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Nonlinear Dynamics and European GNP Data

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Abstract. *The aim of this paper is to assess whether the data-generation process of the GDP can be interpreted by means of a nonlinear model instead of a linear one. We model the first differences of logarithmic real GDP data with constant parameters for those European countries (France, Germany, Italy, U.K., Denmark, Sweden, and Norway) which have long-term time series. Since the linear autoregressive model is rejected, an alternative nonlinear model has been specified: it turns out that the annual European GDPs can adequately be described by means of a nonlinear model with constant parameters.*

Keywords. nonlinearity, data-generation process, European GNP, smooth transition autogressive models

Acknowledgments. We wish to thank an anonymous referee for very useful comments.

1 Introduction

A nonnegligible number of studies on U.S. GNP time series have been carried out from a nonlinear point of view [see e.g., Scheinkman and LeBaron (1989); Potter (1995); Terasvirta (1996)], while only a few analyses of business-cycle data for Europe have adopted the same perspective [Frank, Gencay, and Stengos (1988); Stevenson, Jones, and Manning (1992); Mizrach (1994)]. This is especially troublesome, since many scholars have claimed that U.S. data may be atypical [Blanchard and Summers (1986)]. In discussing the U.S. real per-capita GDP series, some authors argue that there has been a structural break after WWII, because a dramatic change in the volatility of the series has occurred since 1947. For instance, Harvey (1985) finds that there was a strong positive first-order autocorrelation function in the period before 1947, which was very different from that of the subsequent period. In particular, Zarnowitz (1992, p. 215) lists eight reasons why volatility has changed, from economic policy to a more stable aggregate demand composition, from the expansion of the service sector to government expenditure.

When discussing this issue, the econometric literature has developed two approaches: diffusion models with stochastic volatility (Hull and White 1987), and ARCH models (Engle 1992). Nelson (1990) has shown that the two models are not incompatible, since a GARCH process could be interpreted as an approximation of the discrete-time version of the diffusion model. Univariate GARCH models make the conditional variance a function of lagged endogenous and exogenous variables. Nelson (1991) pointed out several limitations of GARCH models. He emphasized that they impose symmetry on the time series and rule out any nonlinear behavior in the volatility, since they typically imply a linear AR equation. Actual time series usually show asymmetric behavior and accelerator-like effects, which can be dealt with using nonlinear models: in this paper we look for them.

On a more technical ground, when analyzing U.S. GNP data, Scheinkman and LeBaron (1989), De Long and Summers (1986), and Durlauf (1989) either divided the sample into two subperiods (pre- and post-WWII), or introduced a time varying residual. All of these models are linear by assumption. Unfortunately, statistical tests do not reject the hypothesis of constancy of the autoregressive parameters, which should follow the structural break. Since the supposed structural break is not supported by the tests, one may suspect that the reduction in volatility may be the effect of an underlying nonlinearity: therefore, we aim to analyze if the data-generation process and its change in volatility can be better interpreted by means of a nonlinear model with constant parameters rather than by a linear model with changing parameters.

We aim to model the first differences of logarithmic real GDP data with constant parameters for those European countries (France, Germany, Italy, U.K., Denmark, Sweden, and Norway) that have secular series. If the linear autoregressive model is rejected, an alternative nonlinear model must be specified. Since there is a volatility change for European countries, a nonlinear framework is needed to describe the asymmetric behavior and the effects of different shocks on the series.

2 Linear Models and Parameter Stability Testing

In this section we adopt the conjecture that the DGP of real GDP is linear and subject to parameter changes. Basically, after having identified the most satisfactory linear AR representation according to the AIC criteria, we will verify if the variance of residuals, found for the U.S., is present also in the European data. Then we will test if there are signs of parameter instability and look for nonlinear specifications.

We use first differences of real output in logs, y (Figures 1–7) or in levels (Figures 8–14), for France (NNP: 1901–1949; GNP: 1950–1989), Germany (NNP: 1850–1913, 1925–1938; West Germany: 1950–1969; GNP: 1970–1989), Italy (GNP: 1861–1989), U.K. (GNP: 1830–1989), Denmark (GNP: 1870–1914, 1921–1989), Sweden (GDP: 1861–1989), and Norway (GDP: 1865–1939, 1946–1989). Table 1 reproduces some descriptive statistics of the series. The data sources are Mitchell (1975) for periods up to 1969, and OECD 1992 for periods from 1969 onward.

The automatic truncation lag procedure yields the results shown in Table 2.

An AR representation adequately describes the DGP with the exception of Sweden, whose output growth rate seems to follow a random walk (a collection of papers on the post-WWII growth performance of several European countries may be found in Crafts and Toniolo (1996); for a longer run analysis see Maddison (1982); Fuà (1981); Bergstrom and Vredin (1995); Federico (1994); Crouset (1993); Persson (1993); Fischer (1997); Feinstein (1997).

We now analyze whether there is a drop in the volatility of output growth between pre- and post-WWII years, similar to what has been found for the U.S. GDP series (Blanchard and Summers 1986; Zarnowitz 1992). Table 3 shows an average decrease by 50% in volatility of standard deviations of y, but the Goldfeld and

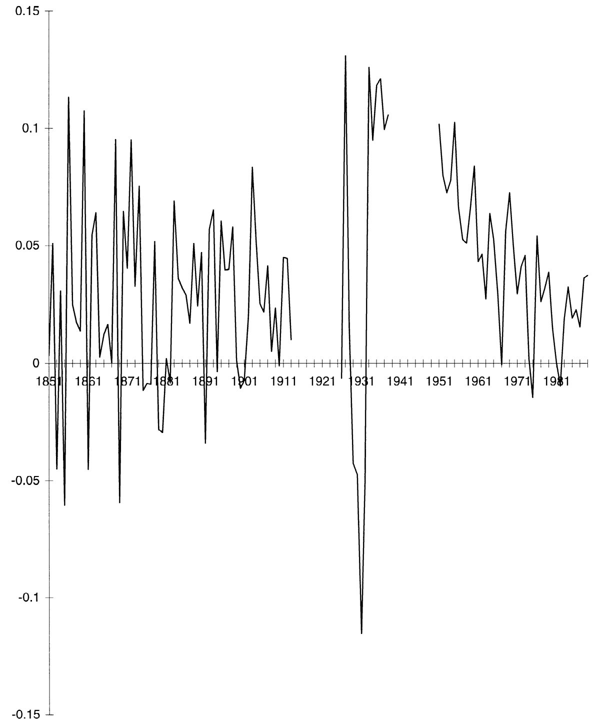

Figure 1
Differences of real output in logs: Germany.

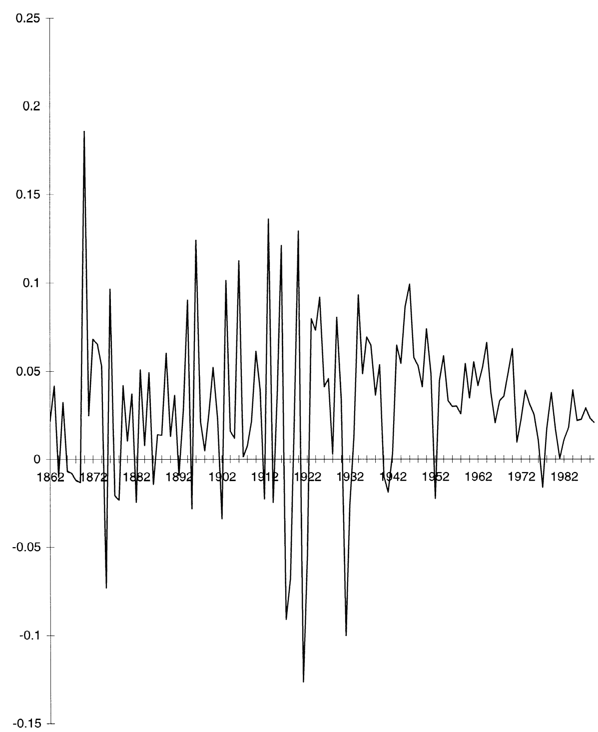

Figure 2
Differences of real output in logs: Sweden.

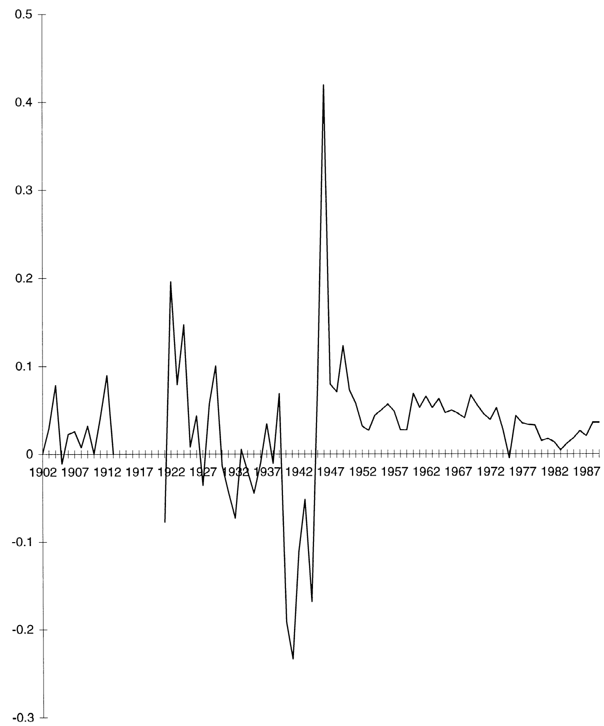

Figure 3
Differences of real output in logs: France.

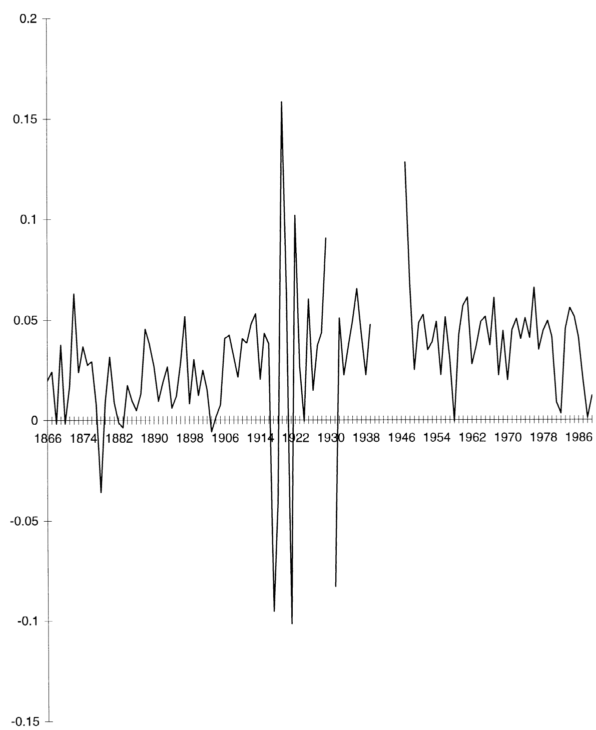

Figure 4
Differences of real output in logs: Norway.

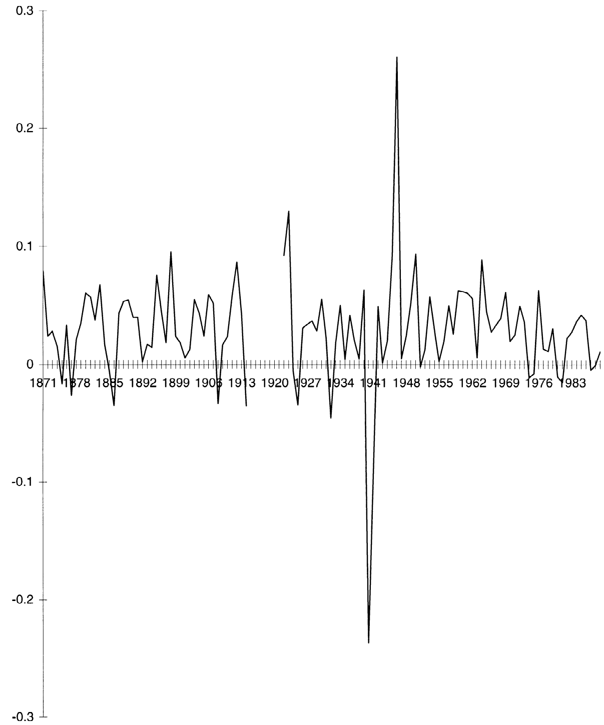

Figure 5
Differences of real output in logs: Denmark.

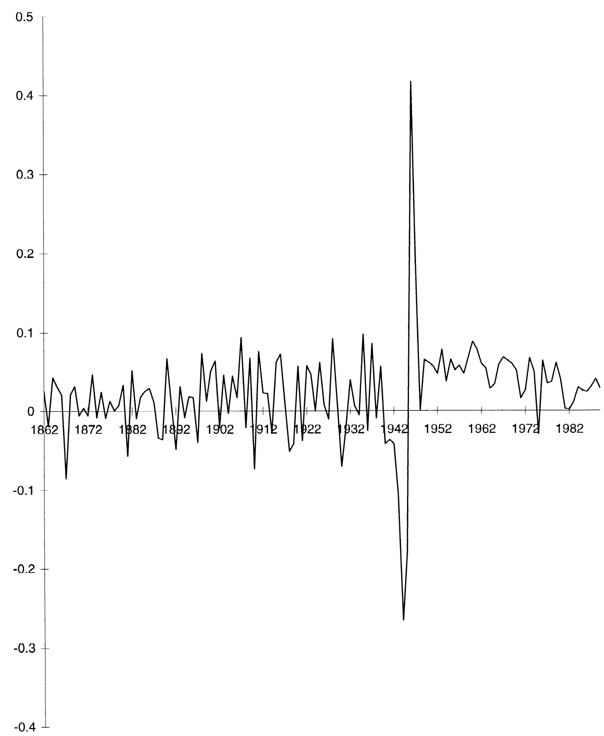

Figure 6
Differences of real output in logs: Italy.

Table 1
Rates of growth for different sample periods (data are given in percentages; parentheses show standard deviations).

Country (first year of observation)	Start–1939	1945–1989	Start–1989	Start–1914	1919–1939
U.K. (1831)	1.778 (3.726)	2.176 (2.459)	1.929 (3.504)	2.057 (3.321)	0.716 (4.956)
Italy (1862)	1.477 (4.075)	5.045 (7.268)	2.290 (6.346)	1.241 (3.743)	1.904 (4.712)
France (1902)	2.332 (5.953)	5.171 (6.080)	2.831 (7.713)	2.603 (3.124)	2.160 (7.281)
Germany (1851)	2.836 (4.963)	4.204 (2.858)	3.300 (4.398)	2.548 (3.866)	4.236 (8.610)
Denmark (1871)	3.043 (3.413)	3.623 (4.385)	2.902 (4.713)	3.046 (3.128)	3.034 (4.128)
Sweden (1862)	2.746 (5.498)	3.511 (2.345)	2.980 (4.569)	2.778 (4.786)	3.192 (6.373)
Norway (1866)	2.356 (3.718)	4.090 (2.183)	3.000 (3.332)	2.102 (1.839)	3.729 (5.633)

Table 2
Automatic truncation lag procedure.

U.K.	$\underset{(8.03)}{y_t = 0.01866} + \underset{(2.42)}{0.14838 y_{t-1}} + \underset{(5.22)}{0.12877\, d_{1839}} - \underset{(-7.07)}{0.18325\, d_{1840}} - \underset{(-6.31)}{0.11381\, d_{1919-20}}$ $+ \underset{(4.14)}{0.10153\, d_{1940}} - \underset{(-4.09)}{0.07181\, d_{1944-45}} - \underset{(-2.18)}{0.05326\, d_{1980}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.02446,\ R^2 = 0.6428,\ T = 158.$
Italy	$y_t = \underset{(2.88)}{0.01472} - \underset{(-3.02)}{0.19242 y_{t-1}} + \underset{(2.71)}{0.19078 y_{t-2}} + \underset{(1.97)}{0.12397 y_{t-3}} + \underset{(3.15)}{0.1858 y_{t-4}}$ $- \underset{(-5.20)}{0.20708\, d_{1943-45}} + \underset{(7.07)}{0.39044\, d_{1946-47}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03966,\ R^2 = 0.6789,\ T = 124.$
France	$y_t = \underset{(2.46)}{0.01173} + \underset{(3.77)}{0.21619 y_{t-1}} + \underset{(2.90)}{0.16688 y_{t-2}} + \underset{(2.10)}{0.11459 y_{t-4}} - \underset{(-9.47)}{0.2173\, d_{1940-41}}$ $- \underset{(-7.07)}{0.18325\, d_{1980}} - \underset{(-3.69)}{0.12828\, d_{1944}} + \underset{(3.88)}{0.14049\, d_{1945}} + \underset{(12.3)}{0.43155\, d_{1946}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03106,\ R^2 = 0.8642,\ T = 73.$
Germany	$y_t = \underset{(3.11)}{0.01677} + \underset{(2.18)}{0.18853 y_{t-1}} + \underset{(2.92)}{0.26064 y_{t-2}} - \underset{(-2.87)}{0.11215\, d_{1931}} + \underset{(3.66)}{0.14896\, d_{1933}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03784,\ R^2 = 0.5183,\ T = 109.$
Denmark	$y_t = \underset{(7.31)}{0.02746} + \underset{(2.77)}{0.17843 y_{t-1}} - \underset{(-2.65)}{0.16711 y_{t-2}} - \underset{(-9.00)}{0.27469\, d_{1940}} + \underset{(7.15)}{0.21958\, d_{1946}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03024,\ R^2 = 0.6948,\ T = 108.$
Sweden	$y_t = \underset{(8.03)}{0.03262} + \underset{(4.47)}{0.15297\, d_{1870}} - \underset{(-3.08)}{0.10555\, d_{1875}} + \underset{(2.67)}{0.09139\, d_{1896}}$ $+ \underset{(3.02)}{0.10341\, d_{1913}} - \underset{(-4.61)}{0.11205\, d_{1917-18}} - \underset{(-5.05)}{0.12289\, d_{1921-22}} - \underset{(-3.88)}{0.13289\, d_{1931}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.0341,\ R^2 = 0.4736,\ T = 128.$
Norway	$y_t = \underset{(14.32)}{0.03593} - \underset{(-3.94)}{0.13682 y_{t-1}} - \underset{(-6.59)}{0.10815\, d_{1917-18}} + \underset{(5.06)}{0.11678\, d_{1919}}$ $- \underset{(-5.62)}{0.12888\, d_{1921}} + \underset{(25.05)}{0.57552\, d_{1930}} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.0228,\ R^2 = 0.9013,\ T = 115.$

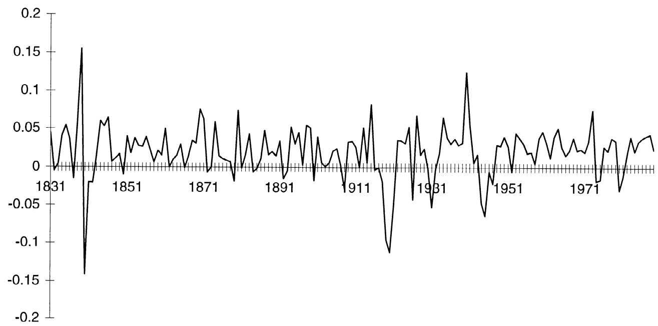

Figure 7
Differences of real output in logs: United Kingdom.

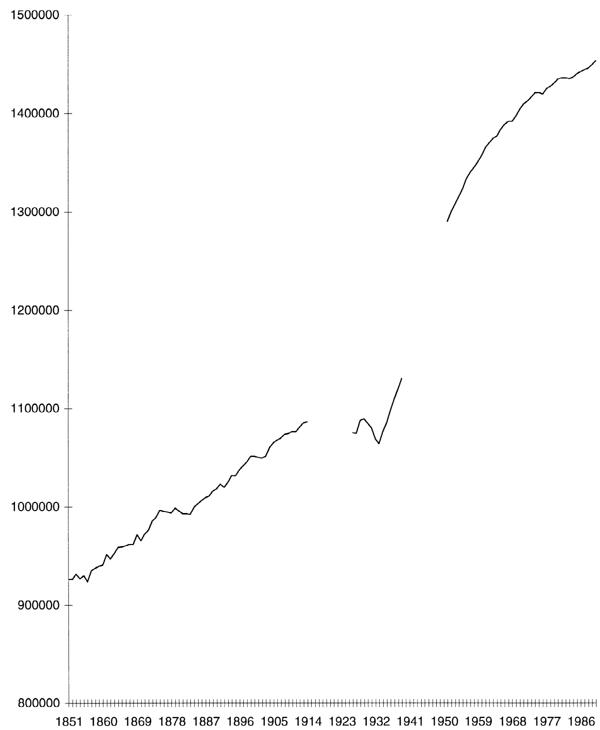

Figure 8
Real output in levels: Germany.

Quandt test (Table 4) shows that there are signs of heteroskedasticity for all the countries but Denmark. Parameter constancy is tested using the method of Lin and Terasvirta (1994): this test rejects the hypothesis of structural instability, except for Italy, as the intercept is concerned (which we interpret as owing to the late Italian economic take-off, dated around 1897), and Norway, as the slope changes over time (owing to a change in the national accounting procedure in 1930).

Since the hypothesis of parameter constancy cannot be rejected, the source of the volatility change could not be due to a structural break of the coefficients, but must lie elsewhere: we suspect that linear models are misspecified (all of the models reject the hypothesis of identically independent distributed residuals according to the BDS test [Brock, Dechert, and Sheinkman 1987]).

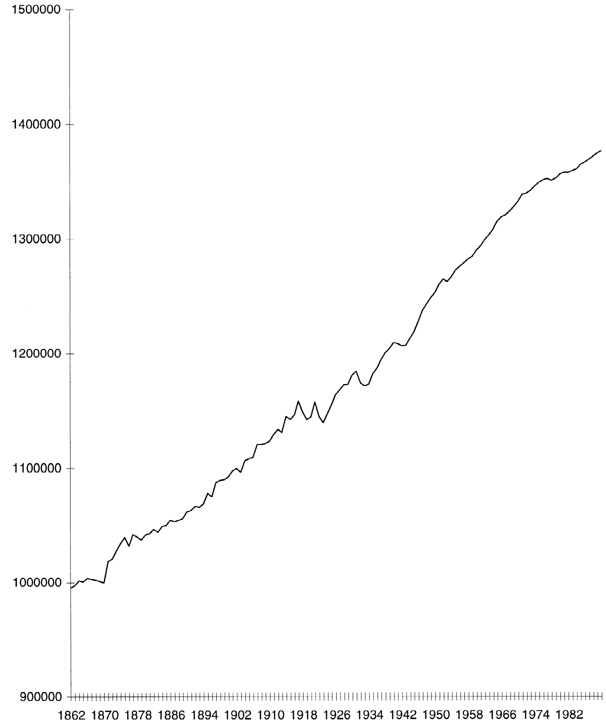

Figure 9
Real output in levels: Sweden.

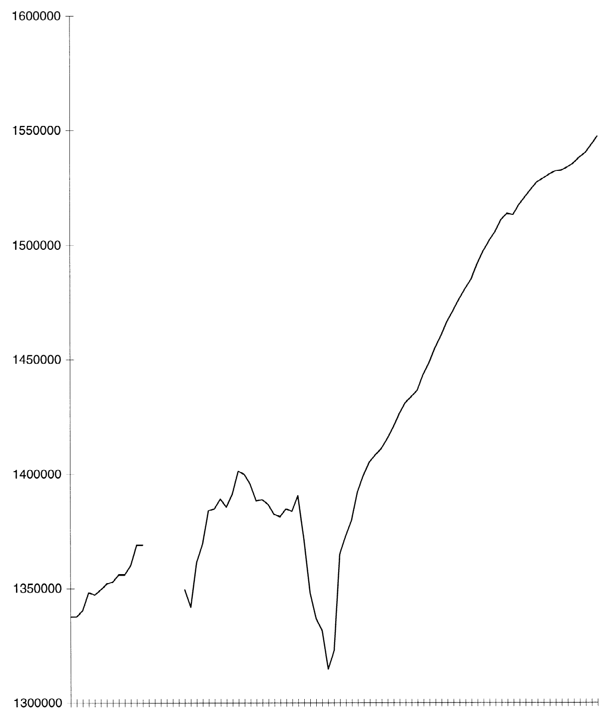

Figure 10
Real output in levels: France.

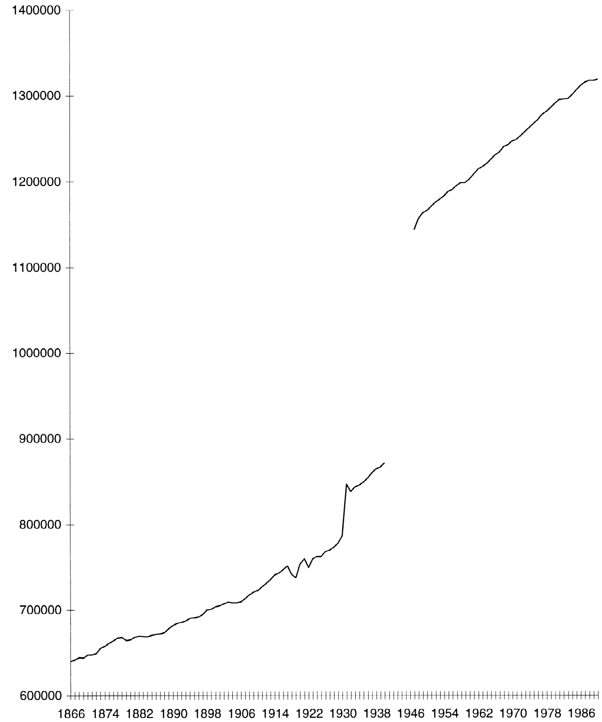

Figure 11
Real output in levels: Norway.

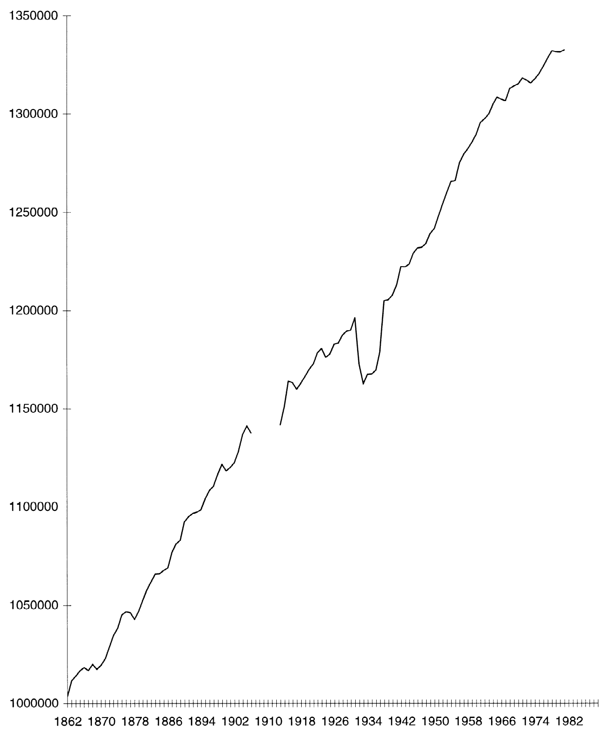

Figure 12
Real output in levels: Denmark.

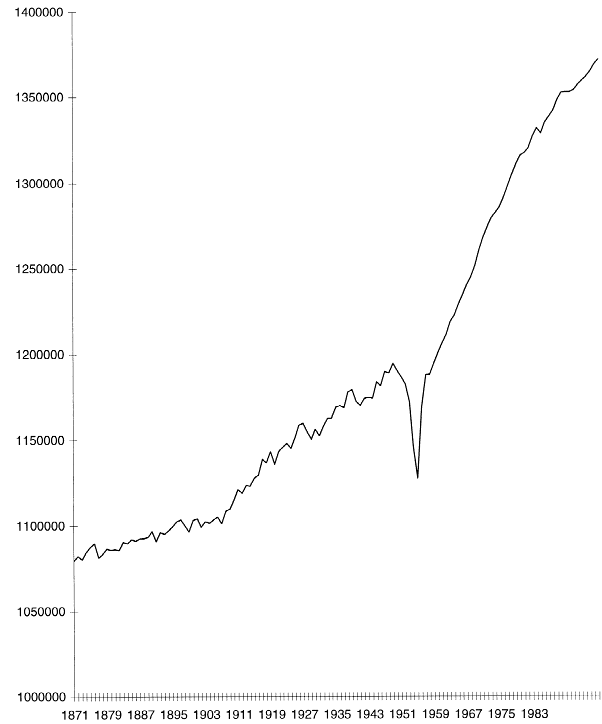

Figure 13
Real output in levels: Italy.

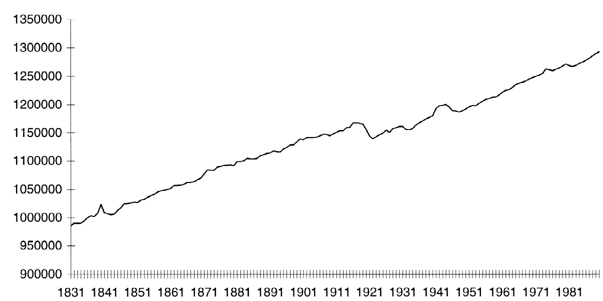

Figure 14
Real output in levels: United Kingdom.

Table 3
Volatility of output growth.

Country	σ_1 (Pre-WWII)	σ_2 (Post-WWII)	Distribution	p-Value
U.K.	2.682	1.895	$F(102, 40)$	0.73
Italy	3.996	2.295	$F(68, 35)$	0.03
France	4.33	1.812	$F(18, 38)$	0
Germany	4.374	2.253	$F(67, 34)$	0
Denmark	2.963	2.667	$F(54, 39)$	24.74
Sweden	3.95	1.917	$F(67, 41)$	0
Norway	2.375	1.67	$F(66, 40)$	0.91

Table 4
Goldfeld and Quandt tests.

Country	Distribution	p-Value, Slope and Intercept	p-Value, Slope	p-Value, Intercept
U.K.	$F(2, 148)$	54.62		
Italy	$F(5, 112)$	1.66	70.99	2.36
France	$F(4, 61)$	13.46		
Germany	$F(3, 101)$	48.87		
Denmark	$F(3, 100)$	10.51		
Sweden	$F(2, 116)$	51.84		
Norway	$F(2, 107)$	0.03	1.41	76.89

The rejection of no ARCH may signify nonconstant conditional variance, as well as a nonlinear conditional mean; therefore, there is a possibility that what looks like a structural change is due to nonlinearity, which can be modeled with a constant-parameter nonlinear model.

3 Nonlinearity of the Series

In this section, we assume that if the series are nonlinear they can be characterized by a smooth-transition autoregressive (STAR) model (Tong 1990; Granger and Terasvirta 1993).[1]

A STAR model is an AR model whose local dynamics depend on lagged values of the series—i.e., the behavior of the model during a recession can be different from the behavior during an expansion—as well as the distance from the transition values. The volatility change can therefore be attributed to a change in regime with constant parameters, rather than to parameter instability (which is not supported by the tests reviewed in Section 2).

The STAR model is defined as:

$$y_t = \pi' w_t + (\theta w_t) F(y_{t-1}) + u_t \tag{1}$$

where

$$\begin{aligned} w_t &= (1, y_{t-1}, \ldots, y_{t-p}), \\ \pi &= (\pi_0, \pi_1, \ldots, \pi_p), \\ \theta &= (\theta_0, \theta_1, \ldots, \theta_p), \text{ and} \\ u_t &\ nid(0, \sigma^2). \end{aligned}$$

The transition function defines the type of the STAR model; in the case of logistic STAR (LSTAR), it is:

$$F(y_{t-d}) = 1 + \exp[-\gamma(y_{t-d} - c)]^{-1}, \tag{2}$$

where $\gamma > 0$. For an exponential STAR (ESTAR) model, the transition function is:

$$F(y_{t-d}) = 1 - \exp[-\gamma(y_{t-d} - c)]^2. \tag{3}$$

Equations (2) and (3) represent nonlinear autoregressive models whose local dynamics change with y_{t-d}. In particular, in an LSTAR model, local dynamics in the case of a recession (low values of y_{t-d}) may differ from

[1]We choose the STAR models since they are widely used in the nonlinear empirical literature: see, e.g., Granger and Terasvirta (1993) and Semmler (1994). Moreover, they are quite simple to identify, and their dynamic properties are very rich.

an expansion, while in an ESTAR model, the dynamic behavior differs depending on (1) how close the system is to c, and (2) the value of c itself. If the transitions are generated by deviations of the transition variable from its linear trend rather than from a fixed value c, the model takes the form of a STAR-deviation (STAR-D) model. One possibility is to use lagged fitted residuals from the linear part of Equation 1 as the transition variable,

$$y_t = y_{t-d} - \pi w_{t-d}. \tag{4}$$

Then, depending upon the form of the transition function, one could have a logistic or exponential STAR-D model.

To identify a STAR model, after having identified a linear AR model, we will test it against a STAR model; if linearity is rejected, we will determine the unknown delay parameter, d. The following steps consist of choosing between logistic and exponential STAR models, and specifying the lag structure of the selected model.

We use the AR models of Section 2 as a basis for linearity testing. To test STAR against linearity, we follow Terasvirta (1994) using the auxiliary regression:

$$y_t = \beta_1 w_t + \beta_2 w_t y_{t-d} + \beta_3 w_t y_{t-d}^2 + \beta_4 w_t y_{t-d}^3 + v_t, \tag{5}$$

$$\begin{aligned} \beta_1 &= (\beta_{10}, \ldots, \beta_{1p}) \\ \beta_j &= (\beta_{j1}, \ldots, \beta_{jp}) \quad j = 2, 3, 4 \end{aligned}$$

where

$$Ev_t = 0 \quad \mathrm{var}(v_t) = \sigma_v^2 \quad \mathrm{cov}(v_t, v_s) = 0 \quad s \neq t.$$

The linearity hypothesis is $H_0 : \beta_2 = \beta_3 = \beta_4 = 0$. When H_0 holds, under stationarity and assuming the existence of the eighth moments, the χ^2 statistic has an asymptotic distribution $\chi^2(3p)$. The choice between ESTAR and LSTAR is made using Equation 4: first test H_{01}: $\beta_4 = 0$, then pass to H_{02}: $\beta_3 = 0 \mid \beta_4 = 0$, and H_{03}: $\beta_2 = 0 \mid \beta_3 = \beta_4 = 0$. The family model will be chosen on the p-value of the tests in the sequence: if the p-value of the test H_{02} is the smallest of the three, we will select ESTAR; otherwise we choose LSTAR.

The results of the linearity tests of the first differences of the logs of output based on the models of Section 2 appear in Table 5. Linearity is strongly rejected, and ESTAR models are the proper choice for every country but Denmark and Norway, whose DGPs are best described by an LSTAR model.

The parameters of the models are estimated using nonlinear least squares. Their values are shown in Table 6.

The residual variance of the STAR models is, on average, only 70% of the corresponding AR models, while R^2 has improved by almost 12%. There is no trace of ARCH in the residuals, which cannot be considered nonnormal. Table 7 shows the BDS test for remaining nonlinearities: it indicates that the STAR models provide an adequate description of the nonlinearity in the series. Moreover, the STAR model parameters are constant, since they pass the heteroskedasticity test at 1%, with the exception of Germany (see Table 8).

We evaluate the forecast performance, based on the mean absolute error, for the sample 1960–1989 of AR, STAR, and random-walk models in Sections 2 and 3.[2] We use the forecast analysis as a further test to discriminate between the AR and the STAR models. Table 9 presents the main results.

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[2]We performed this test to contend with the problem of overfitting the data. Table 10 reports the results of the one-step ahead forecast. Forecasts are obtained by estimating the model up to the t^{th} observation, and forecasting the $t^{\text{th}} + 1$ value. Such a procedure is repeated 30 times for each country. The statistical value we obtain is the mean absolute error (MAE),

$$\frac{1}{30} \sum_{t=1}^{30} |e_t|,$$

where e_t is the one-step ahead forecast error from 1960–1989.

Table 5
Linearity tests.

Country	Hypothesis	Distribution	p-Value $d = 1$	p-Value $d = 2$	p-Value $d = 3$	p-Value $d = 4$	p-Value $d = 5$
U.K.	H_0	$F(6, 103)$	4.85	75.6	26.9	26.63	0.96
	H_{01}	$F(2, 147)$	1.65				34.71
	H_{02}	$F(2, 145)$	17.88				0.14
	H_{03}	$F(2, 143)$	57.10				44.17
Italy	H_0	$F(15, 101)$	1.19	5.64	12.13	64.98	42.3
	H_{01}	$F(5, 111)$	27.41				
	H_{02}	$F(5, 106)$	2.22				
	H_{03}	$F(5, 101)$	34.71				
France	H_0	$F(12, 51)$	5.39	28.34	0.15	8.75	1.13
	H_{01}	$F(4, 59)$			1.42		2.97
	H_{02}	$F(4, 55)$			0.20		5.68
	H_{03}	$F(4, 51)$			50.83		17.48
Germany	H_0	$F(9, 92)$	0.08	24.02	19.68	60.53	15.26
	H_{01}	$F(3, 98)$	14.68				
	H_{02}	$F(3, 95)$	0.18				
	H_{03}	$F(3, 92)$	4.41				
Denmark	H_0	$F(9, 92)$	98.54	1.46	53.45	2.61	8.61
	H_{01}	$F(3, 98)$		5.10		15.37	
	H_{02}	$F(3, 95)$		6.63		0.97	
	H_{03}	$F(3, 92)$		12.63		50.52	
Sweden	H_0	$F(3, 116)$	8.22	44.94	52.58	4.29	86.55
	H_{01}	$F(1, 118)$				66.43	
	H_{02}	$F(1, 117)$				0.55	
	H_{03}	$F(1, 116)$				61.31	
Norway	H_0	$F(6, 101)$	0.02	0.08	0.12	2.47	0.92
	H_{01}	$F(2, 105)$	0	0	0.81	2.07	10.18
	H_{02}	$F(2, 103)$	9.7	73.56	0.44	36.15	55.17
	H_{03}	$F(2, 101)$	75.07	31.62	42.74	9.29	0.38

With the possible exception of the U.K., the nonlinear models perform better than the linear ones on average by 30%.

4 Results and Dynamic Behavior

In this section, we first discuss the interpretation of the ESTAR models and then that of the LSTAR models, emphasizing the dynamic behavior of the systems.

Notice that since the estimates of the location parameters, c, of the ESTAR models are negative and different from zero (with the exception of Italy), local dynamics are asymmetric about zero with expansions longer than contractions. The dynamics of the ESTAR model for the U.K., Germany, and Sweden, and of the LSTAR model for Norway, are locally stationary everywhere. Since these models have a regime switching in c, different shocks will have different effects: a small shock in the neighborhood of c may modify the steady-state growth rate, but shocks of the same size may have no effect on it if the system is far from the threshold. On the other side, a large shock, or a sequence of small shocks with the same sign, may modify the growth rate. The system behaves differently depending on the size of the shocks and/or its past realization: i.e., there is path dependency. On the policy ground, these results suggest that policy has to be case-oriented, since the behavior of the system is affected by the size and the timing of the shock and is therefore dependent on history (Gallegati 1993).

Italy and Denmark are two peculiar cases. As Denmark is concerned, the roots of the mid-regime are stationary; on the other hand, the outer regime contains an explosive root with a period of five years. Because this root induces a cycle, the process tends to return toward the mid-regime where it is locally stationary after first swinging out. Therefore, if the shocks to the economy remain small, the fluctuations in output are very

Table 6
Nonlinear models.

U.K.	$y_t = \underset{(5.54)}{0.02974} - \underset{(-0.97)}{0.14256} y_{t-1} + \underset{(5.29)}{0.12806}\, d_{1839} - \underset{(-7.43)}{0.20328}\, d_{1840} - \underset{(-5.53)}{0.10193}\, d_{1919-20}$ $+ \underset{(4.22)}{0.1016}\, d_{1940} - \underset{(-3.73)}{0.06549}\, d_{1944-45} - \underset{(-2.18)}{0.05282}\, d_{1980}$ $+ \left(\underset{(2.68)}{0.47705} y_{t-1} - \underset{(-2.85)}{0.01834} \right) \left[1 - e^{\underset{(1.31)}{-3319.26} \left(y_{t-5} \underset{(-6.23)}{-0.02155} \right)^2} \right]$ $+ \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.023966,\ R^2 = 0.6693,\ T = 158.$
Italy	$y_t = \underset{(1.82)}{0.02744} - \underset{(-2.20)}{4.16519} y_{t-1} - \underset{(-1.31)}{0.43743} y_{t-2} + \underset{(1.86)}{0.31629} y_{t-3} + \underset{(0.26)}{0.05994} y_{t-4}$ $- \underset{(-7.74)}{0.21098}\, d_{1943-45} + \underset{(3.73)}{0.39443}\, d_{1946}$ $+ \left(\underset{(2.09)}{3.95714} y_{t-1} + \underset{(1.91)}{0.67385} y_{t-2} - \underset{(-1.09)}{0.21602} y_{t-3} + \underset{(0.51)}{0.12510} y_{t-4} - \underset{(-0.55)}{0.00917} \right) \left[1 - e^{\underset{(1.93)}{-4064.42} \left(y_{t-1} \underset{(-0.04)}{+0.00018} \right)^2} \right]$ $+ \varepsilon_t$ $\hat{\sigma}_\varepsilon = 0.03844,\ R^2 = 0.7171,\ T = 124.$
France	$y_t = \underset{(3.30)}{0.02823} + \underset{(4.98)}{1.03759} y_{t-1} - \underset{(-2.51)}{0.54493} y_{t-2}$ $- \underset{(-11.75)}{0.26434}\, d_{1940-41} - \underset{(-4.03)}{0.12713}\, d_{1944} + \underset{(3.69)}{0.09708}\, d_{1945} + \underset{(14.54)}{0.48413}\, d_{1946} - \underset{(-2.18)}{0.05282}\, d_{1980}$ $+ \left(\underset{(-4.66)}{-0.53924} y_{t-1} + \underset{(4.16)}{0.98959} y_{t-2} - \underset{(-1.75)}{0.01904} \right) \left[1 - e^{\underset{(2.59)}{-1130.68} \left(y_{t-3} \underset{(6.23)}{+0.0532} \right)^2} \right] + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.02739,\ R^2 = 0.9022,\ T = 73.$
Germany	$y_t = \underset{(1.83)}{0.01755} + \underset{(2.29)}{0.59247} y_{t-1} - \underset{(-0.44)}{0.07575} y_{t-2} - \underset{(-2.47)}{0.09948}\, d_{1931} + \underset{(3.69)}{0.15049}\, d_{1933}$ $+ \left(\underset{(-1.60)}{-0.53924} y_{t-1} + \underset{(2.25)}{0.62073} y_{t-2} - \underset{(-0.52)}{0.00982} \right) \left[1 - e^{\underset{(1.11)}{-447.83} \hat{\varepsilon}^2_{t-1}} \right] + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03711,\ R^2 = 0.5590,\ T = 109.$
Denmark	$y_t = \underset{(2.29)}{0.03612} - \underset{(-1.00)}{0.31486} y_{t-1} - \underset{(0.35)}{0.6996} y_{t-2} - \underset{(-9.13)}{0.27974}\, d_{1940} + \underset{(7.01)}{0.21516}\, d_{1946}$ $+ \left(\underset{(1.61)}{0.51782} y_{t-1} - \underset{(-1.47)}{0.32606} y_{t-2} - \underset{(-0.28)}{0.00477} \right) \left[1 - e^{\underset{(0.07)}{-2098.65} \left(y_{t-2} + \underset{(2.14)}{0.0528} \right)} \right]^{-1} + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03019,\ R^2 = 0.7105,\ T = 108.$
Sweden	$y_t = \underset{(5.31)}{0.02605} + \underset{(0.12)}{0.11087} y_{t-1} + \underset{(4.56)}{0.15466}\, d_{1870} - \underset{(-2.97)}{0.09961}\, d_{1875} + \underset{(2.68)}{0.09167}\, d_{1896}$ $+ \underset{(3.26)}{0.11022}\, d_{1913} - \underset{(-4.31)}{0.11799}\, d_{1917-18} - \underset{(-4.31)}{0.15568}\, d_{1921-22} - \underset{(-3.86)}{0.12919}\, d_{1931}$ $+ \left(\underset{(-0.41)}{-1.14848} y_{t-1} + \underset{(0.46)}{0.15146} \right) \left[1 - e^{\underset{(2.31)}{-38.24} \left(y_4 + \underset{(1.98)}{0.02131} \right)^2} \right] + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.03324,\ R^2 = 0.6734,\ T = 128.$
Norway	$y_t = \underset{(3.87)}{0.02174} + \underset{(0.10)}{0.01053} y_{t-1} - \underset{(-6.51)}{0.10192}\, d_{1917-18} + \underset{(3.70)}{0.0858}\, d_{1919}$ $- \underset{(-6.35)}{0.13607}\, d_{1921} + \underset{(26.29)}{0.57580}\, d_{1930}$ $+ \left(\underset{(-2.04)}{-0.42626} y_{t-1} + \underset{(3.18)}{0.03908} \right) \left[1 + e^{\underset{(1.22)}{-124.68} \hat{\varepsilon}^2_{t-1}} \right]^1 + \hat{\varepsilon}_t$ $\hat{\sigma}_\varepsilon = 0.01944,\ R^2 = 0.9308,\ T = 115.$

small as well. Since the mid-regime is stationary, the cycle dies out when the output growth returns to mid-regime again.

Self-sustained oscillations have been found for Italy. The Italian ESTAR model contains unstable roots for the inner regime and stable roots for the outer one. If the system explodes leaving the inner regime, when it goes to the other one, it is pushed back because the roots are stable, and the cycle can start again in a chaotic way.

The above results demonstrate that the annual European GDP can adequately be described by means of a nonlinear model with constant parameters.

Table 7
BDS tests.

Country[b]		m = 42	m = 43	m = 4
U.K.	$j = 6$	1.6335	1.1180	0.8398
	$j = 8$	2.1339	1.7018	1.2028
Italy	$j = 6$	0.5449	0.8475	0.8733
	$j = 8$	0.4012	0.4945	0.4905
France	$j = 6$	0.0393	1.051	1.557
	$j = 8$	0.3411	1.4913	1.611
Germany	$j = 6$	2.2485	1.867	1.6433
	$j = 8$	2.0529	1.4446	1.3434
Denmark	$j = 6$	−1.7412	−1.3292	−1.0274
	$j = 8$	−1.5972	−1.3172	−1.0449
Sweden	$j - 6$	0.2794	0.0054	0.0822
	$j = 8$	0.5077	−0.4898	−1.2087
Norway	$j = 6$	−1.9517	−0.83501	−0.23196
	$j = 8$	−1.8824	−0.7727	−0.5629

b. Critical value at 5% = 1.96.

Table 8
Volatility of output growth.

Country	σ_1 (Pre-WWII)	σ_2 (Post-WWII)	Distribution	p-Value
U.K.	2.793	2.153	$F(91, 31)$	5.09
Italy	3.827	2.625	$F(59, 26)$	1.86
Germany	4.374	2.253	$F(62, 27)$	0.06
Denmark	3.007	3.17	$F(47, 33)$	37.96
Sweden	3.825	2.699	$F(60, 30)$	1.98
Norway	2.206	1.867	$F(62, 32)$	30.81

Table 9
Forecast analysis.

Country	Nonlinear Model	Random Walk	Linear Model
U.K.	1.70e-02	1.91e-02	1.36e-02
Italy	2.41e-02	1.97e-02	2.50e-02
France	1.25e-02	1.28e-02	1.33e-02
Germany	1.73e-02	2.07e-02	1.83e-02
Denmark	1.91e-02	2.26e-02	2.01e-02
Sweden	1.35e-02	1.55e-02	1.47e-02
Norway	9.14e-03	1.75e-02	1.75e-02

5 Conclusions

In modeling the logarithmic first differences of the real income of some European countries, we have assumed a unit root in the levels series since the linearity test forming the core of the specifications technique of STAR models is not available if the series is trending.

The results suggest that the effect of random shocks on output is asymmetric and nonlinear, and the linear AR representation is not adequate. Since the detrended series are nonlinear as well, two caveats are in order. First, the linear equations used for testing the unit-root hypothesis against trend stationarity may be misspecified. Second, the permanent versus transitory nature of the shocks loses its meaning: it is the size and the timing that make the difference.

The univariate nature of the STAR models makes them purely descriptive; nevertheless, the implications for econometric and economy modeling are very strong since linearity, rather than being an approximation of the real world, may generate deep misunderstanding and a bad guide for economic policy.

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