identify asymmetric time series.” In *American Statistical Association 1983 Proceedings of the Business and Economics Statistics Section*. Alexandria, VA: American Statistical Association, pp. 390–395.

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